

## PARTITION OF HEAT IN 2D FINITE ELEMENT MODEL OF A DISC BRAKE

Piotr GRZEŚ\*

\*Phd student, Katedra Mechaniki i Informatyki Stosowanej, Wydział Mechaniczny, Politechnika Białostocka, ul. Wiejska 45 C, 15-351 Białystok

[p.grzes@doktoranci.pb.edu.pl](mailto:p.grzes@doktoranci.pb.edu.pl)

**Abstract:** In this paper nine of formulas (theoretical and experimental) for the heat partition ratio were employed to study the temperature distributions of two different geometrical types of the solid disc brake during emergency brake application. A two-dimensional finite element analysis incorporating specific values of the heat partition ratios was carried out. The boundary heat flux uniformly distributed over the circumference of a rubbing path to simulate the heat generated at the pad/disc interface was applied to the model. A number of factors over the heat partition ratio that affects the temperature fields are included and their importance is discussed.

### 1. INTRODUCTION

Frictional heating problem is an important issue during operation of the brake system. When the sliding mutual process occurs the mechanical energy is converted into thermal energy, chemical bonding energy and phase transition energy at the interface of two mating bodies. Nonetheless thermal energy is prevalent. Thus it is essential to develop critical temperature above which various undesirable effects such as softening or sintering of materials, premature wear, friction coefficient fluctuations or breakdown of the system may take place (Olesiak et al., 1997; Yi et al., 2002).

Variou techniques have been so far developed for the calculation of maximum temperatures in sliding systems. Analytical methods for solution of thermal problem of friction during braking are limited to the contact of two semi-spaces or the plane-parallel strip and semi-space (Chichinadze et al., 1979; Balakin 1999; Grylytskyy, 1996; Yevtushenko and Kuciej, 2010). More accurate for the finite object, transform techniques have been used, but numerous mathematical difficulties implies simplifications in geometry. The finite element method among numerical techniques is held as one of the most suitable for thermal problem investigation. Review of FEM-solutions of thermal problems of friction during braking are given in article of Yevtushenko and Grześ (2010).

The calculation of temperature during braking requires appropriate model where sufficient number of variables are included to obtain reliable outcomes. One of the input parameters for FEM-calculations of temperature in pad/disc brake system is the heat partition ratio (Pereverzeva and Balakin, 1992; Evtushenko et al., 2000). The separation of heat between two sliding bodies depends primarily on the relative velocity, the thermophysical properties of materials, the interface contact length, the amount of wear debris (third body) whose magnitude varies during the process. The settlement of the heat partition at the interface of two sliding bodies within the years was somewhat complex and remains in fact unsolved.

The problem of the heat partition in a three-dimensional FE model of a pad/disc brake system subjected to non-axisymmetric thermal load was studied in article of Yevtushenko and Grześ (2011).

In this paper the finite element analysis of frictional heating problem in an axisymmetric arrangement of the disc brake model to assess the impact of separation of total heat generated between members of sliding system was carried out. Irresistible advantages of this numerical technique approach were reported within the past years. Nevertheless several disadvantages, namely partition of heat during frictional heating process became apparent. This study aims to compare the temperatures obtained with the use of nine various formulas, both experimental and theoretical for the heat partition ratio. The comparison of outcomes of the thermal finite element analysis with experimental data of two different circumstances of braking action (Nosko et al, 2009; Zhu et al., 2009) dimensions and properties of materials was accomplished as well.

### 2. FRICTION HEAT DISTRIBUTION BETWEEN A PAD AND A DISC

The thermal energy is generated at the interface as the heat fluxes with the specified intensity, and at the same time is divided into contact surfaces of two bodies. In order to analyze thermal processes, the heat partition ratio denoted  $\gamma$  is employed, e.g. if the heat flux with the intensity of  $q_1 = \gamma q$  enters into body "1", the intensity of heat flux entered into body "2" equals  $q_2 = (1 - \gamma)q$ . Noticeably  $q = q_1 + q_2$ , where the power of friction equals  $q = fVp$ , and  $f$  denotes the coefficient of friction,  $V$  is the relative velocity of sliding bodies,  $p$  is the contact pressure. The heat partition ratio term was introduced in 1937 by Blok (1937), who considered sliding of single roughness with square ( $a \times a$ ) or circular (with radius  $a$ ) shape and the lateral surface of cylinder (narrow layer of contact zone) along surface of semi-space. The dimensions of contact area in comparison to whole dimen-

sions of contacting bodies were insignificant, because the semi-space was considered. It was assumed that the heat generation takes place directly on contact surface, and heat expansion process is unidirectional perpendicular to contact surface. The intensity of heat flux was constant with time and independent of spatial distribution, according to the same law as the contact pressure. Dividing virtually contacting bodies, two problems of frictional heating are obtained, namely: with the surface of semi-space heating with the intensity of  $q_1$  (for roughness), and with the local surface of semi-space heating where the intensity of heat flux  $q_2$  is moving. The magnitude of heat partition ratio for low sliding velocities ( $V \leq 8k_2/25a$  or  $Pe \leq 0.32$ ) Blok specifies as follows

$$\gamma = \frac{K_1}{K_1 + K_2}, \quad (1)$$

where:  $K$  – thermal conductivity, subscripts 1 and 2 indicate the first and the second body, respectively.

It was concluded, that for high sliding velocities ( $V > 8k_2/a$  or  $Pe > 8$ ) of the roughness, the maximum temperature on the friction surface, according to the rod model, is obtained near the edge of the roughness, opposite to sliding direction. In the case of lateral surface of cylinder sliding over the plane surface of semi-space Blok defines high velocity as  $V > 40k_2/a$  ( $Pe \geq 40$ ). In order to find the coefficient  $\gamma$  Blok equates the maximum temperatures on the contact area of roughness and semi-space. As a result, the following formula for the heat partition ratio is obtained:

$$\gamma = \frac{K_1}{K_1 + K_2 \sqrt{\pi Pe/16}} \cong \frac{K_1}{K_1 + 0.44 K_2 \sqrt{Pe}} \quad (2)$$

where:  $Pe$  – Peclet number.

The cases of sliding with constant velocity of semi-infinite rod of rectangular or quadrate profile over the surface of semi-space were considered by Jaeger (1942). Unlike to Blok, Jaeger, defining  $\gamma$ , compared the mean temperatures on the contact surface. In case of the rod of quadrate cross-section ( $a \times a$ ) with a thermally insulated lateral surface, and one tip sliding with constant high speed ( $Pe > 20$ ) over the surface of semi-space, Jaeger obtained the following formula to calculate heat partition ratio

$$\gamma = \frac{1.25 K_1}{1.25 K_1 + K_2 \sqrt{Pe/2}} \cong \frac{K_1}{K_1 + 0.56 K_2 \sqrt{Pe}} \quad (3)$$

From formula (3) it results that an increase in sliding velocity (the Peclet number  $Pe$ ) evokes decrease of  $\gamma$  and, consequently, the amount of heat which heats the rod. Jaeger explains this fact by two sources of heating the semi-space: heat generated as a result of friction, and heat previously heating layers of the rod. At the same time, the tip of the rod is heated only by frictional heating, and cooled by forthcoming “cooler” areas of semi-spaces. The higher speed of sliding there is, the more amount of heat absorbed by the semi-space.

Jaeger also improves formula (3) for the case of convective heat transfer with constant heat transfer coefficient  $h$

between lateral surface of rod and ambient environment:

$$\gamma = \frac{\sqrt{K_1}}{\sqrt{K_1 + 0.67 K_2 \sqrt{Pe/Bi}}} \quad (4)$$

where:  $Bi$  – Biot number.

From the formula (4) it may be concluded that the increase of heat transfer coefficient  $h$  leads to an increase of the amount of heat directed into the rod.

One of typically used formulas to calculate the heat partition ratio in braking systems is the Charron's formula (1943)

$$\gamma = \frac{\sqrt{K_1 \rho_1 c_1}}{\sqrt{K_1 \rho_1 c_1} + \sqrt{K_2 \rho_2 c_2}}, \quad (5)$$

where:  $\rho$  – density,  $c$  – specific heat capacity,

The Charron's formula (5), which is recommended to use for calculating the temperature of clutches and brakes, when the coefficient of mutual overlap  $\eta$  equals approximately to one.

$$\eta = \frac{\theta_0}{2\pi}, \quad (6)$$

where:  $\theta_0$  – cover angle of pad. If  $\eta \ll 1$ , then correction of Charron's formula has the form (Newcomb, 1958-59)

$$\gamma = \frac{\eta K_1 \sqrt{k_2}}{\eta K_1 \sqrt{k_2} + K_2 \sqrt{k_1}} = \frac{\eta}{\eta + \varepsilon} \quad (7)$$

In order to comply real thickness of the friction pair  $d_i$ , which cumulate the heat generated during friction, the following formula for determination of the heat partition ratio was proposed by Hasselgruber (1963):

$$\gamma = \frac{d_1 c_1 \sqrt{k_1}}{d_1 c_1 \sqrt{k_1} + d_2 c_2 \sqrt{k_2}}, \quad d_i \leq \delta_i, \quad i = 1, 2, \quad (8)$$

where the efficient depth of heating  $\delta$  (the distance at which the temperature is equal 5% of the maximum temperature on the contact surface) equals (Chichinadze et al., 1964):

$$\delta_i = 1.73 \sqrt{k_i t_s}, \quad i = 1, 2 \quad (9)$$

If the thicknesses  $d_i$  of the braking elements are greater than thicknesses of thermal layers  $\delta_i$  (9), then it is necessary to replace  $d_i$  on  $\delta_i$ ,  $i = 1, 2$  in the formula (8). Consequently, substituting the thicknesses  $\delta_i$  (9) into the formula (8), we obtain

$$\gamma = \frac{c_1 k_1}{c_1 k_1 + c_2 k_2}, \quad \delta_i > d_i, \quad i = 1, 2. \quad (10)$$

Transformation of formula (8) to comply effective saturated heat of bodies volume  $V_i$ , was made by Chichinadze et al. (1979):

$$\gamma = \frac{V_1 c_1 \sqrt{k_1}}{V_1 c_1 \sqrt{k_1} + V_2 c_2 \sqrt{k_2}}, \quad d_i \leq \delta_i, \quad i = 1, 2, \quad (11)$$

where  $V_i = S_i d_i$ ,  $S_i$  – nominal contact area of body  $i = 1, 2$ . If  $\delta_i > d_i$ , then replacing in the formula (11)  $d_i$  on  $\delta_i$  (8), we find

$$\gamma = \frac{\eta c_1 k_1}{\eta c_1 k_1 + c_2 k_2}, \delta_i > d_i, i = 1, 2, \quad (12)$$

where the coefficient of mutual overlap  $\eta$  was defined by the formula (6).

The frictional heat generation in the pad/disc tribosystem, using the solution of thermal problem of friction for two layers with thickness  $d_i$ ,  $i = 1, 2$  on the assumption that  $q_i = const.$ ,  $i = 1, 2$  and the external surfaces are insulated was studied in article (Ginzburg, 1973). From the condition of equality of temperatures on a surface of contact the time-dependent formula for calculation of the heat partition ratio is obtained:

$$\gamma(t) = \frac{\eta K_1 d_2 \theta(\tau_2)}{\eta K_1 d_2 \theta(\tau_2) + K_2 d_1 \theta(\tau_1)}, 0 \leq t \leq t_s, \quad (13)$$

where

$$\theta(\tau_i) = \frac{1}{3} + \tau_i + 2 \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos(\pi n)}{(\pi n)^2} \exp[-(\pi n)^2 \tau_i] \quad (14)$$

$$\tau_i = \frac{k_i t}{d_i}, d_i \leq \delta_i, i = 1, 2$$

If  $\tau_i > 0.3$ , then the function  $\theta(\tau_i)$  (14) may be written in the following form

$$\theta(\tau_i) \approx \frac{1}{3} + \tau_i, i = 1, 2 \quad (15)$$

As above, in case when  $d_i > \delta_i$ , it is necessary in the formula (13) to replace the thicknesses of friction pair  $d_i$  with appropriate effective thicknesses  $\delta_i$ ,  $i = 1, 2$  (9).

Tab. 1. Heat partition ratios

Curve number	Number of the formula and type of a brake	Author
1	(1) A, B	Blok, 1937
2	(2) A, B	Blok, 1937
3	(3) A, B	Jaeger, 1942
4	(4) A, B	Jaeger, 1942
5	(5) A, B	Charron, 1943
6	(7) A, B	Newcomb, 1958-59
7	(10) A, (8) B	Hasselgruber, 1963
8	(12) A, (11) B	Chichinadze et al., 1979
9	(13) A, B	Ginzburg, 1973

The thermal conductivity of the pad material is considerably less than the thermal conductivity of the disc material, i.e.  $K_1 \ll K_2$ . For that reason, the temperature increases on the external surface of the pad near the moment of standstill  $t_s$ , and it differs slightly from the initial period. As a result, pad may be considered as the semi-infinite body (the semi-space), and disc – as the strip. Then from the formula (13) it follows

$$\gamma(t) = \frac{\eta K_1 d_2 \theta(\tau_2)}{\eta K_1 d_2 \theta(\tau_2) + 2 K_2 d_1 \sqrt{\frac{\tau_1}{\pi}}}, 0 \leq t \leq t_s, \quad (16)$$

The formulas shown in this section are found by various ways and differ significantly from each other. Therefore, comparison of the results of the numerical analysis obtained with their help with appropriate experimental data is the actual problem.

### 3. STATEMENT OF THE PROBLEM

When the friction force acts on the members of brake system being in sliding contact, the energy conversion should be considered as an essential. The work done is converted into heat and the vehicle decelerates with the certain rate. The disc brake system consists primarily of two major parts, namely: rotating axisymmetric disc and immovable non-axisymmetric pads (Fig. 1). The generated thermal energy dissipated by the conduction from disc/pad interface to adjacent components of brake system, and by convection to atmosphere due to Newton's law. Obviously the third of mode of heat transfer takes place as well, nonetheless by virtue of relatively low temperatures attained during slipping and short time of the operation is neglected.

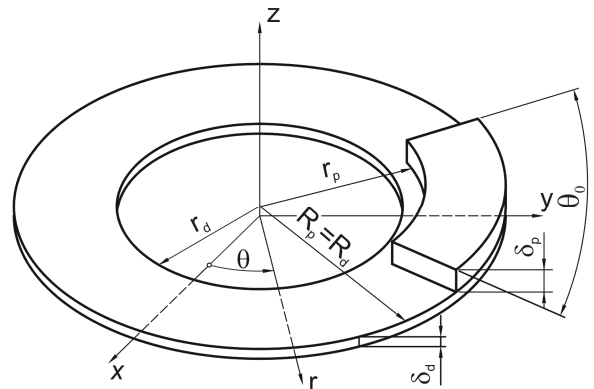


Fig. 1. A schematic diagram of a disc brake system

In actual thermal load of a disc is non-axisymmetric, which stems from the geometry of a pad covering rubbing path partly. Thereby, for the selected spot on the circumference of the friction surface the temperature will differ periodically with time. Such a distribution both in depth and circumferential direction can be obtained by means of the three-dimensional model.

Despite the fact of comprehensive outcomes possible to obtain by means of the spatial model of a disc, hitherto have been made calculations of transient temperatures using two-dimensional configurations of the same phenomenon make a case for their application (Talati and Jalalifar, 2009). The accuracy of such an approximation increases, when the Peclet number is higher for considered tribosystem. In an automotive disc brakes the Peclet numbers almost always are in order of  $10^3$  (Chichinadze et al., 1979). Therefore, the transient heat conduction problem for the disc

and the pad has been analysed in the axisymmetric statement (uniformly distributed heat source over the rubbing path of the disc), assuming boundary thermal flux acting on the lateral surfaces of a disc.

An analytical solution of one-dimensional boundary-value problem of heat conductivity for tribosystem, consisting of a plane-parallel strip and semi-space, was obtained by Nosko et al. (2009). The temperature evolution at mean radius of pad contact surface was illustrated. The frictional heating phenomenon of a brake shoe including spread of heat on its depth during hoist's emergency braking was studied by Zhu et al. (2009). The integral transform method was adopted in the three-dimensional analysis.

In the proposed article two types of a real disc brake system including disc and pad volume during single braking action with a special respect to different heat partition ratios were studied. In order to validate further transient numerical analysis, dimensions, material properties and operating parameters were adopted from the experimental data Nosko et al. (2009) and Zhu et al. (2009).

For the purpose of thermal analysis, it is assumed that the contact pressure  $p$  is constant during entire process of braking and the angular velocity decreases linearly with time

$$\omega(t) = \omega_0 \left( 1 - \frac{t}{t_s} \right), \quad 0 \leq t \leq t_s. \quad (17)$$

where:  $\omega$  – angular velocity,  $\omega_0$  – initial angular velocity,  $t$  – time,  $t_s$  – braking time.

In order to calculate transient temperature distributions in the pad and the disc, the following heat conduction equation for an axisymmetric problem given in the cylindrical coordinate system was employed:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_p} \frac{\partial T}{\partial t}, \quad r_p \leq r \leq R_p, \quad 0 < z < \delta_p, t > 0; \quad (18)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d} \frac{\partial T}{\partial t}, \quad r_d \leq r \leq R_d, \quad 0 < z < \delta_d, t > 0,$$

where:  $T$  – temperature,  $r$ ,  $z$  – radial, axial coordinate respectively,  $k$  – thermal diffusivity,  $r$ ,  $R$  – internal and external radius, respectively. The bottom indexes  $p$  and  $d$ , denote the pad and the disc, respectively.

Taking account of the symmetry of a given problem, the insulation on mid-plane of the disc as well as the inner surface represented by the edge of two-dimensional model was established. On the remained surfaces of the brake models the forced convection takes place with the constant value of heat transfer coefficient. It is also assumed that the material properties of the pad and the disc are isotropic and independent of temperature.

$$q_p(r, t) = \mathcal{F} p r \omega(t), \quad r_p \leq r \leq R_p, \quad 0 \leq t \leq t_s, \quad (19)$$

$$q_d(r, t) = \frac{\theta_0}{2\pi} (1 - \gamma) f p r \omega(t), \quad r_d \leq r \leq R_d, \quad 0 \leq t \leq t_s. \quad (20)$$

where:  $q$  – intensity of the heat flux,  $\gamma$  – heat partition ratio,  $f$  – friction coefficient,  $p$  – contact pressure.

From the formulas (19) and (20) it follows that the intensities of thermal fluxes, which enter the pad and the disc respectively, depends directly on value of the heat partition ratio  $\gamma$ . Consequently, it is essential to specify its influence on the temperature at the pad/disc interface.

#### 4. FE FORMULATION

The understanding of overall formulation (18)–(20) is crucial for the solution of a considered thermal problem of friction by means of the approximate time-stepping procedures for axisymmetric transient governing equations of heat conductivity. The main idea of two-dimensional discretization of the boundary-value heat conductivity problem consists in the following reference (Lewis et al., 2004). Using the Galerkin's method we write Eq. (18) in the matrix form:

$$[\mathbf{C}] \left\{ \frac{d\mathbf{T}}{dt} \right\} + [\mathbf{K}] \{\mathbf{T}\} = \{\mathbf{Q}\}, \quad (21)$$

where:  $[\mathbf{C}]$  – heat capacity matrix,  $[\mathbf{K}]$  – heat conductivity matrix,  $\{\mathbf{Q}\}$  – thermal force matrix.

The solution of the first order ordinary differential equation (21) was obtained using the Crank-Nicolson method with approximation relations (Crank and Nicolson, 1947)

$$\frac{1}{\Delta t} [\{\mathbf{T}\}_{t+\Delta t} - \{\mathbf{T}\}_t] = (1 - \beta) \left\{ \frac{d\mathbf{T}}{dt} \right\}_t + \beta \left\{ \frac{d\mathbf{T}}{dt} \right\}_{t+\Delta t}, \quad (22)$$

where:  $\{\mathbf{T}\}_t$  – temperature vector at time  $t$ . The weight parameter  $0.5 < \beta \leq 1$  was chosen from the conditions of achievement of necessary accuracy of integration and stable scheme. Taking the relation (22) into account, from Eq. (21) we obtain the system of linear algebraic equations

$$([\mathbf{C}] + \beta \Delta t [\mathbf{K}]) \{\mathbf{T}\}_{t+\Delta t} = ([\mathbf{C}] - (1 - \beta) [\mathbf{K}] \Delta t) \{\mathbf{T}\}_t + (1 - \beta) \Delta t \{\mathbf{Q}\}_t + \beta \Delta t \{\mathbf{Q}\}_{t+\Delta t} \quad (23)$$

for determination of the temperature  $\{\mathbf{T}\}_{t+\Delta t}$  in the time moment  $t + \Delta t$ .

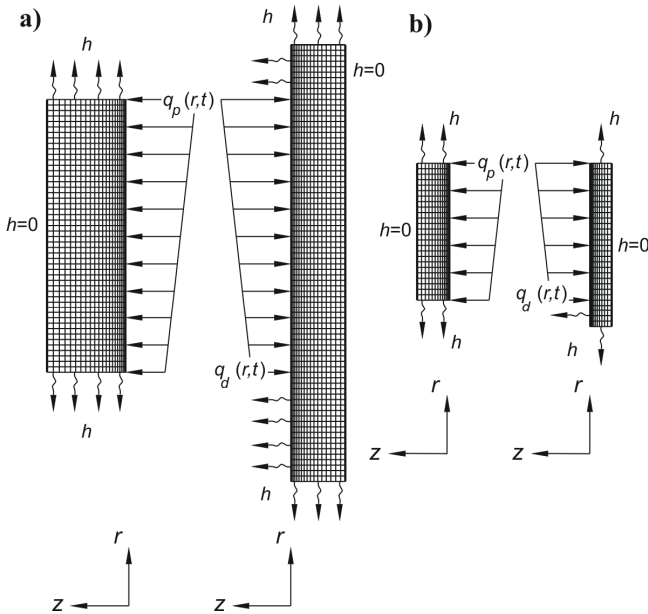
The temperature distributions in the pad and disc were analyzed using the finite element method based programme. In the thermal analysis of disc brake an appropriate finite element division is indispensable. In this study four-node quadratic elements were used for the finite element analysis. In order to avoid inaccurate or unstable results, a proper initial time step  $\Delta t$  associated with spatial mesh size  $\Delta x$  (smallest element dimension) is essential

$$\Delta t = \Delta x^2 \frac{\rho_{p,d} c_{p,d}}{10 K_{p,d}} \quad (24)$$

where:  $\Delta x$  – mesh size (smallest element dimension).

### 5. NUMERICAL ANALYSIS

The temperature distributions of two types of a real disc brake system including disc and pad volume during single braking action regarding different heat partition ratios were studied (Tab. 1).



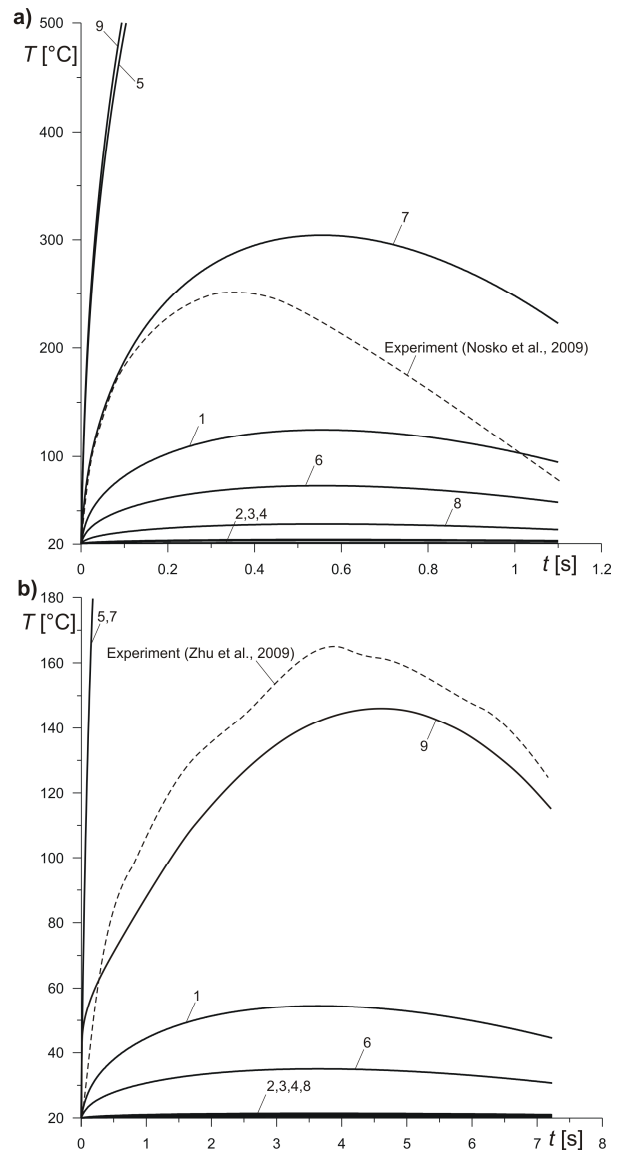
**Fig. 2.** FE models with the boundary conditions, a) A type, b) B Type

The FE element models with the boundary conditions are shown in Fig. 2. In order to validate further transient numerical analysis, the thermophysical material properties, dimensions and operating parameters have been adopted from the experimental data of Nosko et al. (2009) (the A type) and Zhu et al. (2009) (the B type), and are presented in Tab. 2. In the A type the brake pad is made of polymeric material of the type 145-40, and the brake disc is made of cast iron of the type 15-32. The materials of brake pad and brake disc in the B type are asbestos-free and 16Mn, respectively.

The heat transfer coefficient of 100 W/(m<sup>2</sup>K) was assumed. The FE model of A type of a disc brake consists of 3680 elements and 3864 nodes for the disc and 12800 elements and 13065 nodes for the pad, and the B type model consists of 660 elements and 732 nodes for the disc and 2000 elements and 2121 nodes for the pad. The temperature evolutions on the contact surface for two types of disc brake employing nine of formulas for the heat partition ratio were determined and compared with the experimental outcomes for the A type (Nosko et al., 2009) and B type (Zhu et al., 2009). Conformity of numbers of the curves presented on the following figures to formulas for calculation of heat partition ratio, is shown in Tab. 1.

The evolution of temperature on the contact surface of the pad and the disc are shown in Figs. 3 and 4, respectively. The character of evolution of temperature is the following: with the beginning of braking the temperature sharply raises, reaches the maximum value and, after this, it decreases to the minimum level in the stop time moment.

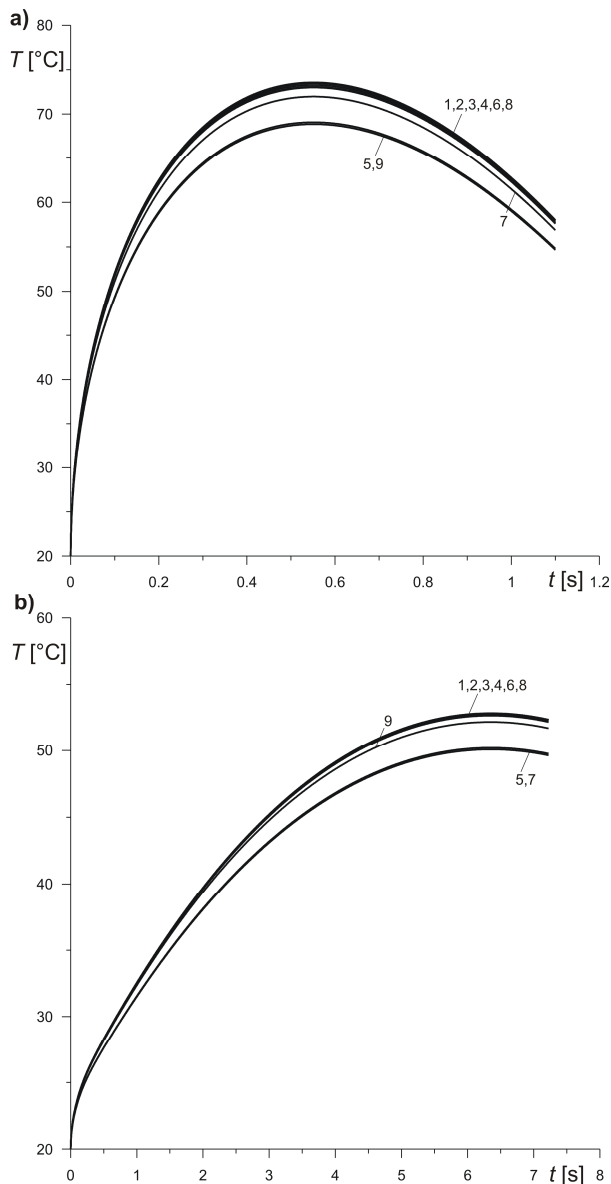
The change of temperature in time on the friction surface of the pad in A type of brake is shown in Fig. 3a. We see, that the evolution of contact temperature calculated with use of Charron’s formula (5) (the curve 5) and of Ginzburg formula (13) (the curve 9) significantly differs, both qualitatively and quantitatively, from experimental curve (Nosko et al. 2009). The curve denoted as 7 (Hasselgruber H., 1963) coincides with the experimental curve from the initial instant of time until  $t = 0.08$  s, then surpasses the experimental values of temperatures. The maximum temperature reached of curve 7 equals  $T = 304.3$  °C, whereas the maximum value of temperature of the experiment equals approximately  $T = 250$  °C, and appears earlier. Then, it decreases considerably to standstill. Most of the solutions illustrated in Fig. 3a range beneath experimental curve of temperature. The cooling conditions have no impact on the temperature values on the average radius, due to comparatively large distance from the outer surface of the disc. A slightly different plot of temperature evolution on friction surface of pad in the B type of brake is observed in Fig. 3b.



**Fig. 3.** Evolution of temperature at the mean radius of contact surface of a pad: a) A type, b) B type

**Tab. 2.** Operation and geometrical parameters

Parameters	A type (Nosko et al., 2009)		B type (Zhu et al., 2009)	
	Disc	Pad	Disc	Pad
thermal conductivity, $K$ (W/mK)	59	0.64	53.2	0.295
specific heat, $c$ (J/kgK)	500	1100	473	2530
density, $\rho$ (kg/m <sup>3</sup> )	7100	2500	7866	2206
inner radius, $r$ (m)	0.03	0.05	0.1325	0.1375
outer radius, $R$ (m)	0.11	0.1	0.1625	0.1625
mean radius of pad, $r_m$ (m)		0.075		0.15
cover angle of pad, $\theta_0$ (rad)		0.384		0.167
thickness, $\delta$ (m)	0.01	0.015	0.004	0.006
pressure, $p$ (Pa)		$4 \times 10^6$		$1.38 \times 10^6$
braking time, $t_s$ (s)		1.1		7.23
initial angular velocity, $\omega_0$ (s <sup>-1</sup> )	200		66.667	
coefficient of friction, $f$		0.38		0.4
initial temperature, $T_0$ (°C)		20		20
ambient temperature, $T_a$ (°C)		20		20
time step, $\Delta t$ (s)	0.0002	0.001	0.0002	0.005
Peclet number, Pe		4136.7		3014.6
Biot number, Bi		0.00777		0.00810

**Fig. 4.** Evolution of temperature at the mean radius of contact surface of a disc: a) A type, b) B type

However, curve denoted 5 is similar to the same one as in Fig. 3a. Previously correct temperature curve 7, currently provides overlapped values relating to the experimental data. The character of the temperature changes is the same as in A type (Fig. 3a), except the curve denoted 9 (Ginzburg, 1973), which results from the variable in time heat partition ratio. In this case the sharp rise of the temperature at the initial period of braking is noticeable. At time  $t = 0.05$  s temperatures of the curve 9 and the experimental one coincide, to separate after this moment till standstill.

In Fig. 4a the temperature evolutions of the disc friction surface of A type obtained from the numerical analysis including different representations of heat partition ratios are plotted against time. For A type of a brake it is important to know the location of the curve 7, because it is the curve, as seen from Fig. 3a, which gives the best coincidence of experimental data. We see in Fig. 4a, that the temperature curve denoted 7 lays between the curves 5, 9 and 1-4, 6, 8. The closeness of the last six curves can be explained by the fact that the value of the heat partition ratio calculated with their help, equals nearly zero. It means, that almost all heat energy, generated on the surface of friction, is absorbed by the disc, whereas influence of a portion of heat entering the pad is negligibly small. The curves 5 and 9 give the worst approximation of the experimental data. Thus, the analysis of evolution of temperature on surfaces of the pad and the disc, allows to come to the conclusion that the most authentic results for A type brake can be obtained with use of the Hasselgruber's formula (10).

The evolutions of temperature on the contact surface of the disc in B type of brake at the mean radius of rubbing path are shown in Fig. 4b. The curve 9, which better coincides with experimental data in Fig. 3b, is located between curves 5 and 7 (the worst coincidence to experimental data) and curves 1-4, 6, 8. The temperature reaches the maximum value after time ranged between  $t = 6.34$  s for curve 9 and  $t = 6.35$  s for curve 3 and decreases slightly after that moment. This may indicate that the disc is heated in the entire

volume, and cooling by the absorption to adjacent area is difficult.

The highest value of temperature on the contact surface of the disc of type B equals  $T = 52.8^{\circ}\text{C}$  (the curve 4) and is reached at time  $t = 6.34\text{ s}$  (the curve 4). The lowest temperature  $T = 50.07^{\circ}\text{C}$  is reached after  $t = 6.34\text{ s}$  (the curve 5). The discrepancy between the peak values of temperatures of the curves 4,8,3,2,6,1 equals 0.34 %. The analysis of evolution of temperature on friction surfaces of the pad and the disc in B type of brake show, that the most authentic distribution of temperature can be find, using the time-dependent ratio of heat partition given by Ginzburg's formula (13).

## 6. CONCLUSIONS

In this paper two-dimensional finite element analysis was carried out to study the effect of use of different heat partition ratios on the contact temperatures of the disc brake components during single braking process. The calculated temperatures on the friction surfaces of the brake system were compared with experimental results (Nosko et al, 2009; Zhu et al., 2009), which allow us make the following conclusions:

- the heat partition ratio is a key factor when analyze the pad friction surface temperature, which is conditioned by the substantial variation of its value in each case of applied formulas;
- the investigation of the pad/disc contact surface temperatures provides an important information about its maximum value reached during frictional heating, nevertheless the obtained results reveal that the material with lower thermal conductivity is more susceptible to the selection of the heat partition ratio included in the expression of the intensity of a heat flux calculation;
- the thickness of the pad plays a significant role as well. Its slender growth may firmly change the actual value of heat partition ratio entering the pad and the disc respectively, whereas in actual, temperatures of the pad alter only in the immediate vicinity of the contact surface;
- relatively small thickness (B type) demonstrates that the entire volume may be nearly uniformly heated from the initial moment of operation without causing significant temperature gradients.

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