# **CONVECTIVE COOLING OF A DISC BRAKE DURING SINGLE BRAKING**

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**Abstract:** This study is primarily focused on the investigation of an impact of heat transfer coefficient on heat dissipation from the solid disc using the finite element method (FEM). The analysis was carried out within four individual cases of braking of the passenger vehicle simulating mountain descent with different velocities and the following release periods, where the convective terms of cooling were dependent on the angular velocity of the disc. For the purpose of confronting contrasting conditions of the braking action, apart from heating region, the whole surface of the disc was insulated. The process was performed under the operation conditions of a real front disc brake, whose dimensions and thermophysical properties of materials were adopted and applied to the FE model. It was concluded that the influence of cooling of the exposed surfaces of the disc during relatively short braking is insignificant. However the period after brake release results in considerable decrease in temperature of the disc.

**Key words:** Braking, Pad/disc System, Finite Element Method, Convective Cooling

### **1. INTRODUCTION**

While considering the mechanical problem of a braking system, the force of friction is the most advisable quantity during the action. The magnitude of total force that opposes motion is influenced by the coefficient of friction, the surface area of a disc remaining in contact with the pad during slipping process and the contact pressure. However, the large amount of heat generated at the pad/disc interface undeniably participates in restriction of the operation range of a brake system. The conditions of friction, in which the critical value of temperature is exceeded, may cause a brake fade, premature wear, thermal cracks, brake fluid vaporization, thermal judder (Gao and Lin, 2002; Gao et al., 2007; Talati and Jalalifar, 2008, 2009). The variety of mechanical, physical and chemical phenomena in the immediate vicinity of contact surface causes difficulties in formulating frictional heating model which would include their sophisticated interactions (Scieszka, 1998).

For the contact temperature evaluation, the amount of kinetic energy is assumed to be entirely converted into thermal energy (Zagrodzki, 1985; Wawrzonek and Bialecki 2008; Grzes, 2009; Grzes, 2010; Adamowicz and Grzes, 2011a, 2011b). For this to happen, the total friction power expressed by the product of friction coefficient, relative linear velocity of the bodies and the contact pressure is applied as the intensity of thermal flux to the surfaces of contact of the friction pair. This approach was implemented by Zagrodzki (1985) to determine the temperature fields and thermal stresses of a multidisc wet clutch after engagement. Two-dimensional model of discs with distributed heat source was used. Thereby the uniform thermal load in the circumference after its separation, set a priori by the heat partition ratio, was applied to the model. In other cases special coefficient determining the energy generated during frictional heating was employed (Scieszka and Zolnierz, 2007a, 2007b).

Analytical models of heating process during braking were proposed in monograph by Awrejcewicz and Pyr'yev (2009). Bauzin and Laraqi (2004) carried out numerical simulation for a problem of sliding contact to calculate the heat flux generated due to friction, the thermal contact conductance and the intrinsic heat partition coefficient. Some analytical models to determine threedimensional temperature filed and the thermal constriction resistance for the problem of moving heat source on semi-infinite bodies were proposed in article by Laraqi et al. (2004).

The heat exchange during the braking process is accomplished by conduction through brake assembly and hub, convection to surrounding air and radiation to adjacent elements. Although conduction is an efficient mode of heat transfer, it may have negative effects, such as brake fluid vaporization or bearing damage (Talati and Jalalifar, 2008, 2009), whereas radiation affects the beading of a tyre.

The finite element analysis of a ventilated disc brake rotor with straight rounded radial vanes, which included macroscopic and microscopic model of frictional heat generation, was developed in article by Talati and Jalalifar (2008). The uniform pressure, as well as the uniform wear conditions, were studied separately for calculation of a boundary heat flux. Empirical correlations were used in order to obtain the heat transfer coefficient. The disc and pad temperature fields versus braking time in axial and radial directions were presented. The convective terms of cooling were concluded to be the most important factors preventing overheating of components of the brake assembly and causing the decrease in friction coefficient.

Adamowicz and Grzes (2011b) studied an influence of convective cooling on three-dimensional temperature fields in a solid disc brake generated during repetitive braking process. The calculations were carried out employing wide range of established a priori values of the heat transfer coefficient of the exposed surfaces of the disc.

The rotational motion of a disc during braking process in relation to stationary pads is a problem of a moving heat source. Nonaxisymmetric thermal load of a brake rotor, for the selected spot on the friction surface, causes periodic heating during pad transition and the following cooling phase for each rotation of a wheel

of a vehicle. In order to overcome the difficulties of geometrical scheme of the disc brake system, where the pad is treated as the heat source and is intrinsically a portion of rubbing path in the subsequent moments of time, three-dimensional representation is required. The problem of a moving heat source in relation to disc brake system was studied in a number of articles (Gao and Lin, 2002; Gao et al., 2007; Wawrzonek and Bialecki, 2008; Adamowicz and Grzes, 2011; Scieszka and Zolnierz 2007a, 2007b). The solution of iterative process of disc heating was developed with an assumption of exponential pressure increase and non-linear angular velocity descent, in order to simulate real conditions after the brake engagement, as proposed in study of Gao and Lin (2002). The review of FE modelling techniques of frictional heat generation in disc brakes and clutches was developed in article by Yevtushenko and Grzes (2010).

Belhocine and Bouchetara (2012) analysed three-dimensional temperature field of both solid and ventilated type of a brake disc incorporating three different cast iron materials. The heat transfer coefficient was calculated separately for specified locations on the free surfaces of the disc and implemented to FE element model.

This paper aims to study the influence of convective heat transfer on the transient temperature distributions of a real disc brake. The process of braking was conducted for the braking with constant velocity of the vehicle. The problem of pad transition based code as a moving heat source was developed and transferred into FE model. The determined temperature distributions of a disc during braking were confronted and compared for the cooling influence assessment.

### **2. HEAT TRANSFER PROBLEM**

The disc brake system consists of a rotating disc and fixed pads. When the braking process occurs, the forces that act on pads, oppose the motion of the system. The accumulated amount of kinetic energy is converted into thermal energy at the interface of the connecting parts according to the first law of thermodynamics. The generated heat is dissipated by forced convection due to enlarged airflow and the natural convection after the full stop, whereas conduction absorbs energy from the interface of the friction pair. However the third mode of heat transfer which takes place has an insignificant influence on the temperature distributions, and therefore has been neglected in this study.



**Fig. 1.** A schematic diagram of a pad/disc brake system

As for the symmetry of the problem, the analysed region was limited to the half of the entire disc of thickness  $\delta_d$  and one pad

(Fig. 1). However only disc temperature distributions were analysed. For the purpose of estimating the influence of convective heat exchange mode on the transient temperatures of a disc, two individual cases of braking process were investigated. The first is related to the problem of non-axisymmetric disc heating with the convective terms of cooling and velocity dependent heat transfer coefficient, whereas in the second, exclusively thermal load without convective boundary conditions was applied to the threedimensional FE model of a disc. It was assumed that the thermophysical properties of materials are isotropic and independent of temperature (Grzes, 2011b). Such an assumption stems from the fact of relatively low temperature attained in the process. The nominal surface of contact of a disc equals the real surface of contact thereby the contact pressure is uniform at every time *t* of braking process.

Assuming that the total friction energy is converted into heat, the total intensity of heat flux generated between the pad and the disc is given by  $q = fVp_0$ , where *f* is the friction coefficient, *V* is the sliding velocity,  $p_0$  is the contact pressure. The separation of heat entering the disc and the pad was adjusted by the heat partition ratio calculated as  $\gamma = 1/(1+\varepsilon)$  (Charron, 1943, Grzes 2011a), where  $\varepsilon = K_d k_p^{1/2}/(K_p k_d^{1/2})$  is the thermal activity coefficient (Luikov, 1968), *K* is the thermal conductivity, *k* is the thermal diffusivity, the subscripts *p* and *d* denote the pad and the disc, respectively.

The amount of the intensity of heat flux that enters the disc is calculated from the formula:

$$
q_d(r,\theta,t)_{z=0} = (1-\gamma)fp_0r\omega_0, r_p \le r \le R_p,
$$
  
\n
$$
0 \le \theta \le 2\pi, 0 \le t \le t_s,
$$
\n(1)

and into pad:

$$
q_p(r, \theta, t)|_{z=\delta_p} = \mathcal{Y}p_0 r \omega_0, r_p \le r \le R_p,
$$
  

$$
0 \le \theta \le \theta_0, 0 \le t \le t_s,
$$
 (2)

where  $\delta$  is the thickness,  $\theta_0$  is the cover angle of pad,  $\omega_0$  is the angular velocity, *t* is the time, *ts* is the braking time, *r* and *R* are the internal and external radii, respectively.

Both, the contact pressure  $p_0$  and angular velocity  $\omega_0$  are constant during the analysis.

# **3. MATHEMATICAL PROBLEM**

The transient heat conduction equation of the rotating disc for non-axisymmetric problem described in cylindrical coordinate system  $(r, \theta, z)$  is given as follows (Nowacki, 1962):

$$
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d} \left( \frac{\partial T}{\partial t} + \omega \frac{\partial T}{\partial \theta} \right),
$$
  
\n
$$
r_d \le r \le R_d, 0 \le \theta \le 2\pi, -\delta_d < z < 0, t > 0
$$
\n(3)

where *T* is the temperature, *r* is the radial coordinate, *z* is the axial coordinate.

Two separate cases were studied. The first simulated the process of real braking with constant velocity, therefore the following boundary conditions including convective mode of heat transfer were established:

− the surface placed in the disc's mid-plane of symmetry – adiabatic condition;

- the internal cylindrical surface, the contact surface free from frictional heating (out of pad contact area), and the external cylindrical surface of a disc – convective cooling terms;
- − the contact surface under relative transition of a pad portion of the total intensity of the heat flux calculated from the known quantity of heat rate, applied as the plane heat source acting with calculated value for the respective elements of the mesh (equation (1)).

$$
K_d \frac{\partial T}{\partial z}\Big|_{z=0} = q_d(r,\theta,t), r_p \le r \le R_p, 0 \le \theta \le 2\pi,
$$
\n
$$
0 \le t \le t_s,
$$
\n
$$
(4)
$$

$$
K_d \frac{\partial T}{\partial z}\Big|_{z=0} = h[T_a - T(r, \theta, t)], r_a \le r \le r_p,
$$
  
0 \le \theta \le 2\pi, z = 0, t \ge 0, (5)

$$
K_d \frac{\partial T}{\partial r}\Big|_{r=R_d} = h[T_a - T(\theta, z, t)], r = R_d,
$$
  

$$
0 \le \theta \le 2\pi, -\delta_d \le z \le 0, t \ge 0,
$$
 (6)

$$
K_d \frac{\partial T}{\partial r}\Big|_{r=r_d} = h[T(\theta, z, t) - T_a], r = r_d,
$$
  

$$
0 \le \theta \le 2\pi, -\delta_d \le z \le 0, t \ge 0,
$$
 (7)

$$
\left. \frac{\partial T}{\partial z} \right|_{z = -\delta_d} = 0, \quad (r, \theta, z), \quad r_d \le r \le R_d,
$$
\n
$$
0 \le \theta \le 2\pi, z = -\delta_d, \quad t \ge 0,
$$
\n(8)

$$
T(r, \theta, z, 0) = T_{\sigma}, r_d \le r \le R_d, 0 \le \theta \le 2\pi, -\delta_d \le z \le 0,
$$
 (9)

where *Ta* is the ambient temperature, *h* is the heat transfer coefficient.

For the second case, unlike the previous case, the thermal insulation was applied on every surface of convective heat exchange. However such an assumption does not correspond with the real conditions of braking process, in this study it is used primarily to confront contrasting terms and to answer whether the average constant heat transfer coefficient is sufficiently precise assumption for single braking.

In order to formulate the following matrix form of the equation (3) the Galerkin method was employed:

$$
\left[\mathbf{C}\right]\frac{\mathrm{d}\{\mathbf{T}\}}{\mathrm{d}t} + \left[\mathbf{K}\right]\{\mathbf{T}\} = \{\mathbf{R}\}\tag{10}
$$

where [**C**] is the matrix of heat capacitance, {**T**} is the matrix of temperature at grid points, [**K**] is the matrix of thermal conductance,  $\{R\}$  is the matrix of applied thermal loads.

In order to solve equation (10) Crank-Nicolson method was used. Based on the assumption that temperature  $\{T\}_t$  and  $\{T\}_{t+\Delta}$ *t* at time *t* and *t*+∆*t* respectively, the following relation is specified:

$$
\frac{1}{\Delta t} \left[ \left\{ \mathbf{T} \right\}_{t+\Delta t} - \left\{ \mathbf{T} \right\}_t \right] = (1 - \beta) \left\{ \frac{\mathrm{d} \mathbf{T}}{\mathrm{d} t} \right\}_t + \beta \left\{ \frac{\mathrm{d} \mathbf{T}}{\mathrm{d} t} \right\}_{t+\Delta t}
$$
(11)

Substituting equation (11) to equation (10) we get the following implicit algebraic equation:

$$
\begin{aligned} ([\mathbf{C}] + \beta \Delta t [\mathbf{K}]) (\mathbf{T})_{t+\Delta t} &= ([\mathbf{C}] - (1 - \beta) [\mathbf{K}] \Delta t) (\mathbf{T})_t \\ &+ (1 - \beta) \Delta t \{ \mathbf{R} \}_t + \beta \Delta t \{ \mathbf{R} \}_{t+\Delta t} \end{aligned} \tag{12}
$$

where  $\beta$  is the factor which ranges from 0.5 to 1 and is given to determine an integration accuracy and stable scheme.

### **4. CONVECTIVE HEAT TRANSFER**

Since the forced convection takes place on the contact surface during every rotation of a disc (out of pad area on the rubbing path) as well as on the cylindrical external and internal surface (Fig. 2), the convective heat transfer coefficient *h* is of the form  $h = KNu/r$ , where *r* is the radial location, which denotes mean radius of the friction surface, Nu is the Nusselt number, *K* is the thermal conductivity of surrounding air.

Furthermore the Nusselt number is related to dimensionless Reynolds number Re and Prandtl number Pr evaluated from the expression (MSC NASTRAN THERMAL, MSC Software Corporation)  $Nu = 0.0267 \text{Re}^{0.8} \text{Pr}^{0.6}$  where Reynolds number is given by Re =  $\rho\omega r^2/\mu$  where  $\rho$  is the density,  $\mu$  is the dynamic viscosity, and Prandtl number is given as  $Pr = c \mu/K$ , where *c* is the specific heat capacity.

The natural convection may affect the convective heat transfer coefficient when the forced convection is relatively weak. The influence of mixed convection was neglected according to the condition given by Mills A. F. (1995):

$$
\operatorname{Re}_D < 4.7 \left( \frac{\operatorname{Gr}_D^3}{\operatorname{Pr}} \right)^{0.137} \tag{13}
$$

where Re*D* and Gr*D* are Reynolds and Grashof number for linear element with dimension *D*, respectively.

#### **5. NUMERICAL ALGORITHM TO EVALUATE TEMPERATURE ON THE CONTACT SURFACE OF THE DISC**

During mutual slipping process of the rotational rotor over the immovable pads, the inboard and outboard surface of a disc is subjected to non-axisymmetric thermal load. In case of braking with constant velocity (such as mountain descent), time of every rotation of the disc is equal in relation to a specific spot on the circumference within the contact region. Two phases – heating and cooling – may be distinguished. The heating time equals  $\theta_0$ /360° of time of one rotation of the wheel, whereas cooling phase equals  $1-\theta_0/360^\circ$  of time of one rotation.



**Fig. 2.** A 1/72 section of FE model of disc with boundary conditions

In the present thermal finite element analysis, the boundary intensity of heat flux, as well as cooling terms, are calculated individually for every element mesh (regarding radial location) for the specific time t. For the convenience purposes the heat exchange through forced convection was assumed to be independent of the radius within the range of maximal and minimal radius of the pad (mean radius of friction surface *rm* was considered).

The numerical algorithm was accomplished by the boundary conditions formulation. The segment of final mesh with moving heat source, accomplished by the thermal flux (pad) entering the disc is shown in Fig. 2. The intensity of heat flux as well as the convective terms are interchangeably calculated within the region of contact surface. Furthermore, to obtain the effect of gradual covering of every element of the mesh on the disc contact surface, specific evolution of the intensity of heat flux was applied by the use of specific boundary conditions. For the second case, in which the adiabatic boundary conditions replace the convective heat transfer, the same algorithm was implemented.

# **6. FE DISCRETIZATION**

In order to calculate the temperature distributions of the disc brake, appropriate mesh division regarding respective coordinates of the model is essential. High temperature gradients require very fine mesh, therefore in the area adjacent to the contact surface, lower dimension of the elements was employed. Nonetheless, for the purpose of validation of the final mesh use, tests of different grids of elements for every direction, with the speed of the vehicle of 25 km/h, were developed. The final mesh consists of 43 200 eight-node hexagonal elements and 33 693 nodes (360 elements in the circumference, 4 in the axial direction, and 20 in the radial direction − Fig. 2). As the temperature gradients in region beneath friction surface were relatively low, the mesh division was of "paver" type (irregular).

### **7. RESULTS AND DISCUSSION**

The 3D finite element analysis of the transient temperture field of the disc for heat conduction problem was conducted. The computations were carried out comprising two different phases. The first is the continuous braking process with the constant velocity lasting *t* = 3.96 s, whereas the second concerns the cooling of the brake system after brake release (*t* = 296.04 s) which is shown in Fig. 3. The thermophysical material properties, dimensions, and parameters of the operation including different values of convective heat transfer coefficient are given in Tab. 1. Because of the magnitude of the obtained differences of temperatures in the model where the convective heat exchange was applied as well as the case with thermally insulated surface out of pad domain, the presentation of the results is restricted exclusively to the friction surface of a disc.

Fig. 4 shows the variations of temperature on the contact surface of a disc at the mean radius of 95 mm during braking with constant velocity, until the moment of brake release. Obviously higher velocity with the same rate thermal load results in higher temperature at every moment of braking. It may be observed that the differences of the temperatures between subsequent velocities for both definite types of heat transfer at the end of braking are of the same range, which indicate their linear dependence.

However, as the cooling impact on the temperature of pad/disc interface appeared insignificant, the areas comprising the moment before the last entire pad transition was enlarged for the purpose of clarity. The highest obtained temperature difference regarding the model with and without convective cooling appeared for the velocity of 100 km/h and equals 1.48°C.







**Fig. 3.** Scheme of four cases of braking and the following release phases



Fig. 4. Evolutions of temperature on the contact surface of a disc at the mean radius of the rubbing path

The comparison of temperature evolutions extended to the succeeding cooling periods after the brake release is shown in Fig. 5. It may be observed that the time significantly affects the temperature values after the phase of heating. As the adiabatic conditions (solid lines) on the friction surfaces were established, the temperature of the entire model equalize after the time of about 50 s. This phenomenon is observable for each of the presented velocities. Furthermore, the same order of temperatures at the end of simulated process  $(t = 300 s)$  regarding subsequent velocities of 25, 50, 75 and 100 km/h presented in Fig. 4, is evident. Nonetheless, convective terms on all of the surfaces of disc, apart from inner cylindrical surface, result in irregular decrease in the temperatures.



**Fig. 5.** Evolutions of temperature on the contact surface of a disc for the mean radius

It may be explained by the influence of convective boundary condition which includes the temperature difference multiplied by convective heat transfer coefficient *h*. Both of the factors correspond to the velocity of the vehicle. Obviously, if the temperature difference is greater, more intense cooling takes place.

The relationship between the braking time and the temperature on the chosen circumferential location  $\theta = 0^{\circ}$  on the contact surface of a disc brake for the velocity of 100 km/h is shown in Fig. 6. Both, temperature of the case with established convective terms dependent on the velocity (dashed lines) and fully adiabatic conditions (solid lines) are confronted for corresponding radial locations. The obtained results for the operation conditions simulating real process of braking during mountain descent reveal that the influence of convective heat transfer is very low. The corresponding temperatures for the maximal radius of the contact surface  $r = 113.5$  mm at the end of operation results in difference of 3.97°C. However, the decrease in temperature difference with the decrease in a radius (enlarged area of the radial position of 113.5 and 95 mm) is observable, which is accounted directly by the expression of the convective heat flux. The even increase in temperature without phases of heating and cooling is observable at the radial location of 66 mm, which indicate insignificant influence of pad transition on the surface out of the rubbing path.

Fig. 7 depicts the temperature evolutions on the contact surface for circumferential position  $\theta = 0^{\circ}$  during braking and after brake release. Four specific radial locations present correspond-

ing values of transient temperature in two cases of threedimensional FE model. The first, where the convection is taken into account, and the second without influence of cooling. It may be observed that after a certain time − about *t* = 100 s, the temperatures for every radius on friction surface of the case with adiabatic condition on the surfaces free from cooling, equalize  $(T = 306.7^{\circ}C)$  and remain on the same level until time of end of the process  $t = 300$  s, whereas the case with convective heat exchange reveals theirs gradual decrease, however equal on each radial position. As can be seen the temperature at the end of cooling phase equals approximately  $T = 165^{\circ}$ C.



**Fig. 6.** Evolution of temperature on the contact surface of a disc during braking with the velocity of 100 km/h



 during braking with the velocity of 100 km/h and after brake release

# **8. CONCLUSIONS**

The aim of this paper was to investigate the temperature distributions caused by the process of braking of a common passenger vehicle with a special emphasis placed on the influence of convective heat exchange from surfaces free from heating.

The obtained results support the claim that in terms of single braking process for the specified dimensions and thermophysical properties of materials the convective heat transfer mode doesn't allow to significantly lower the temperatures of the rotor. The temperature of two studied types of disc brakes almost coincide at every instant of time during short braking period. However the following release period after the braking action when the velocity of the vehicle remains on the same level, results in considerable decrease in temperature. The temperature difference between the case with established convective terms dependent on the velocity and fully adiabatic conditions lowers with the decrease in the velocity.

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