PARAMETRICAL MODELS OF WORKING ROTOR MACHINE BLADE DIAGNOSTICS WITH ITS UNMEASURABLE ENVIRONMENT ELIMINATION

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Abstract: The paper presents the basic of the new method of rotor machine blades during their operation. The method utilizes such diagnostic models as a quotient of diagnostic signal y(t) amplification being a result of blade operation and x(t) signal of its environment as the blade tip approaches the sensor as well as amplification of these signals as the blade tip recedes from the sensor and phase shift difference of these signals as the blade tip approaches and recedes from the sensor. The adopted diagnostic models indirectly take the current blade environment x(t) into account with no necessity of measuring (Kotowski and Lindstedt, 2007; Lindstedt and Kotowski, 2004). Therefore the model is sensitive to blade technical condition changes remaining only slightly sensitive to environment changes. Suggested method may prove very important in diagnostics of rotor blades during operation of rotor machines (turbines, compressors etc.).

1. INTRODUCTION

Rotor machine is complex technical object (combined of multiple parts such as blades, bearings, shields) operating in complex environment (affected by variable stress, pressure, temperature, vibration etc.). The trouble spot of machine are, except bearings, blades. These are technically simple, however hard in matter of use and maintenance mainly due to high risk of aptitude assessment. Experience shows that fall of one blade of several dozen or hundreds almost always leads to serious breakdown of the whole (often very expensive) rotor machine (axis compressor, turbine). This causes huge interests in methods of routine diagnostics (monitoring) of blades technical condition during its operation.

Nowadays multiple methods of working rotor blades technical condition are in use. Diagnostic research in these methods are based on 'non-contact' measurement of values of tip of the blade current moves when in area below dedicated sensor. Many non-contact measuring systems have been designed and introduced. These are commonly known and used measuring system produced by: Hood, Aqilis, Pratt&Whithey (USA), Rolls Royce (UK), Turbocharges (Switzerland), MTU (Germany) as well as other Russian, Chinese and Indian manufacturers (Bovishanskii, 2000; Duan et al., 2005; Von Flotow and Mercadal, 2000; High Cycle Fatigue S & program 1997, 1998, 1999, 2000, 2001, 2002; Klein, 2004; Roberts, 2007; Zieliński and Ziller, 2005).

Polish measuring systems are also used, particularly those designed and introduced by Air Force Institute of Technology (AFIT) in Warsaw. Amongst the non-contact measurement of blade translocation systems manufactured by AFIT the following are to be mentioned (Lindstedt et al., 2009; Szczepanik and Przysowa, 2004; Szczepanik, 1999):

- blade fracture indicator: SPŁ – 29;
- blade excessive vibration indicator: SNDŁ – 2b;
- microwave sensors: MUH, PIT;

These systems are successfully implemented in specific operating technical objects (SO-3 engines).

Diagnostic inference used in hitherto methods are based on elaboration of signals obtained during diagnostic analysis coming as a result of blade activity without sufficient (according to authors) consideration being given to the signals of its variable environment.

Measurement of blade environment signals during operation of rotor machine are often hard or impossible to proceed thus are not sufficiently considered in blades diagnostics. Therefore statement can be made that hitherto methods of rotor blades technical condition in-work assessment do not realize the basic rule of technical diagnostics compelling research and analysis of object technical condition in environment (PN-90/N-04002) thus being not accurate and reliable enough.

Hence the need emerged to elaborate the new method of in-work diagnostics of blade technical condition that would take the environment into consideration and, if possible, would not require the use of unavailable or hard to measure signals. This problem is solved by blade analysis method that base on special diagnostic model allowing to eliminate its practical environment with specific methods.

2. BLADE ACTIVITY IN UNMEASURABLE ENVIRONMENT

Blade, its construction and activity while in variable environment (Lindstedt et al., 2009; Skubacziewskij, 1974) are presented in Fig. 1.

During blade operation its technical condition changes until occurrence of various damage (deformation, fractures). Causes and places of possible occurrence are shown in Fig.2 (Skubacziewskij, 1974).

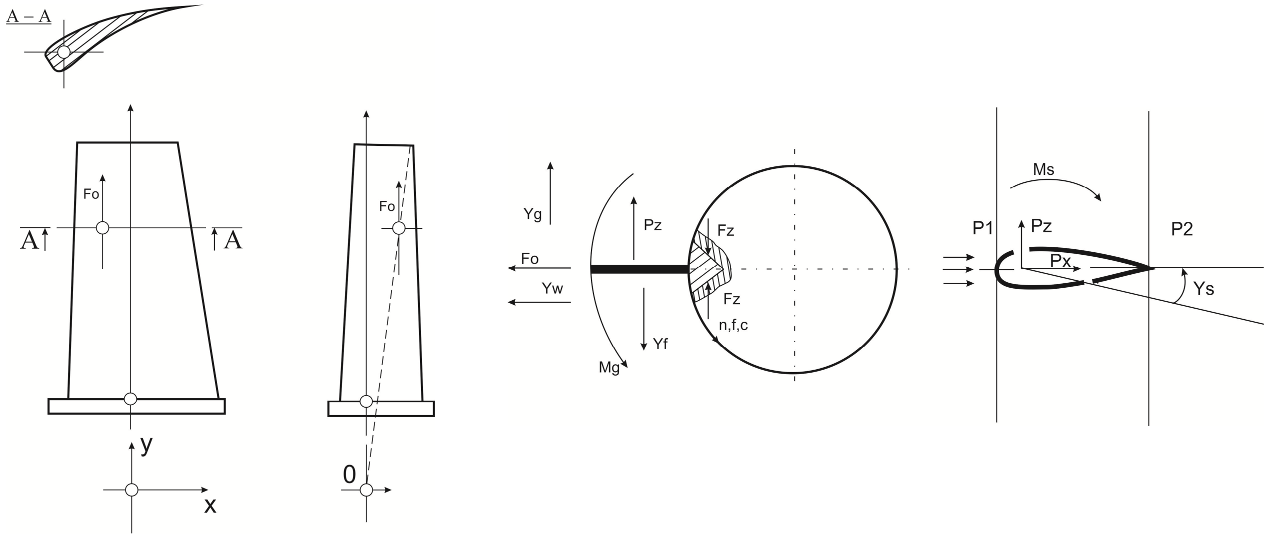

Fig. 1. Rotor machine blade in variable environment (F_0 – centrifugal force; F_z – lock grip force; n – rotational speed; P_z – blade aerodynamic lift; P_x – resisting force; M_s – torque M_g – bending moment, p_1 – gas pressure on blade rim input, p_2 – gas pressure on blade rim output; Y_g – blade deflection; Y_s – blade torsion angle; Y_w – blade lengthwise translocation; Y_f – various vibration signal (bending, lengthwise, torsional); Y_c – thermal deformation; f – vibration signal; f_{ob} – casing vibration signal; c – thermal decomposition signal)

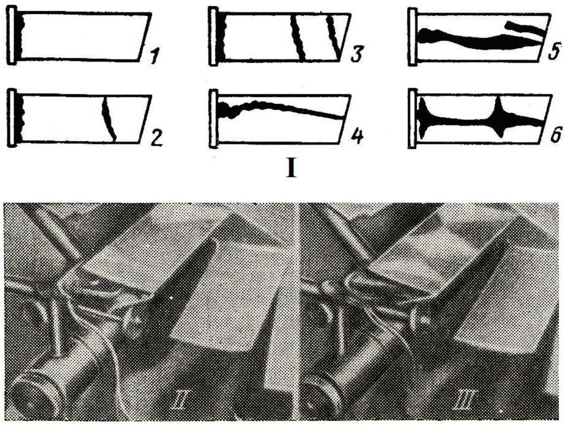

Fig. 2. Forms of vibration and lines indicating traces of vibration nodes (Chart I: 1,2,3 – first, second and third form of bending vibration; 4 – first kind torsional vibration; 5 – second kind torsional vibration; 6 – combined bending-torsional vibration. Chart II: photograph of nodes traces in second form of bending vibration. Chart III: photograph of nodes traces in third form of bending vibration)

Fig. 1. and Fig. 2. show that blade is a technical object with complex activity that need to be described with multidimensional blade deformation state. These deformations originate from environment and are caused by multiple reasons such as:

- centrifugal force stresses F_0 depending on rotational speed that cause lengthwise and bending deformations (Fig.1.) – Y_w, Y_g
- gas stresses P_z and P_x caused by air stream depending on speed and altitude (Fig.1.) – Y_s (in case of rotor machine being an engine turbine or compressor)
- stresses caused by curvilinear flight trajectory (Fig.1.) – Y_g, Y_s
- dynamic stresses accompanying mechanical vibration (particularly in resonant range) caused by pressure p_1, p_2 pulsation, rotation fluctuation etc. (Fig.2.) – Y_f
- blade and casing vibration f, f_{ob} (Fig.1.) and thus Y_g, Y_s
- thermal stresses c caused by uneven thermal distribution (Fig.2. - combined deformation, e.g. I-6) – Y_c.

Comprehensive, blade working status in environment may be described with a signal of tip of the blade translocation y(t) which is a resultant of Y_w, Y_g, Y_s, Y_f, Y_c, (Fig. 1. and Fig. 2.):

$$y(t) = f\left(Y_w,\ Y_g,\ Y_s,\ Y_f,\ Y_c\right) \tag{1}$$

and signal of environment x(t) which is a resultant of: n, F_o, P_z, P_x, P_1, P_2, f, f_{ob}, c (Fig. 1. and Fig. 2.):

$$x(t) = f\left(n,\ F_o,\ P_z,\ P_x,\ P_1,\ P_2,\ f, f_{ob},\ c\right) \tag{2}$$

Technical condition of blade $S_T(\theta)$ according to diagnostic rules, results from relation between activity signal y(t) and environment signal x(t) in the current θ_1 beginning moment of diagnostics θ_0 (while θ is the time of the technical condition changes - evolution)

Therefore, the following may be noted:

$$S_T(\theta) = f(y(t)_{\theta_0,}\ x(t)_{\theta_0,}\ y(t)_{\theta_1,}\ x(t)_{\theta_1,}\ \theta, t) \tag{3}$$

Exploitation experience proved the existence of real difficulties in measuring signals for y(t), especially x(t), thus in blade technical condition in-work assessment as well (Kotowski and Lindstedt, 2007; Lindstedt and Kotowski, 2004; Lindstedt et al., 2009).

3. THEORETICAL BASES OF BLADE DIAGNOSTICS METHOD DURING OPERATION OF ROTOR MACHINE

The matter of blade diagnostics during rotor machine operation is very complex, because only one measurable and yet interfered signal y(t) and actually immeasurable (except for n signal and without Δn) environment signal x(t) are available. Exemplary course of y(t) signal is presented in Fig.3 (Szczepanik and Przysowa, 2004; Szczepanik, 1999). Course of signal x(t), being a resultant of multiple signals is shown in Fig.4. (Dołgolienko, 1984)

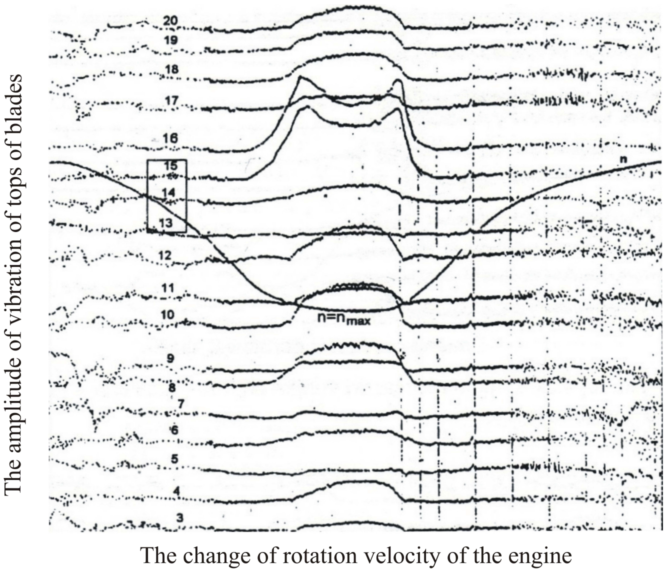

Fig. 3. Signal y(t) course for various blades depending on rotational speed

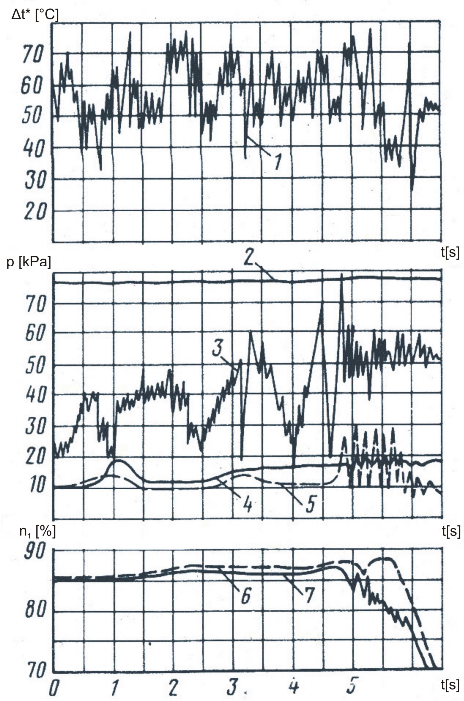

Fig. 4. Change of temperature and pressure of air stream on outlet in specific flight conditions 1 – in lower part of right engine intake, 2 – on side of right engine intake, 3 – in lower part of left engine intake, 4 – on upper part of left engine intake, 5 – on side of left engine intake, 6 – change of rotational speed of right engine rotor, 7 – change of rotational speed of left engine rotor

Tentatively signals x(t) and y(t) are assumed to be temporal, stochastic and interfered courses. In this case switching from space domain „t" of signals x(t) and y(t) to space domain „τ" of correlation function $R_{xx}(\tau)$,$R_{yy}(\tau)$ and $R_{xy}(\tau)$ proves reasonable (Bendat and Piersol, 1976; Kotowski and Lindstedt, 2007; Lindstedt and Kotowski, 2004; Lindstedt et al., 2009; Niederliński, 1985; Szabatin, 2000).

The effects of such approach are:

- noise suppression for signals and possibility of amplifying them
- possibility of simple expressing signals $R_{xx}(\tau)$, $R_{yy}(\tau)$ and $R_{xy}(\tau)$ as analytic functions, what allows further processing to new functions of specific properties in frequency domain (ω), which are functions of density of singular power of signals $S_{xx}(\omega)$ and $S_{yy}(\omega)$ and reciprocal $S_{xy}(\omega)$.

Expressing functions x(t) and y(t) as $S_{xx}(\omega)$, $S_{yy}(\omega)$ and $S_{xy}(\omega)$ allows taking relations between diagnostic signals y(t) and environment signals x(t) into consideration very simply (Fig.3., 4.), as this may be noted:

$$A^2_{T01} = \frac{S^{T01}_{yy}}{S^{T01}_{xx}} \qquad \varphi_{T01} = Arg\frac{S^{T01}_{xy}}{S^{T01}_{xx}} \tag{4}$$

$$A^2_{T12} = \frac{S^{T12}_{yy}}{S^{T12}_{xx}} \qquad \varphi_{T12} = Arg\frac{S^{T12}_{xy}}{S^{T12}_{xx}} \tag{5}$$

where: A^2_{T01}, φ^2_{T01} – amplification and phase shift of signals x and y while the blade approaches the sensor and A^2_{T12}, φ_{T12} – amplification and phase shift of signals x and y while the blade recedes from the sensor.

Furthermore assumption can be made that time of signal T_{12} observation comes shortly (ms) after observation of T_{01} signals.

In such case the following may by assumed:

$$S^{T12}_{xx} = S^{T01}_{xx} \tag{6}$$

Then, on the basis on 4, 5, 6 equations new, abstract and physically interpretable quantity in form of a quotient of amplifications A^2_{T01} and A^2_{T12} can be achieved as well as phase shifts φ_{T01} and φ_{T12}:

$$A^2_{T12,T01} = \frac{A^2_{T12}}{A^2_{T01}} = \frac{\frac{S^{T12}_{yy}}{S^{T12}_{xx}}}{\frac{S^{T01}_{yy}}{S^{T01}_{xx}}} \xrightarrow{S^{T12}_{xx}=S^{T01}_{xx}} \frac{S^{T12}_{yy}}{S^{T01}_{yy}} \tag{7}$$

$$\varphi_{T12,T01} = \varphi_{T12} - \varphi_{T01} = Arg\frac{\frac{S^{T12}_{xy}}{S^{T12}_{xx}}}{\frac{S^{T01}_{xy}}{S^{T01}_{xx}}} = Arg\frac{A_{12}e^{-j\varphi_{T12}}}{A_{01}e^{-j\varphi_{T01}}} =$$

$$= ArgA_{T12T01}e^{-j(\varphi_{T12}-\varphi_{T01})} \xrightarrow{S^{T12}_{xx}=S^{T01}_{xx}} Arg\frac{S^{T12}_{xy}}{S^{T01}_{xy}} \tag{8}$$

Equation $A^2_{T12,T01}$ (7) binds diagnostic signals y(t) to environment signals x(t) and so is a diagnostic model. The distinctive feature of this model is being determined only by measurable signal y(t) in closely following observation periods T_{01} and T_{12} and (what is more important), taking environment into account with no necessity to measure it as well as the noise of signal y(t) was sufficiently suppressed. (Kotowski and Lindstedt, 2007; Lindstedt and Kotowski, 2004; Lindstedt et al., 2009)

Equation $\varphi_{T12,T01}$ (8) binds diagnostic signals y(t) to environment signals x(t) as well and so it is the next diagnostic model. As in case of $A^2_{T12,T01}$ this model is also being determined without necessity of practical measurement of environment signal x(t). To determine signals S^{T12}_{xy}, S^{T01}_{xy} distribution in form of function δ (t, $\hat{t}$) is to be used. The quotient of function of signal y power density and distribution δ (t, $\hat{t}$) sufficiently representing environment signals x is assumed to eliminate real environment of model $\varphi_{T12,T01}$. (Bendat and Piersol, 1976; Lindstedt et al., 2009; Niederliński, 1985; Szabatin, 2000)

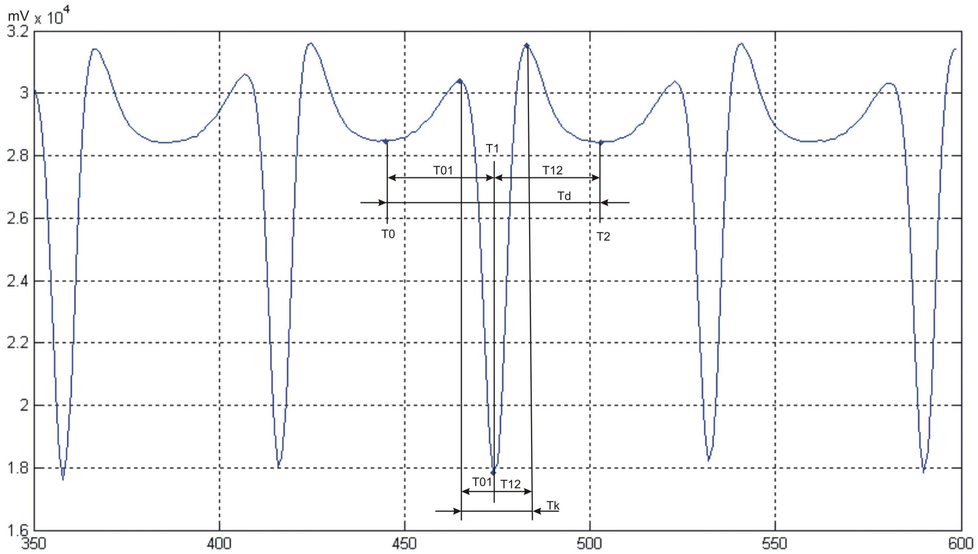

Fig. 5. Inferential sensor signals (T_d, T_k – respectively long and short observation period of tip of a blade staying in sensor area, T_0, T_1, T_2 – particular moments of observation of blade tip below the sensor, T_{01}, T_{12} – observation subperiods of blade tip for T_d and T_k respectively, mV – signal of tip of the blade translocation, uS – blade translocation time)

4. METHOD OF CURRENT ASSESSMENT OF BLADE TECHNICAL CONDITION BASING ON OBSERVATION OF PARAMETERS OF MODEL $A^2_{T12,T01}$

Method of current assessment of rotor machine blade technical condition changes basing on observation of parameters of model $A^2_{T12,T01}$ requires specific diagnostic research.

Afterwards estimates of autocorrelation function R^{*T01}_{yy} and R^{*T12}_{yy} are determined for y(t) translocation in observation periods T_{01} and T_{12} and proper analytic expressions are matched to them. (Bendat and Piersol, 1976; Kotowski and Lindstedt, 2007; Kurowski, 1994; Lindstedt and Kotowski, 2004; Lindstedt et al., 2009; Niederliński, 1985).

Registered signal courses were multiplied by Hanning window and, afterwards, its autocorrelation was calculated. The obtained autocorrelation graphs were approximated with degree 5 polynomial in form of:

$$R_{yy}(\tau) = a_1\tau^5 + a_2\tau^4 + a_3\tau^3 + a_4\tau^2 + a_5\tau + a_6 \tag{9}$$

For approximation of $R^2 > 0{,}997$ described by determination coefficient the following autocorrelation function forms were obtained:

- for blade no. 1.

$$R^{T01}_{yy} = 1557x^5 - 207300x^4 + 9409000x^3 - 1637\cdot10^5x^2 + 4641\cdot10^5x + 9485\cdot10^5 \tag{10}$$

$$R^{T12}_{yy} = 2345x^5 - 285600x^4 + 1182\cdot10^4x^3 - 1876\cdot10^5x^2 + 5033\cdot10^5x + 8727\cdot10^6 \tag{11}$$

- for blade no. 2.

$$R^{T01}_{yy} = 1308x^5 - 182100x^4 + 8594000x^3 - 155\cdot10^6x^2 + 4468\cdot10^5x + 9757\cdot10^6 \tag{12}$$

$$R^{T12}_{yy} = 2043x^5 - 256800x^4 + 1097\cdot10^4x^3 - 1798\cdot10^5x^2 + 4908\cdot10^5x + 9039\cdot10^6 \tag{13}$$

- for blade no. 3.

$$R^{T01}_{yy} = 1448x^5 - 200500x^4 + 918\cdot10^4x^3 - 1607\cdot10^5x^2 + 456\cdot10^6s + 9428\cdot10^6 \tag{14}$$

$$R^{T12}_{yy} = 2548x^5 - 302300x^4 + 123\cdot10^5x^3 - 1931\cdot10^5x^2 + 518\cdot10^6x + 8775\cdot10^6 \tag{15}$$

Basing on analytic forms of self correlation functions R^{T01}_{yy} and R^{T12}_{yy}, the respective power spectral density functions $S^{T01}_{yy}(\omega)$ and $S^{T12}_{yy}(\omega)$ are determined using the Fourier transformation.:

$$F\{R_{yy}\} = \int_{-\infty}^{\infty} R_{yy}(\tau)e^{-j\omega\tau}d\tau \tag{16}$$

$$S^{T01}_{yy}(\omega) = F(R^{T01}_{yy}(\tau)) \tag{17}$$

$$S_{yy}^{T12}(\omega) = F(R_{yy}^{T12}(\tau)) \tag{18}$$

The Fourier transform of polynomial (9) after moving from "jω " space to variable "s" space is as follows:

$$F\left\{R_{yy}\right\} = a_1\frac{120}{s^6} + a_2\frac{24}{s^5} + a_3\frac{6}{s^4} + a_4\frac{2}{s^3} + a_5\frac{1}{s^2} + a_6\frac{1}{s} \tag{19}$$

After the substitution of parameters from equations (10 – 15) to equation (19) the following is obtained:
– for blade no. 1.

$$S_{yy}^{T01}(j\omega) = (9485\cdot 10^6 s^5 + 4641\cdot 10^5 s^4 - 3274\cdot 10^5 s^3 + 56454\cdot 10^3 s^2 - 4975200s + 186840)s^{-6} \tag{20}$$

$$S_{yy}^{T12}(j\omega) = (8727\cdot 10^6 s^5 + 5033\cdot 10^5 s^4 - 3752\cdot 10^5 s^3 + 7092\cdot 10^4 s^2 - 6854400s + 281400)s^{-6} \tag{21}$$

– for blade no. 2.

$$S_{yy}^{T01}(j\omega) = (9757\cdot 10^6 s^5 + 4468\cdot 10^5 s^4 - 31\cdot 10^7 s^3 + 51564\cdot 10^3 s^2 - 4370400s + 156960)s^{-6} \tag{22}$$

$$S_{yy}^{T12}(j\omega) = (9039\cdot 10^6 s^5 + 4908\cdot 10^5 s^4 - 3596\cdot 10^5 s^3 + 6582\cdot 10^4 s^2 - 6163200s + 245160)s^{-6} \tag{23}$$

– for blade no. 3.

$$S_{yy}^{T01}(j\omega) = (9428\cdot 10^6 s^5 + 456\cdot 10^6 s^4 - 3214\cdot 10^5 s^3 + 5508\cdot 10^4 s^2 - 4812000s + 178080)s^{-6} \tag{24}$$

$$S_{yy}^{T12}(j\omega) = (8775\cdot 10^6 s^5 + 518\cdot 10^6 s^4 - 3862\cdot 10^5 s^3 + 738\cdot 10^5 s^2 - 7255200s + 305760)s^{-6} \tag{25}$$

Ultimately new abstract diagnostic model (square of module) might be determined, the parameters of which present the data on technical conditions of diagnosed blade:

$$A_{T12,T01}^2 = \frac{S_{yy}^{T12}}{S_{yy}^{T01}} = k\frac{1 + L_1 s + L_2 s^2 + \ldots}{1 + M_1 s + M_2 s^2 + \ldots} \tag{26}$$

After the substitution of parameters from equations (20 – 25) to equation (26) the following is obtained:

– for blade no. 1.

$$A_{T12,T01}^2 = 1.506(\frac{31012.793s^5 + 1788.577s^4 - 1333.33s^3 + 252.025s^2 - 24.358s + 1}{50765.361s^5 + 2483.943s^4 - 1752.301s^3 + 302.151s^2 - 26.628s + 1}) \tag{27}$$

– for blade no. 2.

$$A_{T12,T01}^2 = 1.562(\frac{3686.799s^5 + 2001.958s^4 - 1466.797s^3 + 268.478s^2 - 25.139s + 1}{62162.334s^5 + 2846.585s^4 - 1975.025s^3 + 328.517s^2 - 27.844s + 1}) \tag{28}$$

– for blade no. 3.

$$A_{T12,T01}^2 = 1.717(\frac{28698.979s^5 + 1694.139s^4 - 1263.082s^3 + 241.366s^2 - 23.728s + 1}{52942.498s^5 + 2560.647s^4 - 1804.807s^3 + 309.299s^2 - 27.02s + 1}) \tag{29}$$

Tab. 1. The parameters of blades for the amplitude amplification

The parameters of blades for the amplitude amplification

		M_5	M_4	M_3	M_2	M_1	k
ROTOR BLADES	**S1T01**	50765.361	2483.943	-1752.301	302.151	-26.628	1.506
	S2T01	62162.334	2846.585	-1975.025	328.517	-27.844	1.562
	S3T01	52942.498	2560.647	-1804.807	309.299	-27.02	1.717
	$S_{śr}$	55290.064	2630.392	-1844.044	313.322	-27.164	1.595
		L_5	**L_4**	**L_3**	**L_2**	**L_1**	**k**
	S1T12	31012.793	1788.577	-1333.33	252.025	-24.358	1.506
	S2T12	36869.799	2001.958	-1466.797	268.478	-25.139	1.562
	S3T12	28698.979	1694.139	-1263.082	241.366	-23.728	1.717
	$S_{śr}$	32193.857	1828.225	-1354.403	253.956	-24.408	1.595

Process of rotor machine blade technical condition is as follows:

- in exploitation time θ_0 (monitoring beginning) function $A^2_{T12,T01}$ parameters: L_{01}, L_{02}, L_{03} ...and M_{01}, M_{02}, M_{03} ... are determined
- in exploitation time θ_1 (another blade monitoring) function $A^2_{T12,T01}$ parameters: L_{10}, L_{11}, L_{12}, ... and M_{10}, M_{11}, M_{12} ... are determined

The technological differences between the blades bight also be determined. In such case the average values of function $A^2_{T12,T01}$: : $L_{1śr}$, $L_{2śr}$, $L_{3śr}$ ...as well as $M_{1śr}$, $M_{2śr}$, $M_{3śr}$

parameters are determined and the diagnosed blades parameters are compared to them. (tab.1 and tab.2)

- the difference between technical condition of consecutive blades is determined basing on relative parameters changes:

$$\Delta \overline{L}_i = \frac{L_{i1} - L_{sr}}{L_{sr}} \text{ ; i = 1,….,n} \tag{30}$$

$$\Delta \overline{M}_i = \frac{M_{i1} - M_{sr}}{M_{sr}} \text{ ; i = 1,….,m} \tag{31}$$

The change of $\overline{L}_i$ and $\overline{M}_i$ parameters as well as a significant number of their configuration allows the identification of multiple various changes of the blade technical condition while the rotor machine is operating (this also applies to assessing the accuracy of performance and installation the new blades).

Tab. 2. Relative differences the parameters of blades for the amplitude amplification

Relative differences the parameters of blades for the amplitude amplification

ROTOR BLADES		ΔM_5	ΔM_4	ΔM_3	ΔM_2	ΔM_1	Δk
	S1T01	-0.082	-0.056	-0.050	-0.036	-0.020	-0.056
	S2T01	0.124	0.082	0.071	0.048	0.025	-0.021
	S3T01	-0.042	-0.027	-0.021	-0.013	-0.005	0.076
		ΔL_5	ΔL_4	ΔL_3	ΔL_2	ΔL_1	Δk
	S1T12	-0.037	-0.022	-0.016	-0.008	-0.002	-0.056
	S2T12	0.145	0.095	0.083	0.057	0.030	-0.021
	S3T12	-0.109	-0.073	-0.067	-0.050	-0.028	0.076

5. METHOD OF ASSESSMENT OF BLADE CURRENT TECHNICAL CONDITION BASING ON MODEL PARAMETERS OBSERVATION $\varphi_{T12,T01}$

Signal y(t) courses in observation times T_{01} and T_{02} are presented in Fig. 5. Environment is assumed to be e.g. high power noise and may be correlated with signal y(t). (Bendat and Piersol, 1976)

As an effect of mathematical operations correlation function estimators and analytic forms of correlation function were obtained:

- for blade no. 1.

$$R_{xy}^{T01} = 3196x^5 - 294900x^4 + 9775 \cdot 10^3 x^3 - 1319 \cdot 10^5 x^2 + 4442 \cdot 10^5 x + 257 \cdot 10^7 \tag{32}$$

$$R_{xy}^{T12} = 4502x^5 - 369500x^4 + 108 \cdot 10^5 x^3 - 1249 \cdot 10^5 x^2 + 2761 \cdot 10^5 x + 2767 \cdot 10^6 \tag{33}$$

- for blade no. 2.

$$R_{xy}^{T01} = 2770x^5 - 258400x^4 + 8621000x^3 - 1155 \cdot 10^5 x^2 + 3525 \cdot 10^5 x + 2649 \cdot 10^6 \tag{34}$$

$$R_{xy}^{T12} = 3578x^5 - 312500x^4 + 9769 \cdot 10^3 x^3 - 1225 \cdot 10^5 x^2 + 3334 \cdot 10^5 x + 2714 \cdot 10^6 \tag{35}$$

- for blade no. 3.

$$R_{xy}^{T01} = 3077x^5 - 285200x^4 + 95 \cdot 10^5 x^3 - 1287 \cdot 10^5 x^2 + 4335 \cdot 10^5 x + 2564 \cdot 10^6 \tag{36}$$

$$R_{xy}^{T12} = 4517x^5 - 370500x^4 + 1082 \cdot 10^4 x^3 - 1248 \cdot 10^5 x^2 + 2713 \cdot 10^5 x + 2794 \cdot 10^6 \tag{37}$$

Further, the following is determined:

- for blade no. 1.

$$S_{xy}^{T01}(j\omega) = (257 \cdot 10^7 s^5 + 4442 \cdot 10^5 s^4 - 2638 \cdot 10^5 s^3 + 5865 \cdot 10^4 s^2 - 7077600s + 383520)s^{-6} \tag{38}$$

$$S_{xy}^{T12}(j\omega) = (2767 \cdot 10^6 s^5 + 2761 \cdot 10^5 s^4 - 2498 \cdot 10^5 s^3 + 648 \cdot 10^5 s^2 - 8868 \cdot 10^3 s + 540240)s^{-6} \tag{39}$$

- for blade no. 2.

$$S_{xy}^{T01}(j\omega) = (2649 \cdot 10^6 s^5 + 3525 \cdot 10^5 s^4 - 2310 \cdot 10^5 s^3 + 51726 \cdot 10^3 s^2 - 6201600s + 332400)s^{-6} \tag{40}$$

$$S_{xy}^{T12}(j\omega) = (2714 \cdot 10^6 s^5 + 3334 \cdot 10^5 s^4 - 2450 \cdot 10^5 s^3 + 58614 \cdot 10^3 s^2 - 75 \cdot 10^5 s + 429360)s^{-6} \tag{41}$$

- for blade no. 3.

$$S_{xy}^{T01}(j\omega) = (2564 \cdot 10^6 s^5 + 4335 \cdot 10^5 s^4 - 2574 \cdot 10^5 s^3 + 57 \cdot 10^6 s^2 - 6844800s + 369240)s^{-6} \tag{42}$$

$$S_{xy}^{T12}(j\omega) = (2794 \cdot 10^6 s^5 + 2713 \cdot 10^5 s^4 - 2496 \cdot 10^5 s^3 + 6492 \cdot 10^4 s^2 - 8892000s + 542040)s^{-6} \tag{43}$$

and in the end:

$$\varphi_{T12,T01} = \frac{S_{xy}^{T12}}{S_{xy}^{T01}} = k\frac{1 + B_1 s + B_2 s^2 + ...}{1 + A_1 s + A_2 s^2 + ...} \tag{44}$$

After the substitution of parameters from equations (38 – 43) to equation (44) the following is obtained:

- for blade no. 1.

$$\varphi_{T12,T01} = Arg1.408(\frac{5121.798s^5 + 511.068s^4 - 462.387s^3 + 119.947s^2 - 16.415s + 1}{6701.085s^5 + 1158.219s^4 - 687.390s^3 + 152.925s^2 - 18.454s + 1}) \qquad (45)$$

- for blade no. 2.

$$\varphi_{T12,T01} = Arg1.292(\frac{6321.036s^5 + 776.504s^4 - 570.617s^3 + 136.515s^2 - 17.468s + 1}{7969.314s^5 + 1060.469s^4 - 694.946s^3 + 155.614s^2 - 18.657s + 1}) \qquad (46)$$

- for blade no. 3.

$$\varphi_{T12,T01} = Arg1.468(\frac{5154.601s^5 + 500.516s^4 - 460.482s^3 + 119.770s^2 - 16.405s + 1}{6943.993s^5 + 1174.033s^4 - 697.107s^3 + 154.371s^2 - 18.537s + 1}) \qquad (47)$$

Blades technical condition is specified by model $\varphi_{T12,T01}$ parameters.

Rotor machine blade technical condition monitoring process is as follows:

- in exploitation time θ_0 (monitoring beginning) function $\varphi_{T12,T01}$ parameters: B_{01}, B_{02}, B_{03} ...and A_{01}, A_{02}, A_{03} ... are determined
- in exploitation time θ_1 (another blade monitoring) function $\varphi_{T12,T01}$ parameters: B_{10}, B_{11}, B_{12}, ... and A_{10}, A_{11}, A_{12} ... are determined

The technological differences between successive blades might also be determined. In such case the average values of function $\varphi_{T12,T01}$: : $A_{1śr}$, $A_{2śr}$, $A_{3śr}$ ...as well as $B_{1śr}$, $B_{2\ r}$, $B_{3śr}$ parameters are determined and the diagnosed blades parameters are compared to them (tab.3 and tab.4).

- the difference between technical condition of consecutive blades is determined basing on relative parameters changes:

$$\Delta\overline{B}_i = \frac{B_{i1} - B_{sr}}{B_{sr}} \text{ ; i = 1,….,n} \qquad (48)$$

$$\Delta\overline{A}_i = \frac{A_{i1} - A_{sr}}{A_{sr}} \text{ ; i = 1,….,m} \qquad (49)$$

The change of $\overline{A}_i$ and $\overline{B}_i$ parameters as well as a significant number of their configuration allows the identification of multiple various changes of the blade technical condition while the rotor machine is operating (this also applies to investigate the new blades).

Tab. 3. The parameters of blades for the phase shift

The parameters of blades for the phase shift

		A_5	A_4	A_3	A_2	A_1	k
ROTOR BLADES	**S1T01**	6701.085	1158.219	-687.39	152.925	-18.454	1.408
	S2T01	7969.314	1060.469	-694.946	155.614	-18.657	1.292
	S3T01	6943.993	1174.033	-697.107	154.371	-18.537	1.468
	$S_{śr}$	7204.797	1130.907	-693.148	154.303	-18.549	1.389
		B_5	**B_4**	**B_3**	**B_2**	**B_1**	**k**
	S1T12	5121.798	511.068	-462.387	119.947	-16.415	1.408
	S2T12	6321.036	776.504	-570.617	136.515	-17.468	1.292
	S3T12	5154.601	500.516	-460.482	119.77	-16.405	1.468
	$S_{śr}$	5532.478	596.029	-497.829	125.411	-16.763	1.389

Tab. 4. Relative differences the parameters of blades for phase shift

Relative differences the parameters of blades for phase shift

		ΔA_5	ΔA_4	ΔA_3	ΔA_2	ΔA_1	Δk
ROTOR BLADES	**S1T01**	-0.070	0.024	-0.008	-0.009	-0.005	0.013
	S2T01	0.106	-0.062	0.003	0.008	0.006	-0.070
	S3T01	-0.036	0.038	0.006	0.000	-0.001	0.057
		ΔB_5	**ΔB_4**	**ΔB_3**	**ΔB_2**	**ΔB_1**	**Δk**
	S1T12	-0.074	-0.143	-0.071	-0.044	-0.021	0.013
	S2T12	0.143	0.303	0.146	0.089	0.042	-0.070
	S3T12	-0.068	-0.160	-0.075	-0.045	-0.021	0.057

6. CONCLUSIONS

Method of current assessment of blade technical condition changes basing on diagnostic models $A^2_{T12,T01}$ and $\varphi_{T12,T01}$ is innovative method of blade diagnostics without environment signal measurements.

Method of blade technical condition monitoring may be based on diagnostic model in form of quotient of output y(t) signal amplifications to environment signal x(t) for observation time T_{01} and T_{12}. This method consists in fact that time T (Fig. 5.) of blade tip movement in sensor area is divided onto two ranges: of blade tip approaching the sensor T_{01} and receding from it T_{12}.

Both observation periods T_{01} and T_{12} of y(t) are so close in time to each other that the environment for these observation periods may be considered identical $S^{T01}_{xx} = S^{T12}_{xx}$.

Method of technical condition monitoring is further distinguished by determining power spectral density S^{T01}_{yy}, S^{T12}_{yy} of signal y(t) through analytic forms of autocorrelation functions R^{T01}_{yy}, R^{T12}_{yy} and S^{T01}_{xy}, S^{T12}_{xy} through reciprocal correlation function R^{T01}_{xy}, R^{T12}_{xy} of signal y(t) and distribution function representing environment x(t). Required match (greater than 0,99) is obtained through proper choice of observation range T_d and T_k (Fig. 5.) and measurement window function: rectangular, Hamming, Hanning etc.

Another distinctive feature of models $A^2_{T12,T01}$ and $\varphi_{T12,T01}$ is no necessity of environment signals measurement although these are indirectly taken into account within special research (two observation periods, determination of diagnostic model as a quotient of models binding diagnostic and environment signals to technical condition parameters).

Method of blade technical condition monitoring may be also based on diagnostic model in form of quotient of phase shifts of output signal y(t) to environment signal x(t) for observation time T_{01} and T_{02}. Observation time T is divided onto two ranges. $\varphi_{T12,T01}$ is determined as a quotient of reciprocal power density S^{T01}_{xy} and S^{T12}_{xy} (to determine S^{T01}_{xy} and S^{T12}_{xy} noise of identical distribution of form is used δ (t, $\hat{t}$)).

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