NUMERICAL ANALYSIS OF CROSSWISE HETEROGENEOUS COVERING STRUCTURES IN 3D CLASS STRUCTURE CONDITIONS

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Summary: The following paper presents the results of analyses of multi-layered elements and thick constructions, as well as simplifications used for solving structures of 2D class models published in specialist literature, and compares them with a different approach involving generalization of pertinent problems into 3D classes. An error estimation method was proposed, together with a procedure of shaping grid's density ensuring necessary computing precision. Solving huge sets of equations allowed for practically continuous values of complex functions of stress states. Several of the presented typical examples indicate the possibility of applying the algorithms, among others, to heterogeneous structures of reinforced concrete constructions.

1. JUSTIFICATION FOR THE PROCESSES OF CHANGES IN THE ANALYSIS OF TYPICAL 2D STRUCTURES

The authors of the present paper aim at presenting an algorithm that could be practically applied in the analysis of complex engineering constructions, or their untypical integral parts. The analysis processes can be useful for studying rigidity and effort in constructing crosswise heterogeneous coverings, as well as for examination of changes in physical properties and mass of materials, and shape of coverings of any given architectural forms. An algorithm for coverings of considerable thickness, for instance thick slabs, turns out to be analogous to another algorithm used to describe multilayered surface coverings with small or huge rises. The problem of approximation of multilayered sandwich-type coatings was only painstakingly solved in Marcinkowski, (2003), despite assumed simplifications such as crosswise symmetry of structure (Fig. 1). By deforming normal, multi-node regular elements, new elements in curvilinear coordinate system were obtained.

It seems that this operation could yield desirable results only for thin-layered coverings of small elevation; but overall, practical value of the solution is evaluated as dissatisfactory. For instance, it is by no means practical nor justified to analyze and design covers of thickness t = 0,635mm with core thickness h_r = 11,430 mm, taking into account production process measurement toleration of any construction materials, as well as their thickness regulation methods.

Moreover, the accepted assumption that thickness of the layers in the covering surface can be achieved with tolerance t = ±0,0001mm is purely academic. Be that as it may, the methods employed for analyzing architectural forms of elevated coverings, such as multi-curve covering of a large spread shown in Fig. 2, based on (Noor and Kim, 1996), are indeed worthwhile and call for further investigation. A covering of any given thickness can be thus studied with well-founded claims for practical applicability of the achieved solutions, even in the cases where variable mass of construction material is an essential and indispensable factor. The authors of the present paper wish to present the analysis algorithms proving positive aspects of analyzing constructions freely heterogeneous in their structure by changing the discretization class from 2D to 3D.

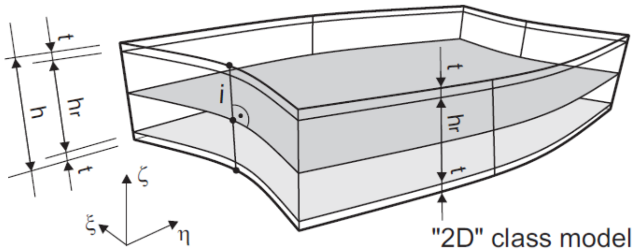

Fig. 1. Scientific interpretation of symmetrically layered sandwich type coating

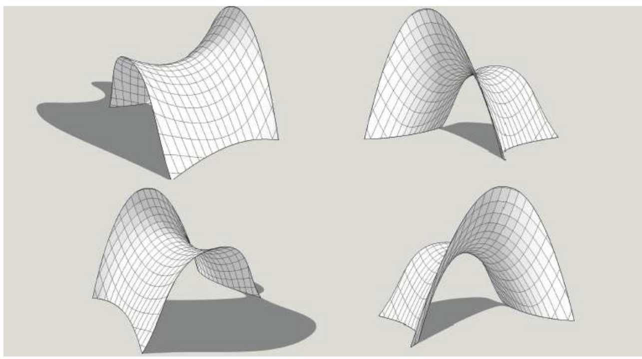

Fig. 2. An elevated covering of a large spread

2. CROSSWISE DISCRETIZATION OF ELEMENTS IN THE PERSPECTIVE OF VARIABLE ANALYSIS PARAMETERS

While the problem of developing methods for analysis of spatial elements remains still relevant, the number of publications offering practically applicable solutions and algorithms is relatively small. Among others, the issue of spatial analysis was addressed in Michalczuk and Tribiłło, (2002). The crucial factor determining the value of results obtained for crosswise heterogeneous structures is the node density in the grid discretizing thickness $h(z)$ of an element. It seems reasonable to generate the grid with constant node distance h_i; accordingly, relying on the fact that $h(z) = h_i \cdot s$, the tests could be created to estimate the discretization error of, among others, a circular slab freely supported along its circumference (Fig. 3). By acting on the middle surface in the layer $h(z) = h_i \cdot (s/2)$ with the load q, a classical closed solution is obtained and the acceptable error margin Δw_d can be established during the discretization to suit the practical applicability of the expected results (Vilberg and Abdulwahab, 1997). For the established error $\Delta w_d \leq 0{,}01 \cdot (gb^4)(64D^2)$, the satisfactory precision is achieved with the thickness parameter $s = 20$, which is proved by the results in Tab.1, and illustrated by the function drawn in Fig. 4.

Essentially, a faultless solution is obtained with the parameter $s = 40$, which is illustrated by the following results:

$\Delta w(\xi{=}0{,}000) = 0{,}001{\cdot}(qb^4)/64D << \Delta w_d$,

$\Delta w(\xi{=}0{,}133) = 0{,}001{\cdot}(qb^4)/64D << \Delta w_d$,

$\Delta w(\xi{=}0{,}583) = 0{,}000{\cdot}(qb^4)/64D << \Delta w_d$,

$\Delta w(\xi{=}0{,}917) = 0{,}000{\cdot}(qb^4)/64D << \Delta w_d$.

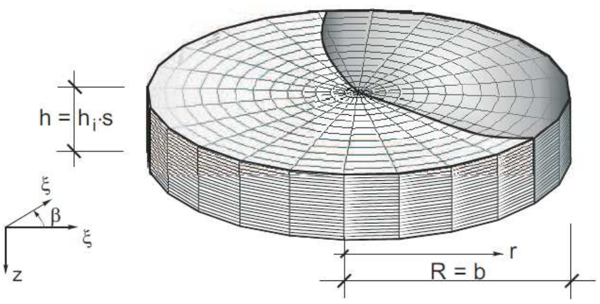

Fig. 3. Discrete model of a circular slab

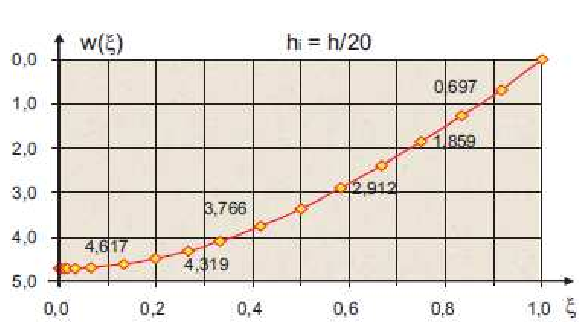

Fig. 4. Results of the solution for the circular slab with the grid density $s = 20$

Tab. 1. Results of the solution for the circular slab with the grid density $s = 20$

ξ = r / R	0,00000	0,00500	0,01000
w (ξ) ·$(qb^4)/64D$	4,720285	4,720138	4,719698
ξ = r / R	0,01667	0,03333	0,06667
w (ξ) ·$(qb^4)/64D$	4,718677	4,713799	4,694335
ξ = r / R	0,13333	0,20000	0,26667
w (ξ) ·$(qb^4)/64D$	4,616848	4,491538	4,318911
ξ = r / R	0,33333	0,41667	0,50000
w (ξ) ·$(qb^4)/64D$	4,100396	3,765554	3,367379
ξ = r / R	0,58333	0,66667	0,75000
w (ξ) ·$(qb^4)/64D$	2,911802	2,406222	1,859450
ξ = r / R	0,83333	0,91667	1,00000
w (ξ) ·$(qb^4)/64D$	1,280466	0,696812	0,00000

3. GENERALIZATION OF A CROSSWISE HETEROGENEOUS STRUCTURE

The principal idea of the generalization involves analyzing a crosswise heterogeneous covering structure made of various materials M_j, $j = 1, 2, 3, \ldots$, each of them having different density, elasticity, and boundary conditions "BC". This kind of approach is graphically presented in Fig. 5.

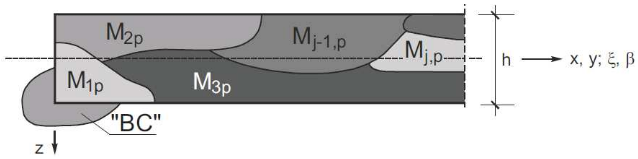

Fig. 5. Generalized structure of a crosswise heterogeneous covering "p" composed of materials M_j

When performing the discretization of a spatial figure of a covering, its construction materials can be interpreted as functions:

$$M_{vj} = M_j(\rho, g, E, \nu)_j \tag{1}$$

of the parameters: ρ_j – material's density; g_z – acceleration resulting from the inertia field in the Z-axis direction, E_j – material's modulus of elasticity, ν_j – Poisson number.

For static problems, the force-balance equations in the nodes consider also gravitational forces, represented by the product of density ρ and acceleration g_z, and marked with the $\mathbf{M_S}$ vector. The elastic features of masses can be expressed by a complex function which takes into account, for instance, material's strength. This is shown by the relation

$$E = F(f_c), \tag{2}$$

f_c denoting here material's compressive stress resistance.

The relation (2) has not been widely studied, but for its interpretation the formula

$$F(f_c) = 9{,}5(f_{ck} + 8)^{1/3} = E_{cm}\,, \tag{3}$$

can be accepted; incidentally, the formula describes the average secant modulus of elasticity of concrete often used in designing processes. Such relationships from the field of concrete mechanics can be employed in numerical analysis algorithms. The value of function (2) can be constant for a wide range of materials. As an example of unchangeable modulus of elasticity E_S, steel of classes A-0, A-I, A-II, A-III, A-IIIN produced in Poland can be mentioned.

Algorithms of numerical analysis of 3D elements formulated in (Michalczuk and Tribiłło, 2002) are used in a generalized form, specifying the approximate norm criteria allowing construction design, usually taking values of generalized *M*, *N*, *Q* type forces or the criteria justified by the stress conditions σ_i = 1, 2, 3, in the three-dimensional space.

Empirical and hypothetical norm stress distribution used to determine cross-section load-bearing capacity can be eliminated by solving the mathematical model taking into account function (1) only. From among several analyzed models, the results of the analysis for the slab shown in Fig. 3 are presented, taking advantage of the previously documented calculations of grid density which would allow for elimination of discretization errors.

The algorithm used with the model is similar for any given coordinate system, and the problem is solved by interpreting stresses in a mixed coordinate system (ξ, β, z). A discretized model of the slab is formed as shown in Fig. 3, with the stress $q(\xi, \beta)$ applied to surfaces of discrete elements of upper layer as can be seen in Fig. 6. The properties of construction materials of the crosswise heterogeneous structure can be expressed for j = 2 as:

$$M_{1p} = M_p(\rho_1, g, E_1, \nu_1)\,, \tag{4}$$

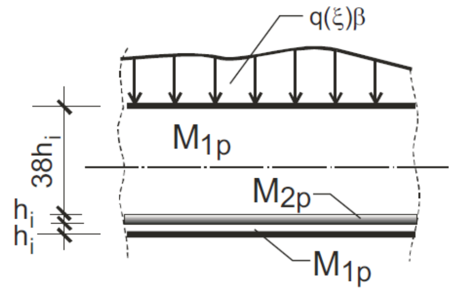

Fig. 6. Section of a crosswise heterogeneous slab with $j = 2$

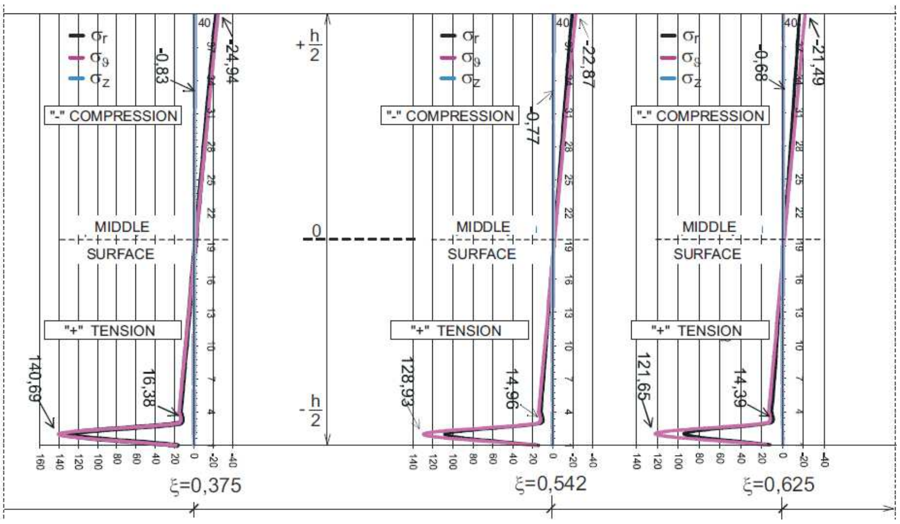

Fig. 7. Stress analysis results $\sigma_r(\xi)$, $\sigma_v(\xi)$, $\sigma_z(\xi)$ in the parameter structure (4) with $j = 2$

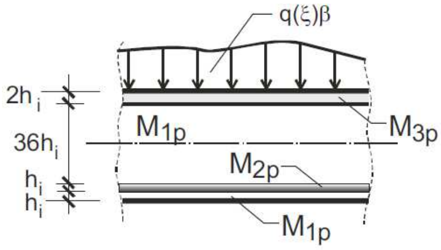

Fig. 8. Crosswise heterogeneous slab with $j = 3$

The equations are solved by calculating stresses σ_r, σ_v, σ_z, in units (q), and the solution is found within $0 \leq \xi \leq 1$ interval for any given value of R. It is assumed that $g = 10\text{m/s}^2$; $\nu_1 = \nu = 0{,}2$; $k = 1$; $E_1 = E$; $c = 8$; $a = 3$; $\rho_1 = \rho$; $\rho_2 = 3\rho$; $q(\xi, \beta) = q$.

The actual stress distribution seems to be different from the empirical norm assumptions in reinforced concrete constructions; this conclusion is further substantiated by the stress functions as drawn in Fig. 7.

From the technological point of view, there is a common problem of designing layered-concrete slabs which are constrained by concerns related to use of erected constructions and not infrequently to transport systems. The mathematical model for the parameter j = 3 clearly justifies non-linear stress distributions in a crosswise heterogeneous structure (Jing Liu and Forster, 2000). Accordingly, the slab presented in Fig. 6 can be augmented with the material M_{3p} = M_p (1,5ρ_j,g,2E,ν) as illustrated in Fig. 8. Practical aspects of the model construction and the parameters M_{3p} are formulated following contemporary, realistic criteria of development of materials technology. The curves of non-linear functions of local stress distributions have increased; values of the function are shown in Fig. 9. The models generated with a grid density s = 40 ensure the positive evaluation of discretization criteria for j = 3, and the grids with higher density are proper for solving problems where j > 3.

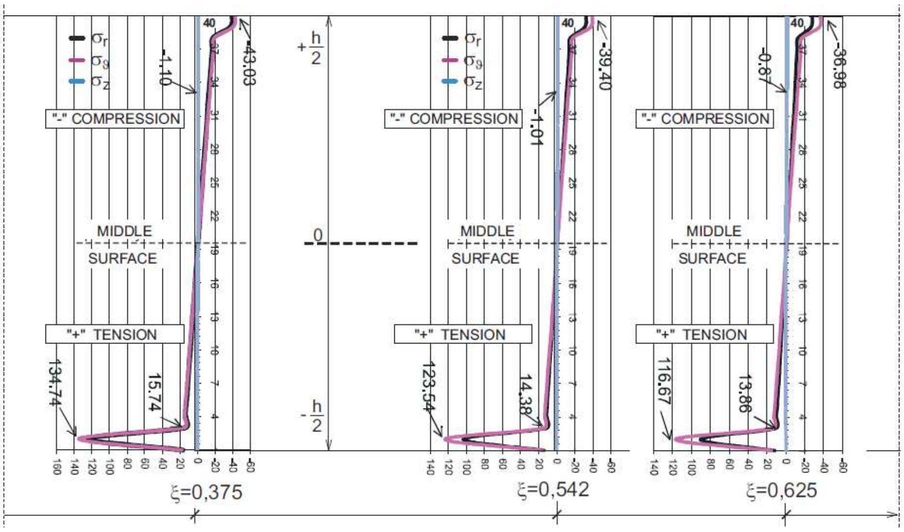

Fig. 9. Stress analysis results $\sigma_r(\xi)$, $\sigma_v(\xi)$, $\sigma_z(\xi)$ in structure (4) augmented with parameters M_{3p}, j $= 3$

The properly-designed and dense enough grid constitutes a practical justification for the inseparability of deformation and tension, and ensuring the force-balancing conditions in several thousands of the spatial grid nodes is, in engineering, a strong basis for practical interpretation of results in terms of continuous functions (Lo, 1985, 1988).

4. SUMMING-UP

The results published in the present paper confirm the claim that analysis of layered elements by changing the 2D class models into the appropriate 3D class models can be effective. The authors have backed up their assumptions with convincing calculations conducted for crosswise heterogeneous structures. The most crucial aspect of the analysis involved obtaining correct and sufficiently precise stress distribution functions in materials of various parameters and elastic properties, also accounting for density of the medium and characteristics of gravitational field. These non-linear functions can lay mathematical foundations for determining of limited load-bearing capacity of the analyzed section, which is often contradictory with empirical criteria used ,for instance, in analysis of atypical, reinforced concrete elements, particularly under complex stress conditions. Contemporary progress in computer technology justifies the change in the point of view on reduction of the model class. The generalizations allowing for conducting of both synthesis and analysis processes with algorithms of transition from 2D into 3D classes involve employing algorithms for solving huge sets of equations.

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ANALIZA STRUKTUR POPRZECZNIE NIEJEDNORODNYCH W WARUNKACH ZMIENNYCH PARAMETRÓW MATERIAŁÓW KONSTRUKCYJNYCH

Streszczenie: Na tle analizowanych w literaturze technicznej elementów warstwowych a także konstrukcji grubych i wykorzystania uproszczeń umożliwiających rozwiązanie struktur modeli klas 2D, opublikowano wyniki rozwiązań uzyskiwanych odmiennie przez uogólnienie problemów do klas 3D. Wskazano na technikę szacowania błędu i kształtowania gęstości siatki umożliwiającej uzyskanie żądanej, uzasadnionej technicznie dokładności obliczeń. W publikowanych kryteriach rozwiązanie wielkich układów równań umożliwiło uzyskanie praktycznie ciągłych wartości funkcji złożonych stanów naprężeń. Zamieszczone niektóre charakterystyczne przykłady wskazują na możliwość wykorzystania algorytmów na przykład w niejednorodnych strukturach konstrukcji żelbetowych.