STICK-SLIP CONTACT PROBLEM OF TWO HALF PLANES WITH A LOCAL RECESS

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Abstract: A plane problem of frictional contact interaction between two elastic isotropic half planes one of which possesses a single shallow recess (depression) is examined in the case of successive application of remote constant normal and shear forces. The loads steps (compression, and next monotonically increasing shear loads) lead to the main contact problem with an unknown stick-slip boundary determined by the Amonton-Coulomb law. It is reduced to a Cauchy-type singular integral equation for the tangential displacement jump in the unknown sliding region. Its size is derived from an additional condition of finiteness of shear stresses at the edges of the slip zone. Considerations are carried out for some general shape of the recess. Analytical results with the characterization of the considered contact are given and illustrated for the certain form of the initial recess.

1. INTRODUCTION

The frictional effects during contact of elastic solids are the subject of the investigation of many authors. Interest to such problems is stimulated by applied requests of engineering, tribology, geophysics, bulding industry and biomechanics. Amonton-Coulomb's classical friction law is used widely in engineering applications involving contact (Kragelsky et al.; 1982). In this law, it is assumed that two contacting bodies either stick ($|s| < fp$) or slip ($|s| = fp$) to each other, where f is the constant coefficient of friction, s and p are the magnitudes of tangential and normal traction due to friction. If the equality $|s| = fp$ is valid for the whole contact region, then we have the case of sliding friction. Realistic frictional contact problems reduce to finding the correct size and location of the stick-slip boundary depending on given loading conditions.

In literature dealing with contact problems (Barber and Ciavarella; 2000) the overwhelming majority of works consider the contact of bodies with non-conforming boundaries (see classification by Johnson; 1985). The problem to be considered is referred to contact frictional problems involving interactions of bodies with conformable boundaries. Such a kind of the interaction taking into account the absence of local contact caused by the presence of local small geometric perturbations of initial boundaries is less investigated although it is quite typical for many contacting joints. In this field basic research regarding frictionless contact has been carried on and documented (see, for example, Shvets et al., 1996; Kaczyński and Monastyrskyy, 2002; Monastyrskyy and Kaczyński, 2010; and references therein). Similar problems involving friction were considered by Martynyak and Kryshtafovych (2000), Kryshtafovych and Matysiak (2001) and in a series of papers by Martynyak et al. (2005, 2006).

The present paper is devoted to analyze the behavior of a complete contact couple formed by two semi-infinite elastic planes with the presence of a small surface recess under the combination of remote normal and shear forces. This is achieved in two steps: first, by solving the full stick contact problem and next, using it to pose and solve the stick-slip problem with an unknown slip zone defined by the Amonton-Coulomb law. Research is performed for some general shape of the recess. The final results are given and illustrated in particular case.

2. FORMULATION OF THE PROBLEM

The problem under study involves the investigation of frictional contact between two homogeneous elastic half-planes D_2 (upper) and D_1 (lower) made of the same isotropic material. Referring to the Cartesian coordinate system Oxy the boundary of D_2 is rectilinear whereas the boundary of D_1 has a small deviation in the form of the sloping recess located in a segment $x \in [-b, b]$ as shown in Fig. 1a. Accordingly, the shape of the lower half-plane boundary is described by the smooth function $r(x)$ given by the formula:

$$r(x) = \begin{cases} -r_0 \left(1 - x^2/b^2\right)^{n+1/2}, & |x| \le b, \\ 0, & |x| > b, \end{cases} \tag{1}$$

where r_0 and $2b$ are maximal depth and length of the recess, $n = 1,2,...$ is a natural parameter, and the assumption $0 < r_0/b \ll 1$ is made.

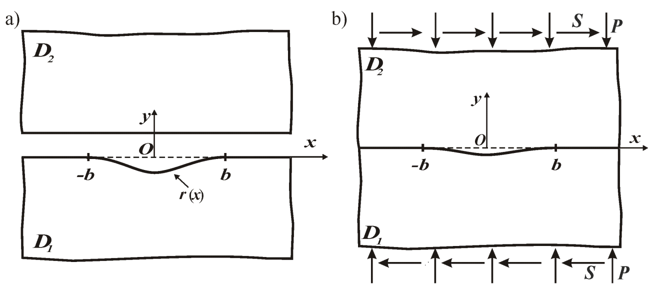

Fig. 1. Sketch of interaction of half-planes:
a) before contact; *b*) in full contact

The following phases of constant loading at infinity are considered: first, normal compressive forces P leading to full contact and subsequently, monotonically increasing shear forces S giving rise to partial sliding.

Similar to the well-known Cattaneo procedure used in partial slip contact under combined normal and tangential loading (Ciavarella; 1998), the full stick contact problem is solved and analyzed first in order to establish correctly the conditions in formulation of the main stick-slip contact problem of interest.

3. FULL-STICK CONTACT PROBLEM

Consider the problem of full-stick contact of the half-planes without slip (Fig. 1b) characterized by the boundary conditions at the interface $y = 0$, $|x| < \infty$:

$$\sigma_y^-(x,0) = \sigma_y^+(x,0), \qquad \tau_{xy}^-(x,0) = \tau_{xy}^+(x,0),$$
$$u^-(x,0) - u^+(x,0) = 0\,,\ v^-(x,0) - v^+(x,0) = -r(x)\,, \tag{2}$$

and at infinity:

$$\sigma_y(x,\pm\infty) = -P, \qquad \tau_{xy}(x,\pm\infty) = S,$$
$$\sigma_x(\pm\infty,y) = 0, \qquad \tau_{xy}(\pm\infty,y) = S. \tag{3}$$

Here and subsequently, σ_y, σ_x, τ_{xy} – the components of stresses; u, v – the components of the displacement vector; superscripts "–" and "+" denote the limit values of functions at the interface of the half planes D_1 and D_2.

Additionally, the requirement of the non-negativity constraint of the contact pressure:

$$p(x) = -\sigma_y^-(x,0) \geq 0\,,\ |x| < \infty \tag{4}$$

has to be used to determine a condition for the complete contact.

Following the solution of the above problem employing the well-known technique of analytical continuation (Muskhelishvili; 1953) and given in Martynyak et al. (2005 a, b), the stresses and displacements in the bodies are expressed by means of the derivative of the function r(x) as follows:

$$\sigma_x(x,y) + \sigma_y(x,y) = 4\,\mathrm{Re}\left[\Phi_l(z)\right] - P,$$
$$\sigma_y(x,y) - i\tau_{xy}(x,y) = \Phi_l(z) - \Phi_l(\bar{z}) + -(z-\bar{z})\overline{\Phi_l'(z)} - P - iS\,,$$
$$2G\frac{\partial}{\partial x}\left[u(x,y) + iv(x,y)\right] = \kappa\Phi_l(z) + \Phi_l(\bar{z}) + -(z-\bar{z})\overline{\Phi_l'(z)} + \frac{3-\kappa}{4}P\,, \tag{5}$$

in which:

$$\Phi_1(z) = -\Phi_2(z) = \frac{(-1)^{l+1}G}{\pi(1+\kappa)}\int_{-b}^{b}\frac{r'(t)\,dt}{t-z}\,,$$
$$z = x + iy \in D_l\,,\quad l = 1,2\,, \tag{6}$$

and G is the shear modulus, ν is Poisson's ratio, $\kappa = 3 - 4\nu$ is Kolosov's constant.

Inserting (1) into (6) and using (5) gives the normal stresses on the contact surface for $|x| \leq b$:

$$\sigma_y^{\pm}(x,0) = \frac{2Gr_0\,(2n+1)}{(1+\kappa)b}\left(\frac{(2n-1)!!}{2^n\,n!} + \mathrm{P}^{(a)}(x)\right) - P \tag{7}$$

and for $|x| > b$:

$$\sigma_y^{\pm}(x,0) = \frac{2Gr_0\,(2n+1)}{(1+\kappa)b}\times$$
$$\times\left(\frac{(2n-1)!!}{2^n\,n!} + (-1)^{n+1}\left|\frac{x}{b}\right|\left(\frac{x^2}{b^2}-1\right)^{n-\frac{1}{2}} + \mathrm{P}^{(a)}(x)\right) - P, \tag{8}$$

where:

$$\mathrm{P}^{(a)}(x) = a_2\left(\frac{x}{b}\right)^2 + a_4\left(\frac{x}{b}\right)^4 + \ldots + a_{2n}\left(\frac{x}{b}\right)^{2n},$$

$$a_{2(j+1)} = \sum_{k=j+1}^{n}\frac{(-1)^k\,n!}{k!(n-k)!}\cdot\frac{(2k-2j-3)!!}{(2k-2j-2)!!}\,,$$

$$j = 0,1,2,\ldots,n-1,\ (-1)!! = 1,\ 0!! = 1.$$

Accordingly, the shear stresses are:

$$\tau_{xy}^{\pm}(x,0) = S\,,\quad x \in (-\infty,+\infty)\,. \tag{9}$$

By observing that the global maximum of RHS in relation (7) is achieved at $x = 0$, we obtain from (4) the inequality for the value of the normal pressure P that satisfies full contact of the bodies

$$P \geq \frac{Gr_0(2n+1)(2n-1)!!}{2^{n-1}n!(1+\kappa)b}\,. \tag{10}$$

According to the Amonton-Coulomb law, the increase in the shear forces S does not affect in sliding if the contact stresses satisfy the condition $|\tau_{xy}| < f|\sigma_y|$, i. e.

$$S < f\left(P - \frac{Gr_0(2n+1)(2n-1)!!}{2^{n-1}n!(1+\kappa)b}\right). \tag{11}$$

Thus, the slip occurs when this condition is violated.

4. STICK-SLIP CONTACT PROBLEM

Let us consider now the case opposite to (11):

$$S \geq f\left(P - \frac{Gr_0(2n+1)(2n-1)!!}{2^{n-1}n!(1+\kappa)b}\right) \tag{12}$$

that is the condition of sliding in the vicinity of the point $x = 0$. So we are faced with the stick-slip problem in which we assume from the loading and geometry symmetry that there exists a region of local sliding $|x| < c$ (see Fig. 2). Note that the half-length of the slip zone c is unknown.

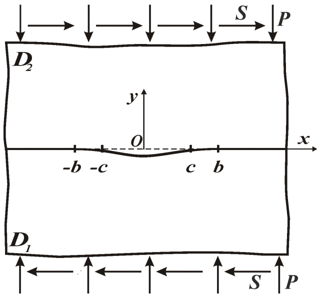

Fig. 2. Interaction of half-planes in stick-slip contact

For the present non-trivial problem we have the same boundary conditions at infinity given by (3) and the following contact condition on $y = 0$:

$$\begin{aligned}
&\sigma_y^-(x,0) = \sigma_y^+(x,0), \quad |x| < \infty, \\
&\tau_{xy}^-(x,0) = \tau_{xy}^+(x,0), \quad |x| < \infty, \\
&\left|\tau_{xy}^-(x,0)\right| = f\left|\sigma_y^-(x,0)\right|, \quad |x| < c, \qquad (13)\\
&u^-(x,0) - u^+(x,0) = 0, \quad |x| \ge c, \\
&v^-(x,0) - v^+(x,0) = -r(x), \quad |x| < \infty.
\end{aligned}$$

Moreover, $sign(\tau_{xy}^-) = signS$ is chosen from the slip behavior.

To determine an unknown coordinate c of the stick-slip boundary, we will use the condition ensuring finiteness of the contact shear stresses at the edges of the slip zone:

$$\lim_{x\to\pm c}\left|\tau_{xy}(x,0)\right| < +\infty . \qquad (14)$$

In dealing with solution to the above posed problem we use the commonly employed method of intercontact gap functions, devised by Martynyak (1985).

First we solve an auxiliary problem with the same boundary conditions of the original problem but instead of $(13)_3$ we set:

$$u^-(x,0) - u^+(x,0) = U(x), \quad |x| \le c. \qquad (15)$$

Results for the normal and tangential stresses at the nominal interface are expressed as (Martynyak et al.; 2005b):

$$\begin{aligned}
&\sigma_y^-(x,0) = \frac{2G}{\pi(1+\kappa)}\int_{-b}^{b}\frac{r'(t)dt}{t-x} - P, \\
&\tau_{xy}^-(x,0) = -\frac{2G}{\pi(1+\kappa)}\int_{-c}^{c}\frac{U'(t)dt}{t-x} + S
\end{aligned} \qquad (16)$$

and comparing with the solution of stick problem we see that the formula for normal stresses doesn't change. Now substitution (16) into relationships $(13)_3$ yields a singular integral equation for the unknown derivative of function $U'(x)$:

$$\frac{1}{\pi}\int_{-c}^{c}\frac{U'(t)dt}{t-x} = \frac{1+\kappa}{2G}(S - fP) + \frac{(2n+1)r_0 f}{b}\left(\frac{(2n-1)!!}{2^n n!} + \mathrm{P}^{(a)}(x)\right), \quad |x| < c \qquad (17)$$

By utilizing the theory of singular equations with Cauchy kernels (Muskhelishvili; 1953), it is possible to obtain the solution of this equation in the class of functions with the natural conditions of continuity of the relative tangential shift $U(\mp c) = 0$. Omitting details, we focus only on the expressions for the tangential stresses at the interface boundary:

$$\begin{aligned}
&\tau_{xy}^-(x,0) = fP - \frac{2G(2n+1)r_0 f}{(1+\kappa)b}\left(\frac{(2n-1)!!}{2^n n!} - \mathrm{P}^{(w)}(x)\right), \quad |x| \le c, \\
&\tau_{xy}^-(x,0) = fP - \frac{2G(2n+1)r_0 f}{(1+\kappa)b}\left(\frac{(2n-1)!!}{2^n n!} - \mathrm{P}^{(w)}(x)\right) + \\
&+\left[S - fP + \frac{2G(2n+1)r_0 f}{(1+\kappa)b}\left(\frac{(2n-1)!!}{2^n n!} - \mathrm{P}^{(d)}(x)\right)\right]\times \\
&\times\frac{|x|}{\sqrt{x^2-c^2}}, \quad |x| > c,
\end{aligned} \qquad (18)$$

where:

$$\mathrm{P}^{(d)}(x) = d_0 + d_2(x/c)^2 + d_4(x/c)^4 + \ldots + d_{2n}(x/c)^{2n},$$

$$d_0 = \sum_{k=1}^{n}\frac{(2k-3)!!}{(2k)!!}a_{2k}(c/b)^{2k},$$

$$d_{2j} = -a_{2j}\left(\frac{c}{b}\right)^{2j} + \sum_{k=j+1}^{n}\frac{(2k-2j-3)!!}{(2k-2j)!!}a_{2k}\left(\frac{c}{b}\right)^{2k}, \quad j = 1,2,\ldots,n-1,$$

$$d_{2n} = -a_{2n}(c/b)^{2n},$$

$$\mathrm{P}^{(w)}(x) = w_0 + w_2(x/c)^2 + w_4(x/c)^4 + \ldots + w_{2n}(x/c)^{2n},$$

$$w_{2j} = \sum_{m=j}^{n}\frac{d_{2m}(2m-2j-1)!!}{(2m-2j)!!}, \quad j = 0,1,2,\ldots,n.$$

In order to close the problem in hand, we have to modify the above expressions to guarantee their finiteness in the vicinity of the point $\mp c$ according to the condition (14). By analyzing relation (18), it is sufficient to fulfil the equation:

$$S - fP + \frac{(2n+1)Gr_0 f(2n-1)!!}{2^{n-1}n!\,b(1+\kappa)} + -\frac{2G(2n+1)r_0 f}{(1+\kappa)b}\mathrm{P}^{(d)}(c) = 0 . \qquad (19)$$

In point of fact, this equation determines the unknown location c of the stick-slip boundary.

5. RESULTS

To analyze and illustrate the behavior of the contact couple on the basis of the obtained analytic solution to the considered problem, calculations are performed for the special form of the recess given by the formula (1) for $n = 3$.

Considering first the stick contact problem, we find the normal contact stresses from relations (7) and (8):

$$\sigma_y^{\pm}(x,0)=\frac{14Gr_0}{(1+\kappa)b}\left(\frac{5}{16}-\frac{15}{8}\frac{x^2}{b^2}+\frac{5}{2}\frac{x^4}{b^4}-\frac{x^6}{b^6}\right)-P,\quad |x|\le b,$$

$$\sigma_y^{\pm}(x,0)=\frac{14Gr_0}{(1+\kappa)b}\times$$

$$\times\left(\frac{5}{16}+\left|\frac{x}{b}\right|\left(\frac{x^2}{b^2}-1\right)^{\frac{5}{2}}-\frac{15}{8}\frac{x^2}{b^2}+\frac{5}{2}\frac{x^4}{b^4}-\frac{x^6}{b^6}\right)-P,\quad |x|>b. \quad (20)$$

Graphs of contact stresses are given in Fig. 3. The solid line corresponds to $f|\bar{\sigma}_y|(\bar{\sigma}_y=\sigma_y/G)$ and the dashed lines 1, 2, 3 – to tangential stresses $\bar{\tau}_{xy}(\bar{\tau}_{xy}=\tau_{xy}/G)$ over the contact line $\bar{x}=x/b$ under pressure $\bar{P}=2\cdot 10^{-4}$ ($\bar{P}=P/G$), friction coefficient $f=0{,}1$, maximal depth of the recess $\bar{r}_0=r_0/b=10^{-4}$ and Poisson's ratio $\upsilon=0{,}2$ for some values of shear stresses $\bar{S}$ ($\bar{S}=S/G$): $1-\bar{S}=2\cdot 10^{-6}$; $2-\bar{S}=6{,}328\cdot 10^{-6}$; $3-\bar{S}=15\cdot 10^{-6}$.

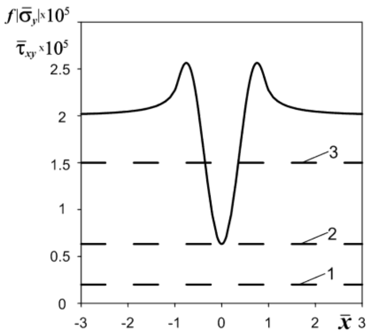

Fig. 3. Distributions of the contact stresses

In view of (11), if $P\ge\frac{35Gr_0}{8b(1+\kappa)}$, then the half planes are in full contact. By analyzing the relations (7) and (8) we see that the increase of the shear forces S does not in sliding if $S<\frac{35Gr_0}{8b(1+\kappa)}$ that follows from the condition $|\bar{\tau}_{xy}<\mathrm{f}|\sigma_y|$. This shows the dashed line 1. However, according to the Amonton-Coulomb law, for $S\ge f(P-\frac{35Gr_0}{8b(1+\kappa)})$ sliding starts in the vicinity of the point $\bar{x}=0$ (see the dashed lines 2, 3).

Now we pass to the main contact problem involving sliding in the unknown region $|x|\le c$ for loads:

$$S\ge f\left(P-\frac{35Gr_0}{8(1+\kappa)b}\right).$$

The governing singular integral equation (17) for the unknown derivative of function $U'(x)$ has the form:

$$\frac{1}{\pi}\int_{-c}^{c}\frac{U'(t)dt}{t-x}=\frac{1+\kappa}{2G}(S-fP)+$$
$$+\frac{7r_0 f}{b}\left(\frac{5}{16}-\frac{15}{8}\frac{x^2}{b^2}+\frac{5}{2}\frac{x^4}{b^4}-\frac{x^6}{b^6}\right),|x|<c. \quad (21)$$

The desired solution is:

$$U(x)=\sqrt{c^2-x^2}\left[-\frac{1+\kappa}{2G}(S-fP)+\frac{fr_0}{b}\left(\frac{5}{16}\frac{c^6}{b^6}-\frac{21}{16}\frac{c^4}{b^4}+\right.\right.$$
$$+\frac{35}{16}\frac{c^2}{b^2}-\frac{35}{16}++\frac{3}{8}\frac{c^4}{b^4}\frac{x^2}{b^2}+\frac{1}{2}\frac{c^2}{b^2}\frac{x^4}{b^4}+\frac{x^6}{b^6}-\frac{7}{4}\frac{c^2}{b^2}\frac{x^2}{b^2}+$$
$$\left.\left.+\frac{35}{8}\frac{x^2}{b^2}-\frac{7}{2}\frac{x^4}{b^4}\right)\right],\ |x|<c. \quad (22)$$

From the equation (19), having now the form:

$$S-fP+\frac{35Gr_0 f}{8b(1+\kappa)}-\frac{14Gr_0 f}{(1+\kappa)b}\left(\frac{15}{16}\frac{c^2}{b^2}-\frac{15}{16}\frac{c^4}{b^4}+\frac{5}{16}\frac{c^6}{b^6}\right)=0,$$

we find the half-length of the slip zone – parameter c:

$$c=b\sqrt{1-\sqrt[3]{\frac{8(1+\kappa)b}{35Gfr_0}(fP-S)}} \quad (23)$$

and then the tangential contact stresses are:

$$\tau_{xy}^{-}(x,0)=fP+\frac{14Gr_0 f}{(1+\kappa)b}\left(-\frac{5}{16}+\frac{15}{8}\frac{x^2}{b^2}-\frac{5}{2}\frac{x^4}{b^4}+\frac{x^6}{b^6}\right),\quad |x|\le c,$$

$$\tau_{xy}^{-}(x,0)=fP+\frac{14Gr_0 f}{(1+\kappa)b}\left(-\frac{5}{16}+\frac{15}{8}\frac{x^2}{b^2}-\frac{5}{2}\frac{x^4}{b^4}+\frac{x^6}{b^6}\right)+$$
$$-\frac{7Gr_0 f}{(1+\kappa)b}\left(\frac{15}{4}-\frac{5}{2}\frac{c^2}{b^2}-5\frac{x^2}{b^2}+\frac{3}{4}\frac{c^4}{b^4}+\frac{c^2}{b^2}\frac{x^2}{b^2}+2\frac{x^4}{c^4}\right)\times$$
$$\times\left|\frac{x}{b}\right|\sqrt{\frac{x^2}{b^2}-\frac{c^2}{b^2}},\quad |x|>c. \quad (24)$$

By assuming the threshold value $S=fP$ we have $c=b$ and the shear contact stresses become:

$$\tau_{xy}^{\pm}(x,0)=fP+\frac{14Gfr_0}{(1+\kappa)b}\left(\frac{x^6}{b^6}-\frac{5}{2}\frac{x^4}{b^4}+\frac{15}{8}\frac{x^2}{b^2}-\frac{5}{16}\right),\quad |x|\le b,$$

$$\tau_{xy}^{\pm}(x,0)=fP+\frac{14Gfr_0}{(1+\kappa)b}\left(\frac{x^6}{b^6}-\frac{5}{2}\frac{x^4}{b^4}+\frac{15}{8}\frac{x^2}{b^2}-\frac{5}{16}+\right.$$
$$\left.-\left|\frac{x}{b}\right|\left(\frac{x^2}{b^2}-1\right)^{5/2}\right),\quad |x|>b. \quad (25)$$

Since the condition $\tau_{xy}=f|\sigma_y|$ is satisfied at any point of the contact region, we have the case of sliding friction.

The results of numerical calculations are performed for the following dimensionless parameters:

$\bar{x}=x/b$, $\bar{r}_0=r_0/b$, $\bar{c}=c/b$, $\bar{U}=U/b$, $\bar{\sigma}_y=\sigma_y/G$, $\bar{\tau}_{xy}=\tau_{xy}/G$, $\bar{P}=P/G$, $\bar{S}=S/G$ and $f=0.1$, $\bar{r}_0=10^{-4}$, $\nu=0.2$.

Fig. 4 shows the distributions of the relative tangential shift of bodies boundaries $\bar{U}$ in the slip zone under the pressure $\bar{P}=2\cdot 10^{-4}$ for the following values of the shear tractions $\bar{S}$:$1-\bar{S}=10^{-5}$; $2-\bar{S}=1{,}4\cdot 10^{-5}$; $3-\bar{S}=1{,}8\cdot 10^{-5}$; $4-\bar{S}=2\cdot 10^{-5}$. The maximum value of the modulus of the relative tangential shift of boundaries increases with the shear tractions and reaches in the centre of the recess.

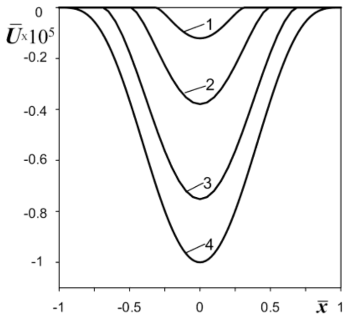

Fig. 4. Distributions of the relative tangential shift of bodies boundaries

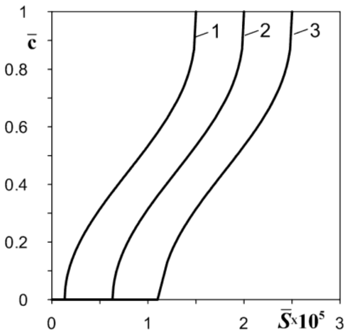

Fig. 5. Half-length of the slip zone versus the external shear forces

The nonlinear dependence of the half length of the slip size $\bar{c}$ on the shear forces $\bar{S}$ for some values of the pressure $\bar{P}$: $1-\bar{P}=1{,}5\cdot10^{-4}$; $2-\bar{P}=2\cdot10^{-4}$; $3-\bar{P}=2{,}5\cdot10^{-4}$ is shown in Fig. 5. The horizontal straight lines of the plots correspond to the case of the stick contact of the bodies. We can see that the zone of sliding becomes greater with increasing shear forces.

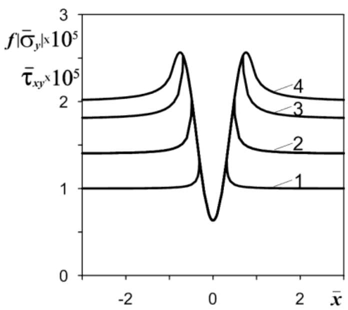

Fig. 6. Contact stresses in the stick-slip problem

Symmetric distributions of stresses for illustrating the behavior of the slip-stick contact are demonstrated in Fig. 6. A graph of $f|\bar{\sigma}_y|$ (curve 4) and graphs of tangential stresses $\bar{\tau}_{xy}$ (curves 1, 2, 3) versus $\bar{x}=x/b$ are given under the pressure $\bar{P}=2\cdot10^{-4}$ for the following values of the shear forces $\bar{S}$:$1-\bar{S}=10^{-5}$; $2-\bar{S}=1{,}4\cdot10^{-5}$; $3-\bar{S}=1{,}8\cdot10^{-5}$. It is seen that the normal stresses have a global maximum near the edges of the recess at the points $\bar{x}=\mp0{,}76$. The maximum of tangential stresses is reached at the ends of the region of sliding. Moreover, the curves coincide in the slip zone $(-\bar{c},\bar{c})$. Outside this interval, the tangential stresses $\bar{\tau}_{xy}$ are less than $f|\bar{\sigma}_y|$ and monotonically decrease to the limiting values at infinity.

6. CONCLUSIONS

In the paper we have investigated the complete frictional contact of two half-planes containing local geometric perturbation of boundaries accounting for frictional slip under sequential remote normal and shear forces. The formulated stick-slip contact problem is reduced to the singular integral equation for the function of the relative tangential shift of bodies boundaries which is next solved with the determination of the size of sliding. On the basis of the analytical solution to the above-mentioned problem the dependences of slip zone length and contact stresses on applied loadings are analyzed.

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