# NUMERICAL-EXPERIMENTAL ANALYSIS OF PLASTIC FLOW BEGINNING PHASE ROUND THE HOLE IN THE THIN SHIELD UNDER TENSION

#### Stefan BUĆKO\*, Henryk JODŁOWSKI\*, Marcin TRZEBICKI\*\*

Institute of Applied Mechanics, Cracow University of Technology, al. Jana Pawła II 37, 30 -864 Kraków, Poland "Institute of Machine Design, Cracow University of Technology, al. Jana Pawła II 37, 30 -864 Kraków, Poland

#### s\_bucko@mech.pk.edu.pl, abies@mech.pk.edu.pl, mtrzeb@mech.pk.edu.pl

**Abstract:** The method of recognition of plastic strains with the use of optical interference was characterized in the work. The idea of the method was shown on the example test for 15HM steel tension. The results of experimental testing of plastic strains initiation in the thin shield with opening of Ø20 diameter under tension, especially considering round the hole zone were presented in further part of the work. The experiment was carried out with the use of method mentioned above. In the experiment the stresses initializing plastic deformations and the shape of plastic zone were determined. The stress state analysis in the hole zone in the specimen under tension was realized with the use of finite element method. The calculations were done for two significantly different hole diameters. The results of FEM calculations allow for certification that for the holes of diameters significantly bigger than the shield thickness round the hole plane stress state occurs.

Key words: Plastic Strains Identification, Optical Interference, Stress Concentration, Experimental Research, FEM Analysis

## 1. INTRODUCTION

The problem of concern is the quantitative description and the experimentally verified interpretation of the initial formation phase of macroscopic plastic deformations in zones with a stress gradient for materials with material unstableness. In materials with a distinct yield point the smallest plastic deformation can be significantly larger than the strains associated to the elastic limit (upper yield point). Occurrence of plastic deformation at the point of maximal effort is not possible at the time of fulfillment in this one point the condition of plasticity, as in the neighboring points stresses are smaller than the yield strength. Occurrence of plastic deformation in the form of plastic slips requires the assembly of a portion of elastic deformation in grains around this point. In the scientific literature so far no convincing answer to the question: how and when the plastic deformation in such cases begins.

The common practice of experimental research into elasticplastic state under conditions of stress gradient i.e. in the vicinity such as holes in the discs or in areas of other notches, depended mostly on the measurement of displacement, strain calculation and use of the yield condition (Durelli and Parks 1970, Theocaris, Marketos 1964) or the use of optically active layer and the condition of plasticity, which is not always synonymous with the start of plastic deformation. Further analysis has usually been conducted by solving the classical equations of the plasticity theory. As a result, the shape of plastic zone depend on the results of measurements and the adopted conditions of plasticity, which has not been the result of the experiment any more.

In that method, it is assumed that the launch of the plastic slides in the homogeneous stress state (pure tensile strength) and in conditions of stress gradient occurs at the same stress level, which assumption is dubious, considering the literature (Brzoska, 1972; Malinin and Rżysko, 1981).

The purpose of this paper is to further verify the effectiveness of previously developed methods for identifying plastic macrostrains in the active process of loading, in steels of material instability, based on the use of optical interference phenomenon (Bucko and Jodłowski, 2006; Jodłowski, 2007) and attempt to estimate the value of the yield strength i.e. at which begins the process of plastic flow - what respond to physical phenomena accompanying the yield strength is pronounced in the static tensile experiment. An important element is the attempt to explain the mechanisms of launching of plastic slips - regarding the plane of maximum shear stress action.

### 2. CHARACTERISTICS OF EXPERIMENTAL RESEARCH

The method of plastic macro-strains identifying in the active process of loading in the active process utilizing known phenomenon of optical interference has been discussed in detail and justified in the previously mentioned works (Bucko and Jodłowski, 2006, 2009; Jodłowski, 2007). A shortened description of the research method is presented below but in the next section the examples of research of verifying correctness and effectiveness of this method in the static tensile test are discussed. The idea of this method of research is as follows:

Disappearance of interference fringes observed previously in the active process of loading shows that plastic deformation occurred.

"Occurrence of interference fringes that means the polished surface ability to reflect light, may in fact occur only on the surface of these grains, in which there was no slip yet malleable, which is in a state of elastic".

The test method is based on the principle formulated above can probably detect only a plastic macro-strains in the scale occurring when the yield stress is pronounced in the static tensile experiment, but it is possible to observe their formation in the active process of loading. The method therefore does not require termination of the experiment in order to perform microscopic observation or realising of the unloading process just to verify the nature of the deformation. The proposed method is not sensitive to the possible displacement of the observed sample surface because the interference fringes number is not important (confirms only a distance between the interference strainer and sample), significant is merely the presence or disappearance of optical interference phenomenon. An important advantage of optical interference method is also possibility to visualize the process of formation and propagation of plastic deformation.



Fig. 1. 15HM steel tensile test diagram



Fig. 2. 15HM steel. A part of the plastic platform of tensile test diagram. Marked points correspond to the times of phenomena occurrence on the video film

In order to illustrate the effects occurred in plastic deformation identification with the use of optical interference method,

the interference fringes images are showed below. The images were recorded during the static tensile test of specimen made of 15HM steel. From the same material the next specimen with the central hole was prepared for the test described in the further part of the work.

Fig. 1 shows a graph of that stretch of steel in a classic form, while Fig. 2 shows an accurate (zoomed) picture of the plastic platform with the registered moments in time attached to the set of pictures interferential fringes registered in the same moments of time. During the study the principle of full synchronization of recording of all parameters and images was adhered.

According to the described above method disappearance of interference fringes means that plastic slips took place. The diagonal bands slopped to the axis of the sample where the interference fringes disappeared are visible on the photographs shown in Fig. 2. Next such bands appear following the development of plastic deformations. In the zones among these bands where the interference fringes are still visible slow disappearance of the fringes is observed.

# 3. EXPERIMENTAL RESEARCH ON DISC WITH AN APERTURE

Experimental studies were carried out for the thin plate of dimensions 550x98.8x4.7 [mm] under tension with symmetric aperture with a diameter of 20 [mm] made of 15HM steel with pronounced yield point , tensile diagram of this steel is shown in Fig. 1 in section 2 of work. Shape of the test plate is shown in Fig. 3.



Fig. 3. Scheme of the shield with strain gauges  $T_{\text{O}}$  i  $T_{\text{B}}$ 

To verify the correctness of the indications of optical interference method in place of extreme stress on the lateral surface of aperture electrical resistance strain gauge  $T_{\rm O}$  was glued, second  $T_{\rm B}$  strain gauge was glued to the side surface of the shield.

The interference fringes images were recorded with the use of camera as well as changes of tensile force and the resistance strain gauge  $T_0$  indications have being registered during the experiment. All these parameters were recorded synchronously on a common timeline. Applied method of registration of characteristic experiment parameters allowed for unambiguous assignment of nominal stresses and maximal strains, strains and stresses calculated according to two methods (analytical and numerical-FEM) to corresponding interference fringes images.

The first signs indicating the occurrence of plastic slip in the disc loading process revealed in the film by a step change of the interference fringes distribution at the opening in the form of a pulse sent to the outer surface of the shield, and by rapid increase of the strain gauge indications placed on the hole side surface - Fig. 3.

One possible and reliable ways to verify the correctness of the method of optical interference in the use to identification of plastic macro-strains is to compare changes in the measured stress value to the maximum stress value calculated for corresponding loads from Howland formula and to the stress calculated by finite element method assuming a linear elasticity.

The maximum value of stress on the hole lateral surface as the first plastic slip was observed calculated from the strain gauge TO indication and the real material Young's modulus value was  $\sigma_{mo}=1.462 R_{eH}$ . Impulse interference fringes on the image appeared in the axis of symmetry perpendicular to the direction of stretching, and plastic zone (disappearance fringes at the surface) was shifted close to half of the thickness of the disc and develop in a direction coinciding with the direction of symmetry axis. This indicates that the initiation of slip occurred in the mid-surface zone of the blade.



Fig. 4. Diagram of strains indicated by resistance strain gauges To i TB



Fig. 5. View of the plastic zone interference fringes in the Ø20 hole neighborhood: a) beginning phase of plastic strains;
b) developed phase of plastic strains

Fig. 4 shows the deformation diagrams on the side surface of the hole ( $T_0$ ) and at the lateral surface of the disc ( $T_B$ ). At the graph of the  $T_0$  strain gauge indication the dramatic increase in deformation display is visible. This corresponds to the impulse

mentioned above what is visible on the interference fringes image.

Two images of plastic zones in various stages of increase are shown in Figs 5a and 5b. To show the pulse starting slipping on the photograph is not possible, because requires the continuous observation of phenomena such as the movie. Plastic zone, visible on the shield surface in the shape of a narrow wedge develops in the direction of the side surface of the plate as shown in the work (Bucko and Jodłowski, 2009) for a disk with a hole of larger diameter. Interference fringes systems on both sides of the plastic zone differ significantly with their density and shape and therefore exhibit a discontinuity. Interference fringes discontinuity indicate on displacement discontinuity on the shield surface, which may indicate the opposite directions of displacements of the surfaces. The fringes are in fact in some ways contour lines of gap size between the mirror and observed shield surface.

# 4. ANALYSIS OF STRESS AND STRAIN STATE IN THE HOLE ZONE

Analytical solutions of the problem of stress distribution around the hole in the thin disk are known for a long time. Theoretical studies of this problem stemmed from the computing needs for riveted connections. For a small hole in an infinite shield is the Kirsch solution of 1898, while for holes of any diameter compared to the shield width the solution has been obtained by Howland (Howland, 1930). Both of the above-mentioned analytical solutions are based on the assumption of plane state of stress and the linear elasticity. Both of the above-mentioned analytical solutions are based on the assumption plane stress state and the linear elasticity obviously. Especially the latter approach became the basis for developing charts of stress concentration factors around holes in thin shields, commonly used in the calculation of fatigue strength. In later years he also obtained a solution based on the equations of the theory of plasticity and also on the assumption of plane stress (Durelli and Parks, 1970; Theocaris and Marketos, 1964). Observations of schemas of plasticising of shields with holes diameters of which were close to the shields thickness suggest that there is significant derogation from the plane stress state in the discs whose thickness is much smaller than the width, which is normally considered to be thin. Analytical solution of the problem of stress state around the hole, taking into account the spatial state of stress is very complex and unlikely to bring success. The work thus takes the concept to analyze the state of stress around the hole using the finite element method allowing for taking into account triaxial stress state in the zone of the hole. Finite element method as an approximate method of calculation provides a number of computational problems especially in boundary zones. These difficulties can be at least partially overcome by using a suitable discretization and a calculation result sufficiently close to a strict solution can be obtained.

For the analysed shield FEM calculations were performed using ALGOR system. In order to show the impact of hole diameter relative to the thickness of the disc at a fixed width below shows the results of calculations of stresses around two openings: one with a value of 1.28 for this parameter and the second for parameter that equals 4.25 for whom results of the experiment are shown in further part of the work. Most important calculation results -from the perspective of the analysis of processes of plastic flow- are depicted in Figs 6, 7, 8 and 9 in the form of maps of distributions of stresses and displacements. Pictures of stress distributions presented in the above Figures relate to the half of the disc thickness i.e. the lowest surface is the midsurface of the shield.

FEM calculation results show that for the test case (hole Ø20 mm) the derogation from the plane state of stress is very small. This is indicated by very small, although nonzero, the value of stress in the direction perpendicular to the shield shown in Fig. 7. The maximum value of the equivalent stress by hypotheses Huber-Mises-Hencky's and the Coulomb-Treska-Guest are almost identical and very close to the maximum stress in the direction of the shield stretching. The maximum equivalent stresses occur in the middle surface of the shield where changes along the shield thickness are small.

The calculation of stresses around the hole  $\emptyset$ 20 with formulas derived analytically by Howland (Howland 1930) with assumption of a plane stress state, give results very close to the values obtained using finite elements method. Shown in Fig. 8 insignificant  $\sigma z$  stress value justifies the assumption of plane stress state.

Numerical and analytical calculations lead to the conclusion that in the area of maximum effort of material stresses  $\sigma_1 > 0$ ,  $\sigma_2 > 0$ , and  $\sigma_3 = \sigma_z \approx 0$ . In this case, the maximum shear stress in the present work are in a plane passing through the y axis and inclined to the plane of the shield at an angle close to  $45^{\circ}$ . The above-mentioned maximum shear stress, according to evidence from Nadai research (Nadai, 1950) on the plasticity of steel (confirmed by studies discussed down in (Bucko and Jodłowski 2006, 2009; Jodłowski 2007) plays a decisive role in the launch of plastic slip in steel with a distinct yield point. This is also confirmed in the experiment shown in this study (Fig. 5).

The results of FEM calculations for a disk with a hole of 20 mm.



Fig. 6.  $\sigma_x$  stress distribution



Fig. 7.  $\sigma_y$  stress distribution



Fig. 8.  $\sigma_z$  stress distribution



Fig. 9. reduced HMH stress distribution

FEM results for the shield with opening of diameter Ø 6 mm



Fig. 10.  $\sigma_x$  stress distribution



Fig. 11.  $\sigma_{\text{y}}$  stress distribution



Fig. 12.  $\sigma_z$  stress distribution



Fig. 13. Reduced HMH stress distribution

As mentioned the calculations of stresses in the disc with a hole diameter of 1.28 times the thickness of the shield (Ø 6 mm) were performed for comparison with earlier results. FEM calculation results indicate that there was a significant departure from the plane state of stress in the zone of the hole. Key results in the form of distributions of stresses in the area of the biggest effort around the hole are shown in Figs. 10, 11, 12 and 13. The  $\sigma_z$ stress valued 8.4% of  $\sigma_x$  stress maximal value occurred in the midsurface of the shield in the zone of the maximal effort. Equivalent stress by hypothesis of Coulomb-Treska-Guest is close to  $\sigma_x$ max and a few percent larger than the equivalent stress by Huber-Mises-Hencky hypothesis. Stress gradient along the shield thickness is clearly larger for the hole Ø 6 than for the hole Ø20.

### 5. COMPARATIVE ANALYSIS EXPERIMENTAL RESULTS AND CALCULATIONS

Calculation of stresses for the load corresponding to the first plastic strains in the disc with a hole Ø20 mm (signaled by the strain gauge TO and simultaneous of impulsive deformation of interference fringes at the hole edge) were realized with the use of Howland formulas and with the use of FEM program. Both calculation methods base on the assumption of a linear relationship between strains and stress. The maximal stress value obtained with the use of Howland formulas is  $\sigma_{Ho} = 1.51 \rm R_{eH}$ , the maximum stress value calculated by finite element method is  $\sigma_{MES} = 1.52 \rm R_{eH}$ , while the stress calculated on the basis of indications of strain gauge TO and Hook's law reached a

value  $\sigma_{mo} = 1.462 R_{eH}$ . The difference between cited above three stress equals of approximately 4%, what confirms the linearity of the relationship between strains and stresses ie. it confirms elasticity of strains to the stresses exceeding the distinct yield point value by almost 50%. At the same time it confirms the effectiveness of the utility of method of optical interference to the identification of plastic macro-strains in steels with a distinct yield point.

Noteworthy is the shape of plastic zone, i.e. the zone in which there is loss of interference fringes. The Plastic zone is wedge-shaped and extends to the edge of the hole i.e. to place of the largest plastic deformations. The interference fringes behave differently on the both sides of plastic zone during increase of the load (see Fig. 5). Above the plastic zone the number of fringes increases with the load increase, while the in area below the wedge the number of fringes decreases. This indicates the opposite direction of the shield surface displacement perpendicularly to it, on both sides of the plastic zone. This effect confirms the occurrence of slips in the plane of action of maximum shear stress, ie in a plane passing through the axis closely to the y-axis sloped to the shield at an angle close to  $45^{\circ}$ .

The results of stress calculations performed by finite element method for hole Ø6 for whom the diameter was comparable to the thickness of the shield, as shown in P.4, justify the conclusion that the patterns of plastic slip for these holes will be different than in cases of large diameter holes such as the hole Ø20 as discussed above. Clarification of the nature of the plastic slides for plates with holes with diameters comparable to the thickness of the disc requires further experimental studies.

#### REFERENCES

- 1. Brzoska Z. (1972), Strength of the materials (in polish), PWN Warszawa.
- Bućko S., Jodłowski H. (2006), Plastic zones formation in materials with physical plastic limits on the base of new research methods (in polish), *Materiały X Krajowej Konferencji Wytrzymałości i Badania Materiałów*, Kudowa Zdrój.
- Bućko S., Jodłowski H. (2009), Inicjacja odkształceń plastycznych w stali z wyraźną granicą plastyczności w warunkach gradientu naprężeń, Acta Mechanica et Automatica, Vol. 3, No. 1.
- Durelli A. J., Parks V.J. (1970), Moire Analysis of Strain, Prentice-Hall, Englewood Cliffs, New Jersey 1970.
- Howland R.C.J. (1930), On the Stresses in Neighborhood of a Circular Hole in a Strip Under Tension, *Phil. Trans. Roy. Soc.* (London), A, Vol. 229 (1929-30), 67.
- Jodłowski H. (2007), Metoda rozpoznawania odkształceń plastycznych w procesie obciążania stali wykazujących niestateczność materiałową, Praca doktorska, Politechnika Krakowska, Kraków.
- 7. Malinin N. N., Rżysko J. (1981), Mechanika materiałów, PWN Warszawa 1981.
- 8. Nadai A. (1950), *Theory of flow and fracture of solids*, t. I, New York 1950, McGraw-Hill.
- Theocaris S., Marketos E. (1964), Elastic-plastic analysis of perforated thin strips of a strain-hardening material, *J. Mech. Phys. Solids.*, 11, 377 – 390.

The scientific work financed by National Center of Science from funds for science 2010-2013 as a research project N N501248838, Decision No 2488/B/T02/2011/.