COMPRESSION OF TWO ROLLERS IN SHEET-FED OFFSET PRINTING MACHINE

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Abstract: The most important units of sheet-fed offset printing machine, like the ink and dampening systems as well as a printing unit, are composed, in the main, of contacting rollers of various sizes (in case of the printing unit they are named cylinders). Adequate setting of the said rollers is very important, because it has big influence on quality of print-outs. The settings are made experimentally by measuring the width of the contact area in the ink and dampening systems or by computing the clamp parameters – in the printing unit. This paper includes analysis of compression of two rollers depending on a width of the contact area, radiuses of the rollers as well as their Poisson's ratios and Young's modules.

1. INTRODUCTION

Printing unit, ink and dampening systems are composed mainly of rollers and cylinders (Fig. 1). A distinctive and extremely important element is the blanket cylinder which is located between the plate cylinder and the impression cylinder. There has been fixed a rubber blanket on a blanket cylinder for the purpose of better conveying the image from metal plate with ink on paper.

The printing unit includes 3 cylinders: plate cylinder, blanket cylinder and impression cylinder (Dejidas and Destree, 2007; Kipphan, 2001). In this part of machine there is a contact between a metal plate fixed onto the plate cylinder and a rubber blanket fixed onto the blanked cylinder as well as between the rubber blanket and the metal impression cylinder.

Plate cylinders are in contact with ink form rollers and with the dampening form roller. Soft – coated with rubber or other artificial materials and hard – metal rollers inside ink unit are in contact, thus being adequately adherent to each other. In the dampening unit, soft – coated with rubber, paper or fabric and hard – metal rollers are in contact alternately.

Setting an inadequate stress between cylinders of the printing unit and rollers of the ink and dampening systems affects 3 aspects of printings, namely: print-outs quality, wear and tear of machine elements and reduction in time of making the printing machines ready for operation. The elimination of printing errors which are observed on print-outs at the beginning and in the course of printing, extends time of making the printing machines ready for operation, increases consumption of waste paper and ink.

Uneven stress between rollers of ink and dampening systems may result in irregular ink and water transmission. Too big stress between ink rollers cause to excessive heat and rubber expansion.

Setting too big stress between ink form rollers and plate cylinder results in bouncing of rollers each from other while their conveying above the channel of the plate cylinder and stroking the front edge of plate. As a result, the ink thickness on plate is changed. It brings about generating smudges on print-outs, faster wear and tear of plate, too much tone value increase. Uneven setting of stress between ink form rollers can cause smudges as well.

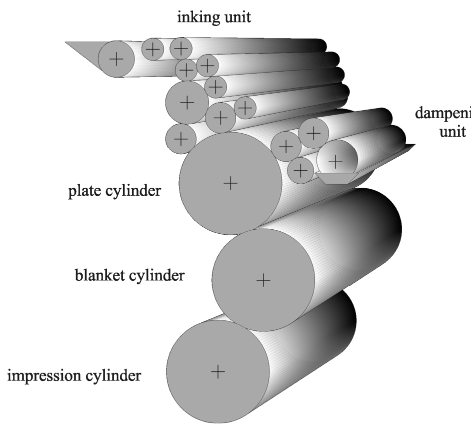

Fig. 1. An exemplary printing unit, ink and dampening systems

Too small stress in the dampening system results in transmission of too much amounts of water on plate. It can cause too much amount of water on plate and ink emulsification. It can involve problems as regards ink drying and ink adhering to ink rollers. In turn, too big stress makes water squeezed from the rollers and on the plate there is to small water film. In the dampening system, stress between the dampening form roller and the distributing roller as well as between the dampening form roller and the plate cylinder should be big enough to distribute water and to quicken the dampening form roller with the distribution roller.

For determining an adequate contact between the rollers and cylinders there is not measured or computed the stress. Printing operators check the stress in the ink and dampening systems regularly with foil stripes which they put in between the rollers and, next, take them out. In the ink system they measure the width of the contact zone between the rollers. As regards the printing unit, a clamp is computed on the basis of height of the plate over bearer rings (hardened metal rings located at the ends of the two cylinders) of the plate cylinder and the height of rubber over bearer rings of the blanket cylinder.

2. STATEMENT OF THE PROBLEM

It becomes apparent that the most essential elements of sheet-fed offset printing machine are rollers which remain in mutual contact (Fig. 2). The contact of two cylinders, the axes of which were compressed to distance d due to unknown vertical compressing forces P, was considered. The contact area has a rectangular shape with unknown width *2a*. A contact stress *p(x)*, which takes place in the contact area $x \in (-a, a)$, is a symmetric, although unknown, function and $p(a) = p(-a) = 0$:

$$\int_{-a}^{a} p(x)dx = \mathrm{P} \tag{1}$$

We assume that in rollers a plane strain takes place which is independent on variable y.

Boundary condition for the contact of the two rollers is determined as follows (Jonson, 1985):

$$u_z^2 + u_z^1 = d_2 + d_1 - f_2(x) - f_1(x)\,,\ x \in (-a, a) \tag{2}$$

where: u_z^1, u_z^2 – displacement of the points located on the contact surfaces of, accordingly, body 2 and body 1 alongside with axis *z*, this displacement being assumed positive, d_2, d_1 – indentation of, accordingly, body 2 and body 1 under loading, $f_1(x) = 0{,}5x^2/R_1$, $f_2(x) = 0{,}5x^2/R_2$ – equations for surface of body 1 and body 2 before indentation.

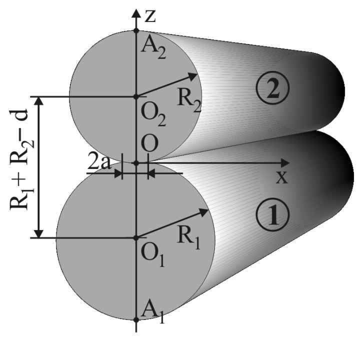

Fig. 2. Two contacting rollers

In the assumption of Herz's conditions (Timoshenko, Goodier, 1962), the problem is to solve the issue of half-space. Assuming that displacement does not depend on the direction *y*, the equation of the theory of elasticity for displacements (Lame) shall be as follows (Nowacki, 1970):

$$\begin{cases} (\lambda + 2\mu)\dfrac{\partial^2 U_x(x,z)}{\partial x^2} + \mu\dfrac{\partial^2 U_x(x,z)}{\partial z^2} + (\lambda + \mu)\dfrac{\partial^2 U_z(x,z)}{\partial x \partial z} = 0 \\ (\lambda + 2\mu)\dfrac{\partial^2 U_z(x,z)}{\partial z^2} + \mu\dfrac{\partial^2 U_z(x,z)}{\partial x^2} + (\lambda + \mu)\dfrac{\partial^2 U_x(x,z)}{\partial z \partial x} = 0 \end{cases} \tag{3}$$

where: λ, μ – Lame parameters, $U_z(x,z), U_x(x,z)$ – displacement alongside with axes *z* and *x*.

The boundary conditions for the considered problem are as follows:

$$\sigma_{zz}(x,z)\,|_{z=+0} = \begin{cases} -p(x), & |x| < a \\ 0, & |x| > a \end{cases} \tag{4}$$

$$\sigma_{xz}(x,0)\,|_{z=+0} = 0\,,\ -\infty < x < \infty \tag{5}$$

$$\sigma_{zz}(x,z)\,|_{z\to\infty} = 0\ \ ,\ \sigma_{xz}(x,0)\,|_{z\to\infty} = 0 \tag{6}$$

where: σ_{zz}, σ_{xz} – normal and shearing stresses:

$$\sigma_{zz} = (\lambda + 2\mu)\frac{\partial u_z}{\partial z} + \lambda\frac{\partial u_x}{\partial x} \tag{7}$$

$$\sigma_{xz}(x,z) = \mu\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \tag{8}$$

3. SOLUTION OF THE PROBLEM

The problem (3)-(6) is solved by using Fourier integral transform (Nowacki, 1970). For $U_x(x,z)$ and $U_z(x,z)$, the following equations are obtained:

$$U_x(x,z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{U}_x(\xi,z)e^{-i\xi x}d\xi \tag{9}$$

$$U_z(x,z) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\tilde{U}_z(\xi,x)e^{-i\xi z}d\xi \tag{10}$$

$$\begin{aligned} \tilde{U}_x = &-i\left[\left(\frac{(\lambda+3\mu)}{(\lambda+\mu)\xi} + z\frac{|\xi|}{\xi}\right)B + \frac{|\xi|}{\xi}A\right]e^{|\xi|z} - \\ &-i\left[\left(\frac{(\lambda+3\mu)}{(\lambda+\mu)\xi} - z\frac{|\xi|}{\xi}\right)D - \frac{|\xi|}{\xi}C\right]e^{-|\xi|z} \end{aligned} \tag{11}$$

$$\tilde{U}_z = (A + zB)e^{|\xi|z} + (C + zD)e^{-|\xi|z} \tag{12}$$

where: $A(\xi), B(\xi), C(\xi), D(\xi)$ resulted from the four conditions namely (4)-(6).

The final solution of the problem (3)-(6) is as follows:

$$U_x(x,z) = \frac{1+\nu}{E\sqrt{2\pi}}\left(\frac{2xz}{r^2} - (1-2\nu)2arctg\frac{x}{z}\right) * p(x) \tag{13}$$

$$U_z(x,z) = -\frac{2(1-\nu^2)}{E\sqrt{2\pi}}\left(\frac{z^2}{(1-\nu)r^2} + \ln(e^{C_0}r^2)\right) * p(x) \tag{14}$$

where: $r^2 = x^2 + z^2$, ν – Poisson's ratio, E – Young's modulus, "*" – the convolution of function

$$g(x) * \varphi(x) = \int_{-a}^{a} g(x-s)\varphi(s)ds\,. \tag{15}$$

and the following relationships were taken into account:

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}\,,\ \ \mu = \frac{E}{2(1+\nu)} \tag{16}$$

The constant C_0 is obtained from the condition $U_z(0,R)=0$. Substitution of normal displacement of half-space's edges $U_z^l(x,0)=u_z^l$, $l=1,2$ in the boundary condition (2) led to the integral equation for a contact stress function $p(x)$. The solution for the considered issue is as follows (Jonson, 1985):

$$p(x)=\frac{aE_0}{2R_0}\sqrt{1-\frac{x^2}{a^2}} \qquad (17)$$

where:

$$\frac{1}{R_0}=\frac{1}{R_1}+\frac{1}{R_2}\,,\ \frac{1}{E_0}=\eta_1+\eta_2\,,\ \eta_l=\frac{1-\nu_l^2}{E_l}\,,\ l=1{,}2 \qquad (18)$$

Substituting (17) in the equation (1) led to the equation of contact area width $2a$ (Jonson, 1985):

$$\mathrm{P}=\frac{\pi a^2 E_0}{4R_0} \qquad (19)$$

The final equation for compression of the two rollers $d=d_1+d_2$ is as follows:

$$d=\frac{\mathrm{P}}{\pi}\left[\eta_1\ln\left(e^{m_1}\frac{4R_1^2}{a^2}\right)+\eta_2\ln\left(e^{m_2}\frac{4R_2^2}{a^2}\right)\right] \qquad (20)$$

where: a – half of the contact zone width, R_1, R_2 – radius accordingly of upper roller and lower roller, v_1, v_2 – Poisson's ratio accordingly of upper and lower rollers, E_1, E_2 – Young's modulus accordingly of upper and lower rollers and

$$m_l=-\frac{\nu_l}{1-\nu_l}+\frac{1}{1-\nu_l}\frac{a^2}{4R_l^2}\,,\ l=1{,}2 \qquad (21)$$

Stresses placed near the contact area resulted from equations known from the literature (Jonson, 1985).

Analogical equation for contact of two spheres has been present in literature for quite a long time (Johnson, 1985; Popov, 2010; Timoshenko and Goodier, 1962).

4. NUMERICAL ANALYSIS AND DISCUSSION

The literature provides for various experimental researches and equations on compression of cylinders. Many divergences can be avoided if loading areas are defined accurately. Fig. 2 shows points O, O_1, O_2, A_1, A_2 where cylinders may be loaded. If cylinders are loaded in the points A_1, A_2 the coefficient m_l equals $m_l=\ln 4-1$ (Loo, 1958; Jonson, 1985; Zhuravlev, Karpenko, 2000). The last two of these authors used the solution consisting in compression of the cylinders by two forces (Muskhelishvili, 1963) and then they obtained compression for any radii. The papers (Birger, Panovko, 1968; Jarema, 2006) shows the coefficient $m_1=m_2=0{,}814$, which is most often used. But in reality this value of coefficients m_l takes place only for $v_1=v_2=0{,}3$. The authors (Zhuravlev, Karpenko, 2000) paid attention to this fact. Generally, the coefficient m_l equals $m_l=\ln 4-1+v_l(1-v_l)$.

If cylinders are loaded in the points O_1, O_2, coefficient m_l for $\eta_1=\eta_2$ equals $m_1=m_2=2/3$ (Chandrasekaran, 1987). The same coefficient m_l is given in the papers (Dinnik, 1952; Galin, 1976), where none of the said papers includes correct citation.

Using equations (19) and (20) we will come to dimensionless relation between compression of rollers $d_*=d/R_2$ and the contact area width $a_*=a/R_2$.

$$d_*=\frac{a_*^2}{2}\frac{R_*+1}{R_*(\eta_*+1)}\left[\eta_*\ln\left(e^{m_1/2}\frac{2R_*}{a_*}\right)+\ln\left(e^{m_2/2}\frac{2}{a_*}\right)\right] \qquad (22)$$

where $\eta=\eta_1/\eta_2$, $R_*=R_1/R_2$, as well as to dimensionless relation between compression of rollers $d_*=d/R_2$ and the dimensionless radius $R_*=R_1/R_2$:

$$d_*=\frac{\mathrm{P}_*}{\pi}\left[\eta_*\ln\left(e^{m_1}\frac{\pi R_*(1+R_*)}{(1+\eta_*)\mathrm{P}_*}\right)+\ln\left(e^{m_2}\frac{\pi(1+R_*)}{(1+\eta_*)\mathrm{P}_*R_*}\right)\right] \qquad (23)$$

where $P_*=P\eta_2R_2$.

Fig. 3 shows dependence (23) of dimensionless compression of the distance between axes of cylinders $d_*=d/R_2$, an indentation of lower cylinder $d_1=R_2$ and an indentation of upper cylinder $d_2=R_2$ on the dimensionless radius $R_*=R_1/R_2$ for steel cylinders $E_l=2{,}15\cdot 10^5$Mpa, $v_l=0{,}3$, $l=1{,}2$, $P=2{,}15\cdot 10^7$N/m ($P_*=0{,}84\cdot 10^{-2}, \eta_*=1$).

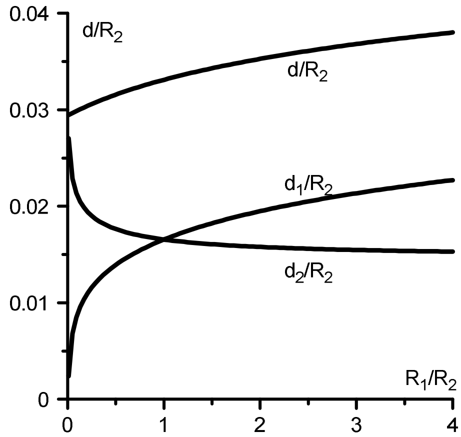

Fig. 3. Dependence of compression of cylinders' axes
$\boldsymbol{d/R_2=d_1/R_2+d_2/R_2}$
and its components on $\boldsymbol{R_*=R_1/R_2}$

From Fig. 3 it can be seen that an increase of cylinder's indentation is on relation in increase of this cylinder's radius. The increase in radius of the bigger cylinder ($R_*>1$) causes the increase of compression $d_*=d/R_2$. The reduction in radius of the smaller cylinder ($R_*<1$) causes reduction of the cylinders' compression $d_*=d/R_2$, although an indentation of the bigger cylinder increases. Reduction in distance between the cylinders depends directly on an increase in the cylinders' loading.

Whenever contact takes place between the steel cylinder (body 1) and the rubber blanket (body 2), $\eta_*=0$ can be assumed. The equation (22) shall then take the following form:

$$d_* = \frac{a_*^2 (R_* + 1)}{2R_*} \ln\left(e^{m_2/2} \frac{2}{a_*} \right) \qquad (24)$$

Fig. 4 shows the obtained dependence between dimensionless compression of cylinders' axes d/R_2 and the dimensionless contact area a/R_2 for various values $\boldsymbol{R_* = R_1/R_2}$.

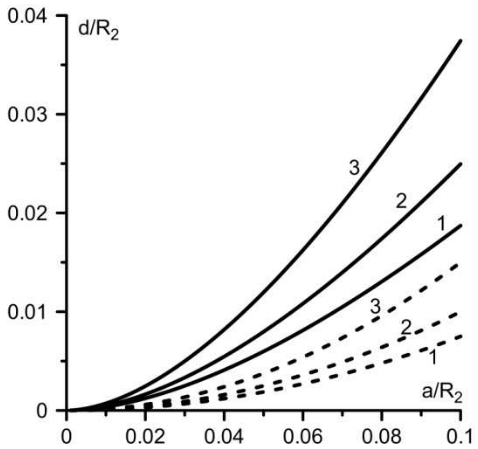

Fig. 4. Dependence of compression of cylinders' axes $\boldsymbol{d/R_2}$ on contact area $\boldsymbol{a/R_2}$. Curve $\boldsymbol{1 - R_* = 2}$, $\boldsymbol{2 - R_* = 1}$, $\boldsymbol{3 - R_* = 0,5}$

Analogical dependence which is used in the printing technical literature (Chehman i inni, 2005) $d/R_2 = 0{,}5 a_*^2 (R_* + 1)/R_*$ is shown in the Fig. 4 as a dashed curve. A significant divergence of results can be seen.

5. CONCLUSSIONS

The obtained equation will be used in printing with the aim to calculate and verify the width of the contact area between the two rollers which remain in contact. Until now, in the printing industry the said contact area used to be determined only experimentally. The equation (20) allows for calculating the width of the contact zone depending on compression of the cylinders' axes, their radii as well as their Poisson's ratios and Young's modulus.

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