GENERAL RESPONSE FORMULA FOR FRACTIONAL 2D CONTINUOUS-TIME LINEAR SYSTEMS DESCRIBED BY THE ROESSER MODEL

Krzysztof ROGOWSKI*

*Phd student, Faculty of Electrical Engineering, Białystok University of Technology, Wiejska 45 D, 15-351 Białystok

k.rogowski@doktoranci.pb.edu.pl

Abstract: A new class of fractional two-dimensional (2D) continuous-time linear systems is introduced. The general response formula for the system is derived using a 2D Laplace transform. It is shown that the classical Cayley-Hamilton theorem is valid for such class of systems. Usefulness of the general response formula to obtain a solution of the system is discussed and illustrated by a numerical example.

1. INTRODUCTION

The most popular models of two-dimensional (2D) linear system are the ones introduced by Roesser (1975), Fornasini and Marchesini (1976, 1978) and Kurek (1985). An overview of 2D linear systems theory is given in (Bose, 1982, 1985; Kaczorek, 1985, 2001; Gałkowski, 2001, Farina and Rinaldi, 2000).

Mathematical fundamentals of fractional calculus and its applications are given in the monographs (Oldham and Spanier, 1974; Nashimoto, 1984; Miller and Ross, 1993; Podlubny, 1999, Ostalczyk, 2008).

The notion of fractional 2D discrete-time linear systems was introduced by Kaczorek (2008a) and extended in (Kaczorek, 2008b, 2009, Kaczorek and Rogowski, 2010, Rogowski, 2011). An overview in state of the art in 1D and 2D fractional systems is given in the monograph (Kaczorek, 2011).

In this paper a new 2D continuous-time fractional Roesser type model will be introduced. The general response formula for the system will be derived using the 2D Laplace transform method (Section 2). Moreover the classical Cayley-Hamilton theorem will be extended to fractional 2D continuous-time systems in Section 3. In Section 4 usefulness of the general response formula to obtaining the solution of the system will be discussed and illustrated by a numerical example. Concluding remarks are given in Section 5.

To the best knowledge of the author 2D continuous-time fractional linear systems have not been considered yet.

2. FRACTIONAL 2D STATE EQUATIONS AND THEIR SOLUTION

Let $R^{n\times m}$ be the set of $n\times m$ real matrices and $R^n := R^{n\times 1}$. The set of nonnegative integers will be denoted by Z_+ and the $n\times n$ identity matrix will be denoted by I_n.

We introduce the following definition of fractional partial derivative of a 2D continuous function $f(t_1,t_2)$ of two independent variables $t_1,t_2\geq 0$.

Definition 1. The α_i order partial derivative of a 2D continuous function $f(t_1,t_2)$ is given by the formula

$$D_{t_i}^{\alpha_i} f(t_1,t_2)=\frac{\partial^{\alpha_i}}{\partial t_i^{\alpha_i}} f(t_1,t_2) = \frac{1}{\Gamma(N_i-\alpha_i)}\int_0^{t_i}\frac{f_{t_i}^{(N_i)}(\tau_i)}{(t_i-\tau_i)^{\alpha_i-N_i+1}}d\tau_i, \quad (1)$$

where $i=1,2$; $N_i-1<\alpha_i<N_i\in N=\{1,2,\ldots\}$, $\alpha_i\in R$ is the order of fractional partial derivative,

$$\Gamma(x)=\int_0^\infty e^{-t}t^{x-1}dt \quad (2)$$

for $x\geq 0$ is the gamma function and

$$f_{t_i}^{(N_i)}(\tau_i)=\begin{cases}\dfrac{\partial^{N_1} f(\tau_1,t_2)}{\partial\tau_1^{N_1}} & \text{for} \quad i=1\\[2ex] \dfrac{\partial^{N_2} f(t_1,\tau_2)}{\partial\tau_2^{N_2}} & \text{for} \quad i=2.\end{cases} \quad (3)$$

Consider the fractional 2D continuous-time system described by the state equations

$$\begin{bmatrix} D_{t_1}^{\alpha_1} x^h(t_1,t_2)\\ D_{t_2}^{\alpha_2} x^v(t_1,t_2)\end{bmatrix} = \begin{bmatrix} A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix} x^h(t_1,t_2)\\ x^v(t_1,t_2)\end{bmatrix}+\begin{bmatrix} B_1\\ B_2\end{bmatrix}u(t_1,t_2), \quad (4a)$$

$$y(t_1,t_2)=C\begin{bmatrix} x^h(t_1,t_2)\\ x^v(t_1,t_2)\end{bmatrix}+Du(t_1,t_2), \quad (4b)$$

where $x^h(t_1,t_2)\in R^{n_1}$, $x^v(t_1,t_2)\in R^{n_2}$ $(n=n_1+n_2)$ are the horizontal and vertical state vectors, respectively, $u(t_1,t_2)\in R^m$ is the input vector, $y(t_1,t_2)\in R^p$ is the output vector and $A_{kl}\in R^{n_k\times l}$, $B_k\in R^{n_k\times m}$ for $k,l=1,2$; $C\in R^{p\times n}$; $D\in R^{p\times m}$.

The boundary conditions for (4) are given in the form

$$x_{t_1}^{h(k)}(0,t_2)=\left[\frac{\partial^k x^h(t_1,t_2)}{\partial t_1^k}\right]_{t_1=0} \tag{5a}$$

for $k=0,1,\dots,N_1-1$ and $t_2 \geq 0$,

$$x_{t_2}^{v(l)}(t_1,0)=\left[\frac{\partial^l x^v(t_1,t_2)}{\partial t_2^l}\right]_{t_2=0} \tag{5b}$$

for $l=0,1,\dots,N_2-1$ and $t_1 \geq 0$.

In the following theorem the Riemman-Liouville formula of fractional integration of a function $f(t)$ will be used (Podlubny, 1999)

$$I_t^{\alpha} f(t)=\frac{1}{\Gamma(\alpha)}\int_0^t f(\tau)(t-\tau)^{\alpha-1}d\tau, \tag{6}$$

where $\alpha>0$ is the fractional (real) order of the integration. Similarly, we may define the 2D fractional integral of function $f(t_1,t_2)$

$$\begin{aligned} I_{t_1,t_2}^{\alpha,\beta} f(t_1,t_2) &= I_{t_1}^{\alpha}\left[I_{t_2}^{\beta} f(t_1,t_2)\right]=I_{t_2}^{\beta}\left[I_{t_1}^{\alpha} f(t_1,t_2)\right] \\ &= \frac{1}{\Gamma(\alpha)\Gamma(\beta)}\times \\ &\int_0^{t_1}\int_0^{t_2}(t_1-\tau_1)^{\alpha-1}(t_2-\tau_2)^{\beta-1} f(\tau_1,\tau_2)d\tau_2 d\tau_1, \end{aligned} \tag{7}$$

where $\alpha,\beta>0$.

Theorem 1. The solution to the equation (4a) with the boundary conditions (5) is given by

$$\begin{aligned} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} &= \sum_{i=0}^{\infty}\sum_{j=1}^{\infty} T_{ij}\left\{\sum_{k=1}^{N_1}\frac{t_1^{k+i\alpha_1-1}}{\Gamma(k+i\alpha_1)} I_{t_2}^{j\alpha_2}\begin{bmatrix} x_{t_1}^{h(k-1)}(0,t_2) \\ 0 \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} I_{t_1,t_2}^{(i+1)\alpha_1,j\alpha_2} u(t_1,t_2)\right\} \\ &+ \sum_{i=0}^{\infty} T_{i0}\left\{\sum_{k=1}^{N_1}\frac{t_1^{k+i\alpha_1-1}}{\Gamma(k+i\alpha_1)}\begin{bmatrix} x_{t_1}^{h(k-1)}(0,t_2) \\ 0 \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} I_{t_1}^{(i+1)\alpha_1} u(t_1,t_2)\right\} \\ &+ \sum_{i=1}^{\infty}\sum_{j=0}^{\infty} T_{ij}\left\{\sum_{l=1}^{N_2}\frac{t_2^{l+j\alpha_2-1}}{\Gamma(l+j\alpha_2)} I_{t_1}^{i\alpha_1}\begin{bmatrix} 0 \\ x_{t_2}^{v(l-1)}(t_1,0) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} I_{t_1,t_2}^{i\alpha_1,(j+1)\alpha_2} u(t_1,t_2)\right\} \\ &+ \sum_{j=0}^{\infty} T_{0j}\left\{\sum_{l=1}^{N_2}\frac{t_2^{l+j\alpha_2-1}}{\Gamma(l+j\alpha_2)}\begin{bmatrix} 0 \\ x_{t_2}^{v(l-1)}(t_1,0) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} I_{t_2}^{(j+1)\alpha_2} u(t_1,t_2)\right\}, \end{aligned} \tag{8}$$

where

$$T_{ij}=\begin{cases} I_n & \text{for } i=0,\ j=0 \\ T_{10}T_{i-1,j}+T_{01}T_{i,j-1} & \text{for } i+j>0, (i,j\in Z_+) \\ 0 \text{ (zero matrix)} & \text{for } i<0 \text{ and/or } j<0 \end{cases} \tag{9}$$

and

$$T_{10}=\begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix},\quad T_{01}=\begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}. \tag{10}$$

Proof. Let $F(p,t_2)$ $(F(t_1,s))$ be the Laplace transform of a 2D continuous function $f(t_1,t_2)$ with respect to t_1 (t_2) defined by

$$\begin{aligned} F(p,t_2) &= \mathcal{L}_{t_1}\left[f(t_1,t_2)\right]=\int_0^{\infty} f(t_1,t_2)e^{-pt_1}dt_1 \\ \left(F(t_1,s)\right. &= \left.\mathcal{L}_{t_2}\left[f(t_1,t_2)\right]=\int_0^{\infty} f(t_1,t_2)e^{-st_2}dt_2\right). \end{aligned} \tag{11}$$

The 2D Laplace transform of $f(t_1,t_2)$ will be denoted by $F(p,s)$ and defined by

$$\begin{aligned} F(p,s) &= \mathcal{L}_{t_1}\left\{\mathcal{L}_{t_2}\left[f(t_1,t_2)\right]\right\}=\mathcal{L}_{t_2}\left\{\mathcal{L}_{t_1}\left[f(t_1,t_2)\right]\right\} \\ &= \mathcal{L}_{t_1,t_2}\left[f(t_1,t_2)\right] \\ &= \int_0^{\infty}\int_0^{\infty} f(t_1,t_2)e^{-pt_1-st_2}dt_1 dt_2. \end{aligned} \tag{12}$$

Applying (12) to (1) for $i=1$ and taking into account that (Kaczorek, 2011)

$$\mathcal{L}_{t_1}\left[t_1^{\alpha}\right]=\frac{\Gamma(\alpha+1)}{p^{\alpha+1}} \tag{13}$$

and

$$\mathcal{L}_{t_1}\left[f_{t_1}^{(N)}(t_1)\right]=p^N F(p,t_2)-\sum_{k=1}^{N}p^{N-k}f_{t_1}^{(k-1)}(0,t_2) \tag{14}$$

for $N=0,1,\dots$; we obtain

$$\begin{aligned} \mathcal{L}_{t_1,t_2}\left[D_{t_1}^{\alpha_1} f(t_1,t_2)\right] &= \mathcal{L}_{t_2}\left\{\mathcal{L}_{t_1}\left[D_{t_1}^{\alpha_1} f(t_1,t_2)\right]\right\} \\ &= \frac{1}{\Gamma(N_1-\alpha_1)}\mathcal{L}_{t_2}\left\{\mathcal{L}_{t_1}\left[\int_0^{t_1}\frac{f_{t_1}^{(N_1)}(\tau_1)}{(t_1-\tau_1)^{\alpha_1-N_1+1}}d\tau_1\right]\right\} \\ &= \frac{1}{\Gamma(N_1-\alpha_1)}\mathcal{L}_{t_2}\left\{\mathcal{L}_{t_1}\left[t_1^{N_1-\alpha_1-1}\right]\mathcal{L}_{t_1}\left[f_{t_1}^{(N_1)}(\tau_1)\right]\right\} \\ &= p^{\alpha_1}F(p,s)-\sum_{k=1}^{N_1}p^{\alpha_1-k}F_{t_1}^{(k-1)}(0,s), \end{aligned} \tag{15}$$

where

$$F_{t_1}^{(k)}(0,s)=\mathcal{L}_{t_2}\left\{\left[\frac{\partial^k f(t_1,t_2)}{\partial t_1^k}\right]_{t_1=0}\right\} \tag{16}$$

for $k=0,1,\dots$.

Similarly, for $i = 2$ in (1) we have

$$\mathcal{L}_{t_1,t_2}\left[D_{t_2}^{\alpha_2} f(t_1,t_2)\right] = s^{\alpha_2} F(p,s) - \sum_{l=1}^{N_2} s^{\alpha_2 - l} F_{t_2}^{(l-1)}(p,0), \tag{17}$$

where

$$F_{t_2}^{(l)}(p,0) = \mathcal{L}_{t_1}\left\{\left[\frac{\partial^l f(t_1,t_2)}{\partial t_2^l}\right]_{t_2=0}\right\} \tag{18}$$

for $l = 0, 1, \ldots$

Taking into account (15) and (17) we obtain the 2D Laplace transform of the state equation (4a)

$$\begin{bmatrix} p^{\alpha_1} X^h(p,s) - \sum_{k=1}^{N_1} p^{\alpha_1 - k} X_{t_1}^{h(k-1)}(0,s) \\ s^{\alpha_2} X^v(p,s) - \sum_{l=1}^{N_2} s^{\alpha_2 - l} X_{t_2}^{v(l-1)}(p,0) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X^h(p,s) \\ X^v(p,s) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} U(p,s). \tag{19}$$

Premultiplying (19) by the matrix

$$\text{blockdiag}\left[p^{-\alpha_1} I_{n_1}, s^{-\alpha_2} I_{n_2}\right]$$

we obtain

$$\begin{bmatrix} X^h(p,s) \\ X^v(p,s) \end{bmatrix} = G^{-1}(p,s) \times \left\{ \begin{bmatrix} \sum_{k=1}^{N_1} p^{-k} X_{t_1}^{h(k-1)}(0,s) \\ \sum_{l=1}^{N_2} s^{-l} X_{t_2}^{v(l-1)}(p,0) \end{bmatrix} + \begin{bmatrix} p^{-\alpha_1} B_1 \\ s^{-\alpha_2} B_2 \end{bmatrix} U(p,s) \right\}, \tag{20}$$

where

$$G(p,s) = \begin{bmatrix} I_{n_1} - p^{-\alpha_1} A_{11} & -p^{-\alpha_1} A_{12} \\ -s^{\alpha_2} A_{21} & I_{n_2} - s^{\alpha_2} A_{22} \end{bmatrix}. \tag{21}$$

Let

$$G^{-1}(p,s) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} T_{ij} p^{-i\alpha_1} s^{-j\alpha_2}. \tag{22}$$

From

$$G^{-1}(p,s) G(p,s) = G(p,s) G^{-1}(p,s) = I_n,$$

using (21) and (22), it follows that

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \left\{T_{ij} - T_{10} T_{i-1,j} - T_{01} T_{i,j-1}\right\} p^{-i\alpha_1} s^{-j\alpha_2} = I_n \tag{23}$$

where T_{10} and T_{01} are defined by (10).

Comparing the coefficients at the same powers of p and s we obtain (9).

Substituting the expansion (22) into (20) we obtain

$$\begin{bmatrix} X^h(p,s) \\ X^v(p,s) \end{bmatrix} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} T_{ij} \left\{ \begin{bmatrix} p^{-(i+1)\alpha_1} s^{-j\alpha_2} B_1 \\ p^{-i\alpha_1} s^{-(j+1)\alpha_2} B_2 \end{bmatrix} U(p,s) + \begin{bmatrix} \sum_{k=1}^{N_1} p^{-k-i\alpha_1} s^{-j\alpha_2} X_{t_1}^{h(k-1)}(0,s) \\ \sum_{l=1}^{N_2} p^{-i\alpha_1} s^{-l-j\alpha_2} X_{t_2}^{v(l-1)}(p,0) \end{bmatrix} \right\}, \tag{24}$$

Taking into account (Kaczorek, 2011)

$$\mathcal{L}^{-1}\left[p^{-\alpha} F(p)\right] = I_t^{\alpha} f(t), \tag{25}$$

where $\alpha > 0$ and $\mathcal{L}^{-1}$ denotes the inverse Laplace transform, it is easy to show that

$$\mathcal{L}_{t_1,t_2}^{-1}\left[p^{-\alpha_1} s^{-\alpha_2} F(p,s)\right] = I_{t_1,t_2}^{\alpha_1,\alpha_2} f(t_1,t_2), \tag{26}$$

where $\alpha_1, \alpha_2 > 0$.

Applying the inverse 2D Laplace transform to (24) and taking into account (26) we obtain the formula (8).

3. EXTENSION OF CAYLEY-HAMILTON THEOREM

Theorem 2. Let

$$\det G(p,s) = \begin{vmatrix} I_{n_1} - p^{-\alpha_1} A_{11} & -p^{-\alpha_1} A_{12} \\ -s^{\alpha_2} A_{21} & I_{n_2} - s^{\alpha_2} A_{22} \end{vmatrix} = \sum_{k=0}^{n_1} \sum_{l=0}^{n_2} a_{n_1-k,n_2-l} p^{-k\alpha_1} s^{-l\alpha_2} \tag{27}$$

be the characteristic polynomial of the system (4). Then the transition matrices T_{ij} satisfy the equality

$$\sum_{k=0}^{n_1} \sum_{l=0}^{n_2} a_{k,l} T_{k+m_1,l+m_2} = 0, \tag{28}$$

where $m_1, m_2 = 0, 1, \ldots$.

Proof. From the definition of the inverse matrix, as well as (22) and (27), we have

$$\text{Adj}G(p,s) = \left(\sum_{k=0}^{n_1} \sum_{l=0}^{n_2} a_{n_1-k,n_2-l} p^{-k\alpha_1} s^{-l\alpha_2}\right) \times \left(\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} T_{ij} p^{-i\alpha_1} s^{-j\alpha_2}\right) = \sum_{k=0}^{n_1} \sum_{l=0}^{n_2} \sum_{i=-k}^{\infty} \sum_{j=-l}^{\infty} a_{kl} T_{i+k,j+l} p^{-(i+n_1)\alpha_1} s^{-(j+n_2)\alpha_2}, \tag{29}$$

where $\mathrm{Adj}G(p,s)$ denotes the adjoint matrix of $G(p,s)$.

Comparing the coefficients at the same powers of p and s for $i \geq 0$ and $j \geq 0$we obtain (28) since $AdjG(p,s)$ is a polynomial matrix of the form

$$\mathrm{Adj}G(p,s) = \sum_{\substack{i=0 \\ i,j \neq n_1,n_2}}^{n_1} \sum_{j=0}^{n_2} D_{ij} p^{-i\alpha_1} s^{-j\alpha_2}, \tag{30}$$

where $D_{ij} \in R^{n \times n}$ are some real matrices.

Theorem 2 is an extension of the well-known classical Cayley-Hamilton theorem to fractional 2D continuous-time systems.

4. NUMERICAL EXAMPLE

Example 1. Consider fractional 2D system (4) with $\alpha_1 = 0{,}7$, $\alpha_2 = 0{,}9$ and matrices

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -0.9 & 0.7 \\ 0 & -0.3 \end{bmatrix}, \quad \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = [0]. \tag{31}$$

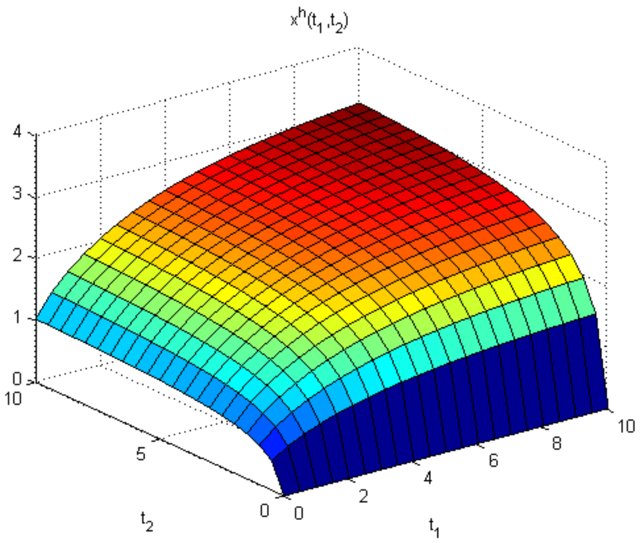

Fig. 1. State variable $x^h(t_1,t_2)$ of the system

Find a step response of the system (4) with the matrices (31), i.e. $y(t_1,t_2)$ for $t_1,t_2 \geq 0$ and

$$u(t_1,t_2) = H(t_1,t_2) = \begin{cases} 0 & for \quad t_1 < 0 \quad and\,/\,or \quad t_2 < 0 \\ 1 & for \quad t_1,t_2 \geq 0 \end{cases} \tag{32}$$

and zero boundary conditions

$$x^h(0,t_2) = 0, \qquad x^v(t_1,0) = 0. \tag{33}$$

Note that in this case from (31) and (4) it follows that

$$y(t_1,t_2) = \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix}.$$

It is well-known that (Podlubny, 1999)

$$I_t^{\alpha} H(t) = \frac{t^{\alpha}}{\Gamma(1+\alpha)}. \tag{34}$$

From (34) and (7) it is easy to show that

$$I_{t_1,t_2}^{\alpha_1,\alpha_2} u(t_1,t_2) = I_{t_1,t_2}^{\alpha_1,\alpha_2} H(t_1,t_2) = \frac{t_1^{\alpha_1} t_2^{\alpha_2}}{\Gamma(1+\alpha_1)\Gamma(1+\alpha_2)}. \tag{35}$$

Using (8) for $N_1, N_2 = 1$ and taking into account (31), (32) (33) and (35) we obtain

$$\begin{aligned} \begin{bmatrix} x^h(t_1,t_2) \\ x^v(t_1,t_2) \end{bmatrix} = & \sum_{i=0}^{\infty} \sum_{j=1}^{\infty} T_{ij} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{t_1^{(i+1)\alpha_1} t_2^{j\alpha_2}}{\Gamma[1+(i+1)\alpha_1]\Gamma(1+j\alpha_2)} \\ & + \sum_{i=0}^{\infty} T_{i0} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \frac{t_1^{(i+1)\alpha_1}}{\Gamma[1+(i+1)\alpha_1]} \\ & + \sum_{i=1}^{\infty} \sum_{j=0}^{\infty} T_{ij} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{t_1^{i\alpha_1} t_2^{(j+1)\alpha_2}}{\Gamma[1+i\alpha_1]\Gamma[1+(j+1)\alpha_2]} \\ & + \sum_{j=0}^{\infty} T_{0j} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{t_2^{(j+1)\alpha_2}}{\Gamma[1+(j+1)\alpha_2]}, \end{aligned} \tag{36}$$

where transition matrices T_{ij} are given by (9).

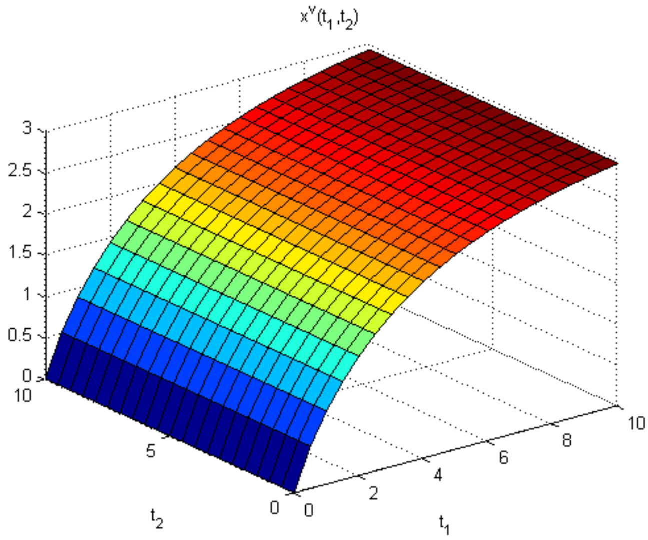

Fig. 2. State variable $x^v(t_1,t_2)$ of the system

Formula (36) describes the step response of the system (4) with the matrices (31). It is easy to show that the coefficients $1/\Gamma(.)$ strongly decrease when i and j increase. Therefore, in numerical analysis we may assume that i and j are bounded by some natural numbers L_1 and L_2.

The plots of the step response (36) where $L_1 = 50$ and $L_2 = 50$ are shown on Fig. 1 and 2.

5. CONCLUDING REMARKS

A new class of fractional 2D continuous-time linear systems described by the Roesser model has been introduced. The general response formula for such systems has been derived (Theorem 1) using the 2D Laplace transform. The classical Cayley-Hamilton theorem has been extended to fractional 2D continuous-time systems (Theorem 2). It has been shown that using the general response formula we are able to obtain the step response of the fractional 2D continuous-time system. The considerations have been illustrated by a numerical example.

The above considerations can be extended for general 2D model (Kurek, 1985). An open problems are the positivity and stability of fractional 2D continuous-time systems.

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