MEAN STRESS VALUE IN SPECTRAL METHOD FOR THE DETERMINATION OF FATIGUE LIFE

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Abstract: The paper presents a proposal of account of mean stress value in the process of the determination of the fatigue life, using the spectral method. The existing approaches have been described and some chosen stress models used to take into account the influence of the mean value in the process of the determination of fatigue life have been introduced. Those models, referring to their linear character, have been used to determine the power spectral density function (PSD) of the transformed stress taking into account the mean value. The method introduced by the authors allows a wide usage of many formulas used to predict the fatigue life by means of the spectral method.

Key words: Mean Stress, Fatigue Life, Spectral Method

1. INTRODUCTION

Structures and machine components being subjected to variable loads require constant monitoring during operation due to the emerging phenomenon of material fatigue. Also, when designing new constructions or modification of nodes of machine elements, it is required to check their load capacity and fatigue life before finally being put into operation. The verifications of this type are performed in laboratories carrying out the strength of materials fatigue tests, or if it is not possible because of e.g. the size and cost, then calculations are made only with a view to the best possible estimate of fatigue life. The way of calculations depends of the character of the load. In the case of load-amplitude with no mean value, the expected number of cycles to fatigue crack initiation can be read directly from graphs of fatigue, for example, Wöhler curve. If there are evident mean values in the course, then their effect must be taken into account on fatigue. For this purpose you can use the charts to take account of the influence of the mean load, for example, Smith chart or Wöhler graphs drawn up for various cycle asymmetry coefficients $R = \sigma_{min}/\sigma_{max}$.

2. MEAN VALUE OF RANDOM LOADING

The assignation of fatigue under variable amplitude or random load is generally done in the time domain using an algorithm determining the course of the cycles of variable amplitude, using a chosen model to circumscribe the impact of the mean load and the hypothesis of summation of fatigue damage. The papers by Łagoda et al., (1998, 2001) present the results of fatigue tests under uniaxial random tension-compression with the mean value of samples made of steel 10HNAP. They proposed an algorithm for calculating the fatigue life using the rain flow cycle counting method and the hypothesis of summation of fatigue damage by Palmgren-Miner. The authors of this work have analyzed three paths to take into account the influence of the mean value, that are:

- I – not taking into calculations the mean value,
- II – taking into calculations the influence of the mean value by transforming each of the cycle amplitude on the basis of their local mean value determined while cycle counting,
- III – taking into calculations the influence of the mean value by transforming the whole load course on the basis of its global mean value.

Fig. 1 shows an diagram of the algorithm of the calculation of fatigue life taking into calculations the mean stress value. In this work the K coefficient has been introduced, which allows you to calculate the transformed amplitude according to the method II:

$$\sigma_{aTi} = \sigma_{ai} \cdot K_i(\sigma_{mi}), \tag{1}$$

for the i-th cycle emphasised by the rain flow algorithm from a random course with amplitude σ_{ai} and the mean value σ_{mi}. Method III is based on the principle of the transformation of the entire random stress course using the global mean value:

$$\sigma_T(t) = [\sigma(t) - \sigma_m] \cdot K(\sigma_m). \tag{2}$$

Amplitude of the transformed cycle σ_{aTi} for this case is obtained directly by counting cycles of the course $\sigma_T(t)$ using the cycle counting algorithm. Summation of fatigue damage is done according to the formula:

$$D = \sum_{i=1}^{n} \frac{n_i}{N(\sigma_{aTi})} \tag{3}$$

where: D – fatigue damage parameter, n_i – the number of cycles of amplitude σ_{aTi}, $N(\sigma_{aTi})$ – the number of cycles determined from the Wöhler diagram for the transformed amplitude σ_{aTi}. Fatigue life N_{cal} expressed in cycles is determined from the formula:

$$N_{cal} = \frac{N_{blok}}{D} \tag{4}$$

where N_{blok} is the number of distinguished cycles of the analyzed section of the stress course.
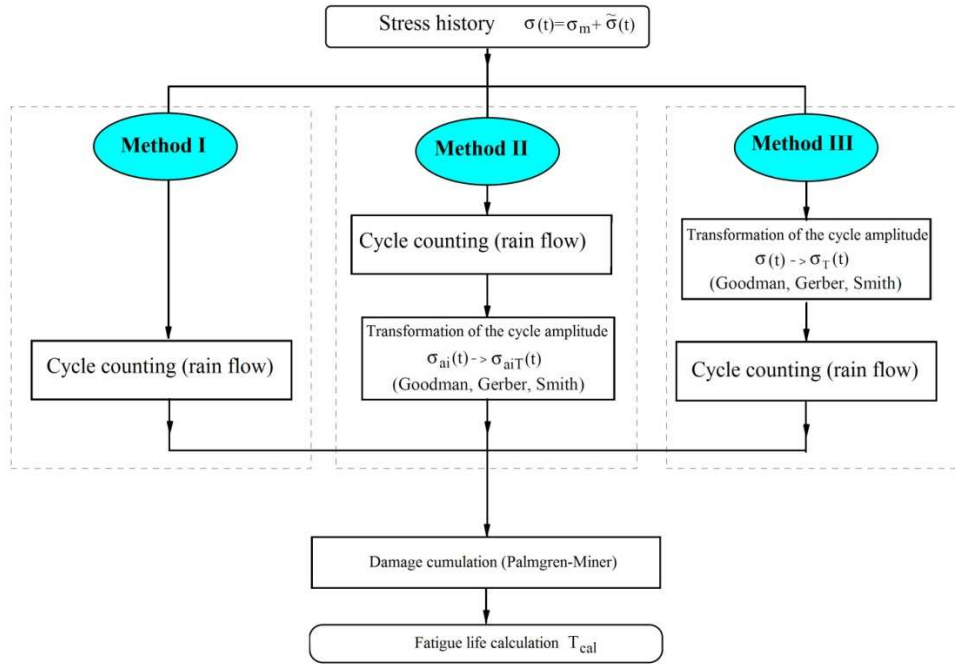
Fig. 1. Three paths to calculate the fatigue life T_{cal} (Łagoda et al., 2001)

The study showed that the course of a stationary random and symmetrical distribution of values of the instantaneous probability methods II and III are equivalent and can be used interchangeably in the calculations. In special cases, the K coefficient is determined from the formulas derived on the basis of the adopted model to take account of the mean stress. In literature you will find a significant number of models of this type (Łagoda et al., 2001; Pawliczek, 2000; Böhm, 2010) for which the K coefficient takes the form:

$$K_S = \frac{1}{1 - \dfrac{\sigma_m}{R_e}} \tag{5}$$

$$K_{Go} = \frac{1}{1 - \dfrac{\sigma_m}{R_m}} \tag{6}$$

$$K_M = \frac{1}{1 - \dfrac{\sigma_m}{\sigma'_f}} \tag{7}$$

$$K_{Ge} = \frac{1}{1 - \left(\dfrac{\sigma_m}{R_m}\right)^2} \tag{8}$$

$$K_K = \frac{1}{\exp\left(-\alpha \cdot \dfrac{\sigma_m}{R_m}\right)} \tag{9}$$

where: K_s, K_{Go}, K_M, K_{Ge}, K_K – coefficients determined on the basis of appropriate models of Soderberg, Goodman, Morrow, Gerber and Kwofie, σ_m – mean cycle value of the stress course, R_e – plasticity limit, R_m – tensile strength, σ'_f – fatigue strength coefficient, α – mean stress sensitivity of the material.

Fatigue life can be assigned also in the frequency domain using a stochastic analysis of random processes, the so-called spectral method. Taking into account the mean stress in this case is hard because the stress in the course of this method is represented by a power spectral density function, which contains information about the occurring locally and globally mean value in a way that is difficult to use in practice. In literature, however, we can find only a few suggestions on this issue. Kihl and Sarkani (1999) show the effect of the mean value on fatigue life of welded steel joints. The tests were set to be run under both cyclic and random loadings with non-zero and zero mean stress value. The authors derived a formula to find the expected number of cycles to initiation of fatigue cracks in the case of random loads with extremes of Rayleigh distribution with a nonzero mean value of stress:

$$N_{cal} = \left(1 - \frac{\sigma_m}{R_m}\right)^{-B} \frac{2^{\frac{B}{A}} \sigma_x^B A}{\Gamma\left(1 - \dfrac{B}{2}\right)} \tag{10}$$

where: N_{cal} – number of cycles to fatigue crack initiation, A and B – life axis and slope of the constant amplitude Wöhler curve, σ_x – is the RMS stress of the narrowband random loading, $\Gamma(\cdot)$ – is the gamma function, σ_m – global mean stress value of the random stress history, R_m – tensile strength. It is easy to notice that in the formula (10), the part being responsible for taking into account the mean value is $(1 - \sigma_m/R_m)^{-B}$, which modifies the cycle number till the initiation of the fatigue crack determined by the Miles formula (Niesłony and Macha, 2007).

3. PSD OF A RANDOM FUNCTION WITH THE MEAN VALUE

Let us analyse an example of one-dimensional stationary random process $x(t)$ showing the characteristics of ergodicity. The assumption that $x(t)$ represents the physical signal is often convenient to present as the sum of the static x_m and dynamic $x_d(t)$ or fluctuant component:

$$x(t) = x_m + x_d(t). \tag{11}$$

Static component can be described by the mean value given by the formula:

$$x_m = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)dx, \tag{12}$$

and the dynamic component by the signals variance:

$$\mu_x = \lim_{T \to \infty} \frac{1}{T} \int_0^T [x(t) - x_m]^2 dt. \tag{13}$$

The variance, however, does not describe the spectral structure of a random process, and this information is essential for the proper estimation of the number of cycles and the amplitude distribution of the load during the fatigue calculations. Therefore for this purpose the power spectral density (PSD) function is being used. Power spectral density of the signal describes the overall structure of a random process using the frequency spectral density of mean values of the physical signal in question. This value can be determined for the interval from f to $f + \Delta f$ using a central-pass filter with a narrow band and averaging the square on the output of the filter (Bendat and Piersol, 1976):

$$\Psi_x(f, \Delta f) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x^2(t, f, \Delta f)dt, \tag{14}$$

where: Ψ_x – the mean square value of the process $x(t)$, T – time of observation, $x(t, f, \Delta f)$ – component of $x(t)$ in the frequency range from f to $f + \Delta f$. For small values of Δf the formula (14) shows the one-sided PSD function.

$$G_x(f) = \lim_{\Delta f \to 0} \frac{\Psi_x(f, \Delta f)}{\Delta f} = \lim_{\Delta f \to 0} \frac{1}{\Delta f} \left[\lim_{T \to \infty} \frac{1}{T} \int_0^T x^2(t, f, \Delta f)dt \right]. \tag{15}$$

A characteristic property of the $G_x(f)$ function is the relation to the autocorrelation function. In particular, for stationary signals, these functions are closely related by the Fourier transformation:

$$G_x(f) = 2 \int_{-\infty}^{\infty} R_x(\tau)e^{-j2\pi f\tau} d\tau, \tag{16}$$

where:

$$R_x(\tau) = \lim_{\tau \to \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau)dt, \tag{17}$$

is the autocorrelation function of the signal $x(t)$. Mean value x_m of the random process can be determined from the autocorrelation function:

$$x_m = \sqrt{R_x(\infty)}, \tag{18}$$

and the mean value of $x(t)$ is a function of the PSD presented as a Dirac function at zero frequency

$$x_m = \sqrt{\int_{0^-}^{0^+} \delta(0)G_x(f)df}. \tag{19}$$

The formula (19) shows that the mean value is equal to the positive square root of the "surface" underlying the Dirac function.

This is an abstract space, as Dirac function takes the value $+\infty$ for an infinite small interval. For this reason, the direct use of formula (19) to determine the mean value on the basis of a PSD function of a random function is virtually impossible. Numerical algorithms to estimate the PSD functions are limited to the basic frequency resolution and the value of the function $R_x(0)$ results from the mean value $x(t)$ and from the mean square value of a random process from the interval (0, Δf). Proper separation of these two values is impossible without additional information such as of the static value. Therefore, in practice, we analyze those two values separately, the dynamic and static component of the random process according to equation (11).

4. PSD FUNCTION OF A TRANSFORMED STRESS COURSE

The crossing of the signal $x(t)$ by a linear system with constant parameters determined by the impulse response $h(\tau)$ and the transfer function $H(f)$ describes the following relationships (Bendat and Piersol, 1976, 1980; Kirsten, 2002):

$$y(t) = \int_0^{\infty} h(\tau)x(t-\tau)d\tau, \tag{20}$$

$$G_y(f) = |H(f)|^2 G_x(f), \tag{21}$$

where: $y(t)$ – output signal of the system, $G_x(f)$ and $G_y(f)$ – respectively PSD input and output. From the equation (21) we can notice that the power spectral density of the output signal can be calculated knowing the gain factor $|H(f)|$ of the system. Fig. 2(a) shows schematically the signal pass through a linear system. The spectral method of determining the fatigue life using the PSD function is used to describe the course of stress directly in the frequency domain. If the stress course includes a static and a fluctuant component then the transformed course should be designated according to equation (2). Treating the fluctuant component of the course $[\sigma(t) - \sigma_m]$ as an input signal of an linear system with constant coefficient of strengthening $|H(f)| = K(\sigma_m)$ we can determine the PSD of a transformed strain course:

$$G_{\sigma T}(f) = K(\sigma_m)^2 G_{\sigma}(f), \tag{22}$$

where $G_\sigma(f)$ are the power spectral density of a centered stress course. Fig. 2(b) presents the interpretation of the linear process of strain transformation due to the mean value, which can be compared to the transition signal by a linear system, Fig. 2(a). Formula (22) allows the use of different forms of $K(\sigma_m)$-factor, for example, described by equations (5)-(9), in the process of determining the fatigue life by means of spectral method taking into account the static stress component. The main advantage of the proposed solution is that the transformation is subjected to power spectral density function before using the models to determine the fatigue life. This gives the possibility of applying fatigue formulas in the spectral method for the waveforms developed for narrow-band frequency and the more universal solutions correctly describing most of the random waveforms used in the calculation of fatigue (Niesłony and Macha, 2007; Niesłony, 2003, 2008).

a)

b)
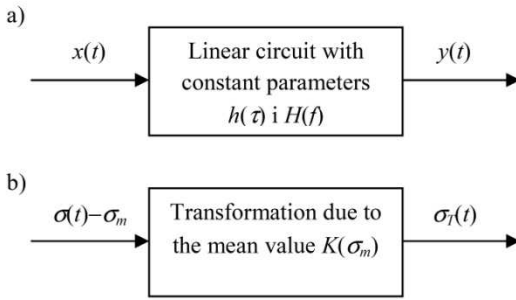
Fig. 2. Single entry linear system (a) and interpretation of the linear process of transformation strain due to the average value (b)

Such a method is proposed by Dirlik (1985) which is developed by using the empirical formula describing the probability distribution of amplitudes ranges.

$$p(\Delta\sigma) = \frac{1}{2\sqrt{m_0}}\left[\frac{K_1}{K_4}e^{\frac{-Z}{K_4}} + \frac{K_2 Z}{R^2}e^{\frac{-Z^2}{2R^2}} + K_3 Z e^{\frac{-Z^2}{2}}\right], \quad (23)$$

where: Z, K_1, K_2, K_3, K_4, R – factors which are functions of the first five moments m_k ($k = 0, …, 4$) of the transformed PSD stress function:

$$m_k = \int_0^\infty G_{\sigma T}(f)f^k df . \quad (24)$$

Fatigue life is calculated using the selected hypotheses of fatigue damage accumulation, e.g. for a linear Palmgren-Miner hypothesis with the amplitude below the fatigue limit we obtain:

$$N_{cal} = \frac{1}{\int_0^\infty \frac{p(\Delta\sigma)}{N(\Delta\sigma)}d\Delta\sigma} , \quad (25)$$

where the number of cycles for a range of amplitudes is calculated on the basis of the characteristics of the material fatigue:

$$N(\Delta\sigma) = \sigma_{af}^m N_0\left(\frac{\Delta\sigma}{2}\right)^{-m} . \quad (26)$$

5. CALCULATION ALGORITHM

In order to calculate the fatigue life using the spectral method and taking into account the impact of the mean stress on fatigue life you should follow these steps:
a) Assigne or define the PSD of the fluctuant component of the stress course $G_\sigma(f)$ and its static part σ_m,
b) Calculate the coefficient $K(\sigma_m)$ according to the right model, formulas (5)-(9), and the choice of model depends of the mean stress value sensitivity of material,
c) Calculate PSD of then transformed stress $G_{\sigma T}(f)$ according to the equation (22),
d) Calculate the fatigue life using spectral method formulas i.e. (23) and (25) (Niesłony A. and Macha E., 2007).

6. CONCLUSIONS AND OBSERVATIONS

Based on the literature research it can be noticed, that there are no papers that would propose the transformation of the power spectral density function of the stress, to take into account the influence of the mean value on the fatigue life. The proposed formula (21) allows the calculation of the PSD of the transformed stress, using models that are well known and widely verified in experimental researches. The proposal of Kihl and Sarkani (1999) uses a Rayleigh amplitude distribution approximation, which reduces the area of application of the formula (9) only to narrowband processes. The method introduced by the authors doesn't have this limitation and therefore allows a wide usage of many formulas used to predict the fatigue life by means of the spectral method. Compared with the time domain fatigue life prediction methods, the spectral method shows greater efficiency and it can be used there, where a multiplicand fatigue calculation is required (constructions optimization, fatigue damage maps etc.). The experimental verification should be performed to verify the correctness of the fatigue calculations evaluated according to the proposed method, nevertheless the transformation of the PSD function in the spectral method is equivalent to the formula (2) in the time domain.

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