POSITIVE SWITCH 2D LINEAR SYSTEMS DESCRIBED BY THE GENERAL MODELS

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Abstract: The positive switched 2D linear systems described by the general models are addressed. Necessary and sufficient conditions for the asymptotic stability of the positive switched system are established for any switching. The considerations are illustrated by numerical examples.

1. INTRODUCTION

In positive systems inputs, state variables and outputs take only non-negative values. Examples of positive systems are industrial processes involving chemical reactors, heat exchangers and distillation columns, storage systems, compartmental systems, water and atmospheric pollution models. A variety of models having positive linear behavior can be found in engineering, management science, economics, social sciences, biology and medicine, etc.

Positive linear systems are defined on cones and not on linear spaces. Therefore, the theory of positive systems is more complicated and less advanced. An overview of state of the art in positive systems theory is given in the monographs Farina and Rinaldi (2000) and Kaczorek (2009).

The most popular models of two-dimensional (2D) linear systems are the discrete models introduced by Roesser (1975), Fornasini and Marchesini (1976, 1978) and Kurek (1985). These models have been extended for positive systems in Kaczorek (1996; 2005) and Valcher (1997). An overview of standard and positive 2D systems theory is given in Bose (1985), Gałkowski (2001) and Kaczorek (1985) and some recent results in positive systems have been given in Kaczorek (1996, 2001, 2002, 2005, 2007a, b, 2009). The stability of switched linear systems has been investigated in many papers and books (Colaneri, 2009; Liberzon, 2003, 2009; Sun and Ge, 2004). The disturbance decoupling problem for switched linear continuous-time systems by state-feedback has been considered in Otsuka (2010) and the stabilizer design of planar switched linear systems in Hu and Cheng (2008).

In this paper the positive switched 2D linear system described by the general models will be considered. We shall analyze the following question: When is a positive switched 2D linear system defined by linear general models and a rule describing the switching between them asymptotically stable. It is well known (Liberzon, 2003, 2009) that a necessary and sufficient conditions for stability under arbitrary switching is the existence of a common Lyapunov function for the family of subsystems. This result will be extended for positive switched 2D linear systems described by the general models.

The paper is organized as follows. Preliminaries and the problem formulation are given in section 2. The main results of the paper are presented in section 3, where necessary and sufficient conditions are established for the asymptotic stability of the positive switched 2D linear systems described by the general model for any switching. Illustrating examples are presented in section 4. Concluding remarks are given in section 5. In Appendix the definition of equilibrium point is given and the formula determining the point is derived.

To the best of the author's knowledge the positive switched 2D linear systems have not been considered yet.

2. PRELIMINARIES AND THE PROBLEM FORMULATION

Let $\Re^{n\times m}$ be the set of $n\times m$ real matrices. The set $n\times m$ matrices with nonnegative entries will be denoted by $\Re_+^{n\times m}$ and $\Re_+^n = \Re_+^{n\times 1}$. A matrix $A=[a_{ij}]\in\Re^{n\times m}$ (vector x) is called strictly positive and denoted by $A > 0$ ($x > 0$) if $a_{ij} > 0$ for $i = 1,\dots,n$; $j = 1,\dots,m$. The et of nonnegative integers will be denoted by Z_+ and the n x n identity matrix will be denoted by I_n.

The general model of 2D linear system has the form (Kurek, 1985):

$$\begin{aligned} x_{i+1,j+1} &= \overline{A}_0 x_{i,j} + \overline{A}_1 x_{i+1,j} + \overline{A}_2 x_{i,j+1} \\ &\quad + \overline{B}_0 u_{i,j} + \overline{B}_1 u_{i+1,j} + \overline{B}_2 u_{i,j+1} \end{aligned} \tag{2.1a}$$

$$y_{i,j} = \overline{C} x_{i,j} + \overline{D} u_{i,j},\ i, j \in Z_+. \tag{2.1b}$$

where $x_{i,j}\in\Re^n$, $u_{i,j}\in\Re^m$ and $y_{i,j}\in\Re^p$ are the state, input and output vectors and $\overline{A}_k\in\Re^{n\times n}$, $\overline{B}_k\in\Re^{n\times m}$, $k=0,1,2$; $\overline{C}\in\Re^{p\times n}$, $\overline{D}\in\Re^{p\times m}$.

Boundary conditions for (2.1a) have the form

$$x_{i0} \in \Re^n, i \in Z_+ \text{ and } x_{0j} \in \Re^n, \ j \in Z_+ \tag{2.2}$$

The model (2.1) is called (internally) positive general model if $x_{i,j} \in \Re^n_+$, and $y_{i,j} \in \Re^p_+$, $i, j \in Z_+$ for any non-negative boundary conditions

$$x_{i0} \in \Re^n_+, i \in Z_+, \ x_{0j} \in \Re^n_+, \ j \in Z_+ \tag{2.3}$$

and all input sequences $u_{i,j} \in \Re^m_+$, $i, j \in Z_+$.

Theorem 2.1. [13] The general model is positive if and only if

$$\begin{aligned} &\overline{A}_k \in \Re^{n\times n}_+, \quad \overline{B}_k \in \Re^{n\times m}_+, \quad k = 0,1,2, \\ &\overline{C} \in \Re^{p\times n}_+, \quad \overline{D} \in \Re^{p\times m}_+ \end{aligned} \tag{2.4}$$

The positive general model (2.1) is called asymptotically stable if for any boundary conditions (2.3) and zero inputs $u_{i,j} = 0$, $i, j \in Z_+$

$$\lim_{i,j\to\infty} x_{i,j} = 0 \tag{2.5}$$

Theorem 2.2. (Kaczorek, 2009) The positive general model (2.1) is asymptotically stable if and only if there exists a strictly positive vector $\lambda \in \Re^n_+$ such that

$$[\overline{A} - I_n]\lambda < 0 \text{ , and } \overline{A} = A_0 + A_1 + A_2 \tag{2.6}$$

Theorem 2.3. (Kaczorek, 2009a, b) The positive general model (2.1) is asymptotically stable if and only if the positive 1D linear system

$$x_{i+1} = \overline{A} x_i \tag{2.7}$$

is asymptotically stable, where matrix $\overline{A}$ is given by (2.6).

Theorem 2.4. (Kaczorek, 2009) The positive general model is asymptotically stable if and only if one of the following equivalent conditions is met:

– Eigenvalues $z_1, \ldots, z_n$ of the matrix $\overline{A}$ have modules less than 1, i.e.

$$|z_k| < 1 \text{ for } k = 1,\ldots,n \tag{2.8a}$$

– All coefficients a_i, $i = 0,1,\ldots,n-1$ of the characteristic polynomial

$$p(z) = \det[I_n(z+1) - \overline{A}] = z^n + a_{n-1}z^{n-1} + \ldots + a_1 z + a_0 \tag{2.8b}$$

are positive, i.e. $a_i > 0$, $i = 0,1,\ldots,n-1$.

– All principal minors of the matrix

$$\hat{A} = I_n - \overline{A} = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \ldots & \hat{a}_{1n} \\ \hat{a}_{21} & \hat{a}_{22} & \ldots & \hat{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{a}_{n1} & \hat{a}_{n2} & \ldots & \hat{a}_{nn} \end{bmatrix} \tag{2.8c}$$

are positive, i.e.

$$\hat{a}_{11} > 0, \quad \begin{vmatrix} \hat{a}_{11} & \hat{a}_{12} \\ \hat{a}_{21} & \hat{a}_{22} \end{vmatrix} > 0, \ldots, \det \hat{A} > 0 \tag{2.8d}$$

Consider the switched positive system consisting of q autonomous positive general models of the form

$$x_{i+1,j+1} = A_{l0}x_{i,j} + A_{l1}x_{i+1,j} + A_{l2}x_{i,j+1}, \ l = 1,\ldots,q \tag{2.9}$$

It is assumed that in the point (p_i, t_i), $p_i, t_i \in Z_+$, $i = 1,\ldots,q$ the matrices of the general model jump instantaneously from A_{ik} to A_{jk} for some $i \neq j$, $i, j = 1,\ldots,q$; $k = 0,1,2$. The following question arises: when the switched positive general model (2.9) is asymptotically stable for every switching if every positive general model of the set is asymptotically stable.

3. PROBLEM FORMULATION

To simplify the notation it is assumed $q = 2$. In this case the switched positive system consists of two positive general models

$$x_{i+1,j+1} = A_{10}x_{i,j} + A_{11}x_{i+1,j} + A_{12}x_{i,j+1} \tag{3.1a}$$

$$x_{i+1,j+1} = A_{20}x_{i,j} + A_{21}x_{i+1,j} + A_{22}x_{i,j+1} \tag{3.1b}$$

where $A_{lk} \in \Re^{n\times n}_+$, $l = 1,2$; $k = 0,1,2$ and the switching between them occur in the points

$$(p_1, t_1), \ldots, (p_k, t_k), (p_{k+1}, t_{k+1}), \ldots \tag{3.2}$$

satisfying the condition

$$p_{k+1} \ge p_k, \ t_{k+1} \ge t_k \text{ and } p_{k+1} + t_{k+1} > p_k + t_k, \quad k = 1,2,\ldots \tag{3.3}$$

Theorem 3.1. (Kaczorek, 2001) The solution of the autonomous ($u_{i,j} = 0$, $i, j \in Z_+$) positive general model (3.1a) with boundary conditions (2.3) is given by

$$\begin{aligned} x_{i,j} &= T_{i-1,j-1}A_0 x_{00} + \sum_{t=1}^{i-1} T_{i-t-1,j-1}A_0 x_{t0} \\ &+ \sum_{v=1}^{j-1} T_{i-1,j-v-1}A_0 x_{0v} + \sum_{t=1}^{i} T_{i-t,j-1}A_1 x_{t0} \\ &+ \sum_{v=1}^{j} T_{i-1,j-v}A_2 x_{0v} \end{aligned} \tag{3.4}$$

where the transition matrix T_{ij} is defined by

$$T_{ij} = \begin{cases} I_n & \text{for } \quad i = j = 0 \\ A_0 T_{i-1,j-1} + A_2 T_{i-1,j} + A_1 T_{i,j-1} & \text{for } i, j \ge 0 \ (i + j > 0) \\ 0 & \text{for } \quad i < 0 \text{ or } j < 0 \end{cases} \tag{3.5}$$

Using (3.4), (3.5) and the boundary conditions (2.3) we can compute the state vector x_{ij} for $i, j \in Z_+$.

Theorem 3.2. The switched positive system (3.1) is asymptotically stable for any switching (3.2) satisfying (3.3) only if both positive models (3.1) are asymptotically stable.

Proof. Without loss of generality we may assume that the first model (3.1a) is asymptotically stable and the second (3.1b) is unstable. For any bounded boundary conditions (2.3) using (3.4), (3.5) and the first model we may compute boundary conditions for the second model which will be bounded since the first model is asymptotically stable. In the similar way we may compute the boundary conditions for the first model but those boundary conditions will be greater than those for the second model since it is unstable. Therefore, the switched positive general model will be unstable.

In (Kaczorek, 2007) it was shown that for positive model (2.1a) as a Lyapunov function a linear form $V(x_{ij}) = \lambda^T x_{ij}$ can be chosen, where $\lambda \in \Re_+^n$ is strictly positive.

For the switched positive system consisting of positive models (3.1) we choose Lyapunov functions in the form

$$V_1(x_{ij}) = \lambda_1^T x_{ij} \text{ and } V_2(x_{ij}) = \lambda_2^T x_{ij} \tag{3.6}$$

where the strictly positive vectors λ_1 and λ_2 satisfy the equations

$$\lambda_1 = A_1\lambda_1 + 1_n,\ \lambda_2 = A_2\lambda_2 + \lambda_1,\ 1_n = [1\ \ 1\ \ \dots\ \ 1]^T \in \Re_+^n \tag{3.7}$$

and $A_k = A_{k0} + A_{k1} + A_{k2}$, k = 1,2.

If A_1 and A_2 are Schur matrices then from (3.7) we have

$$\lambda_1 = [I_n - A_1]^{-1} 1_n \text{ and } \lambda_2 = [I_n - A_2]^{-1}\lambda_1 \tag{3.8}$$

Remark 3.1. From the comparison of (3.3) and (A.2), (A.3) it follows that as λ_1 we can choose equilibrium point x_e for $Bu = 1_n$ and as λ_2 the vector x_e for $Bu = \lambda_1$.

Lemma 3.1. The function

$$V_2(x_{ij}) = \lambda_2^T x_{ij} \tag{3.9}$$

is a common Lyapunov function for the both positive general models (3.1) if

$$A_1A_2 = A_2A_1 \tag{3.10}$$

Proof. The function (3.9) for both positive Roesser models (3.1) for strictly positive $\lambda_2 \in \Re_+^n$ is positive if and only if $x_{ij} \neq 0$. Note that the dual general models

$$x_{i+1,j+1} = A_{10}^T x_{i,j} + A_{11}^T x_{i+1,j} + A_{12}^T x_{i,j+1} \tag{3.11a}$$

$$x_{i+1,j+1} = A_{20}^T x_{i,j} + A_{21}^T x_{i+1,j} + A_{22}^T x_{i,j+1} \tag{3.11b}$$

are positive and asymptotically stable if and only if the corresponding general models (3.1) are positive and asymptotically stable (Kaczorek, 2007). Taking into account Theorem 2.3 and using (3.9) for the positive general model (3.1a) we obtain

$$\Delta V_2(x_{ij}) = \lambda_2^T [A_1 - I_n]x_{ij} \tag{3.12}$$

and

$$\lambda_2^T = \lambda_1^T [I_n - A_2]^{-1} \tag{3.13}$$

since $\lambda_2 = A_2^T \lambda_2 + \lambda_1$.

Substitution of (3.13) into (3.12) yields

$$\begin{aligned}\Delta V_2(x_{ij}) &= \lambda_1^T [I_n - A_2]^{-1}[A_1 - I_n]x_{ij} \\ &= \lambda_1^T [A_1 - I_n][I_n - A_2]^{-1} x_{ij} \\ &= -1_n^T [I_n - A_2]^{-1} x_{ij} < 0\end{aligned} \tag{3.14}$$

for every $x_{ij} \in \Re_+^n$, $x_{ij} \neq 0$, since (3.10) implies $[A_1 - I_n][I_n - A_2]^{-1} = [I_n - A_2]^{-1}[A_1 - I_n]$ and the sum of entries of each column of the matrix $1_n^T[I_n - A_2]^{-1}$ is positive for the positive asymptotically stable general model (3.1b) .

Similarly, using (3.9) for positive general model (3.11b) we obtain

$$\Delta V_2(x_{ij}) = \lambda_2^T [A_2^T - I_n]x_{ij} < 0 \tag{3.15}$$

for every $x_{ij} \in \Re_+^n$, $x_{ij} \neq 0$, since $\lambda_2^T [A_2^T - I_n] < [0\ \ \dots\ \ 0]$.

Theorem 3.3. Let the matrices A_1 and A_2 of (3.1) satisfy the conditions (3.10). The positive switched system (3.1) is asymptotically stable for any switching (3.2) satisfying (3.3) if and only if the positive models (3.1) are asymptotically stable.

Proof. Necessity follows immediately from Theorem 3.2. If the condition (3.10) is met and the models (3.1) are asymptotically stable then by Lemma 3.1 the function (3.9) is a common Lyapunov function for the positive models (3.1) which satisfies the conditions (3.14) and (3.15). Therefore, the positive switched system (3.1) is asymptotically stable.

Remark 3.2. It is well-known (Kaczorek, 2001) that substituting $B_1 = B_2 = 0$ into (2.1) we obtain the first Fornasini-Marchesini model and substituting $A_0 = 0$ and $B_0 = 0$ we obtain second Fornasini-Marchesini model.

Consider the positive 2D Roesser model (Kaczorek, 2001):

$$\begin{bmatrix} x_{i+1,j}^h \\ x_{i,j+1}^v \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + \begin{bmatrix} B_{11} \\ B_{22} \end{bmatrix} u_{i,j} \tag{3.16a}$$

$$y_{i,j} = [C_1 \ \ C_2]\begin{bmatrix} x_{i,j}^h \\ x_{i,j}^v \end{bmatrix} + Du_{i,j} \quad i,j \in Z_+ \tag{3.16b}$$

where $x_{i,j}^h \in \Re_+^{n_1}$ and $x_{i,j}^v \in \Re_+^{n_2}$ are the horizontal and vertical state vectors at the point (i,j) $u_{i,j} \in \Re_+^m$ and $y_{i,j} \in \Re_+^p$ are the input and output vectors and $A_{kl} \in \Re_+^{n_k \times n_l}$, $k,l = 1,2$, $B_{11} \in \Re^{n_1 \times m}$, $B_{22} \in \Re^{n_2 \times m}$, $C_1 \in \Re^{p \times n_1}$, $C_2 \in \Re^{p \times n_2}$, $D \in \Re^{p \times m}$.

The positive 2D Roesser model (3.16) is a particular case of the positive second Fornasini-Marchesini model for

$$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}, \quad A_2 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}, \tag{3.17a}$$

$$B_1 = \begin{bmatrix} 0 \\ B_{22} \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_{11} \\ 0 \end{bmatrix} \tag{3.17b}$$

Therefore, the presented results are also valid for the positive Fornasini-Marchesini models and the positive Roesser model.

4. ILLUSTRATING EXAMPLES

Example 4.1. Consider the positive switched system consisting of two general models (3.1) with the matrices

$$A_{10} = \begin{bmatrix} 0.5 & 0.3 \\ 0 & 0.1 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 1 & 0.5 \\ 0 & 0.2 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.1 \end{bmatrix},$$
$$A_{20} = \begin{bmatrix} 0.05 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0 & 0 \\ 0.4 & 0.05 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.05 & 0 \\ 0.3 & 0.05 \end{bmatrix} \tag{4.1}$$

and the boundary conditions

$$x_{00} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad x_{i0} = x_{0j} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad i, j = 1,2,... \tag{4.2}$$

The switching occurs between them in the points

$$(2,1), (3,3), (5,4), (6,6), ... \tag{4.3}$$

The first model is unstable since the matrix $A_1 = A_{10} + A_{11} + A_{12} = \begin{bmatrix} 2 & 1 \\ 0 & 0.4 \end{bmatrix}$ has one diagonal entry greater than 1 (Kaczorek, 2009) and the second model is asymptotically stable. By Theorem 3.2 the positive switched system is unstable since $\lim x_{i,j} = \infty$ (Fig.1).

Example 4.2. Consider the positive switched system consisting of two general models (3.1) with the matrices

$$A_{10} = \begin{bmatrix} 0.1 & 0.05 \\ 0.1 & 0 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} 0.2 & 0.05 \\ 0.1 & 0.1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix},$$
$$A_{20} = \begin{bmatrix} 0.3 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix} \tag{4.4}$$

and the boundary conditions (4.2).

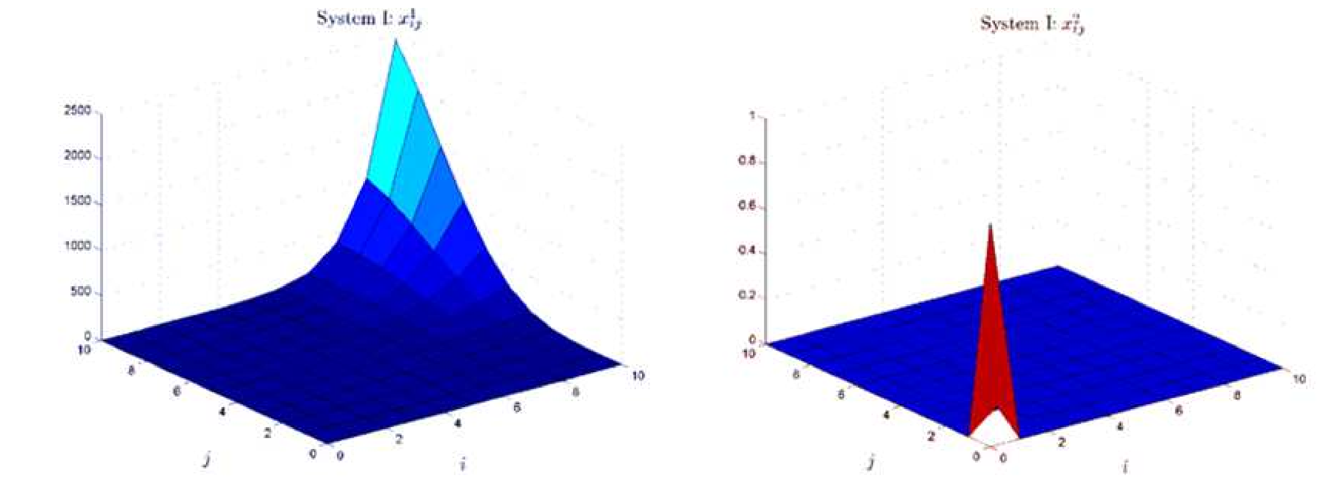

Fig. 1a. State variables of the first system with A_1

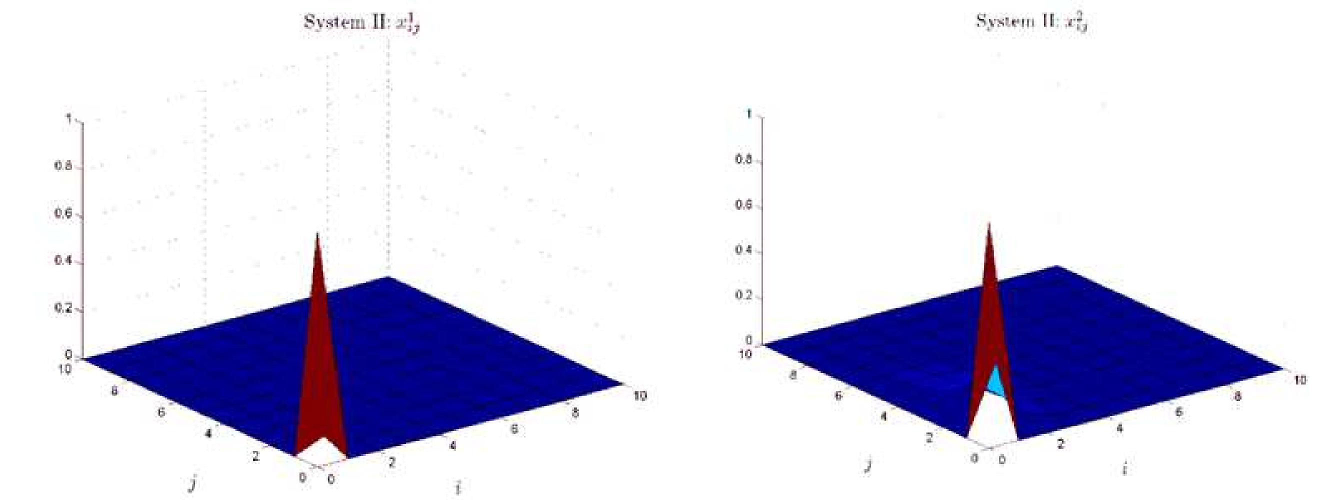

Fig. 1b. State vector for second system with A_2

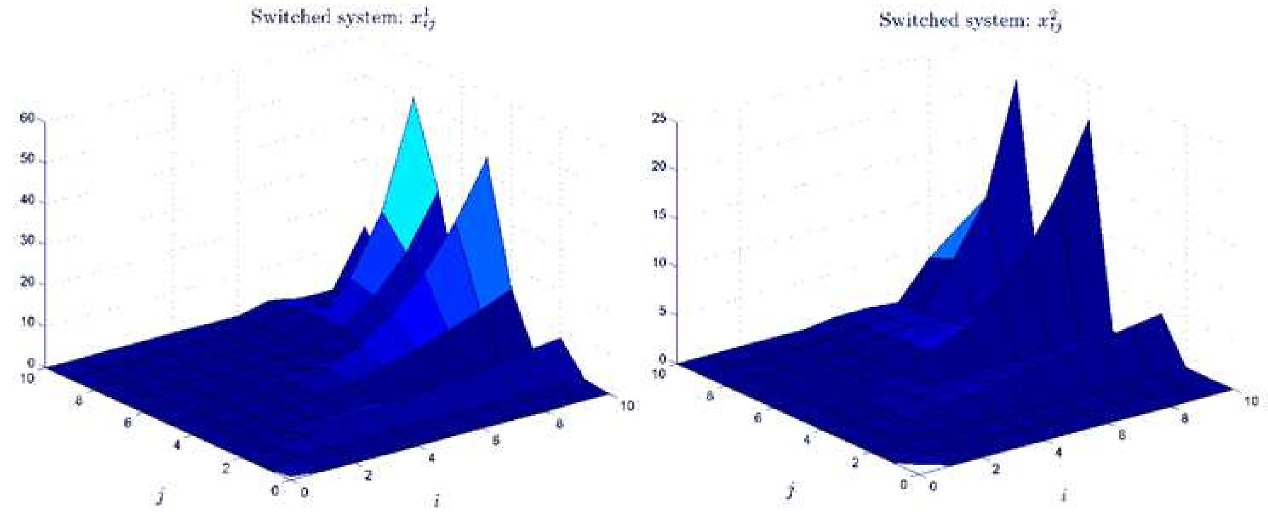

Fig. 2. State variables of the system with switches

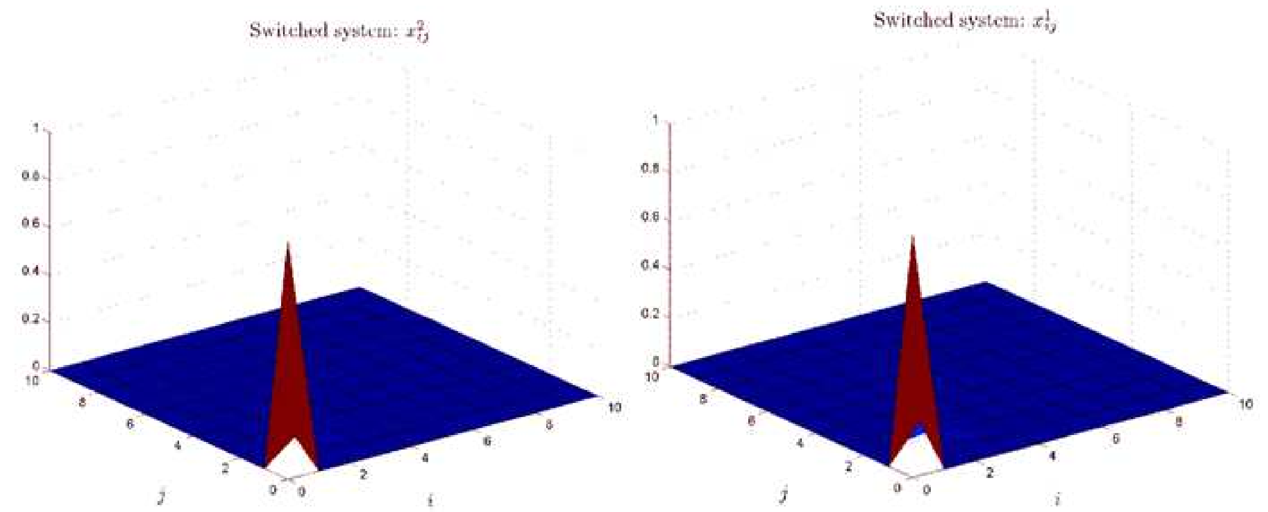

Fig. 3. State variables of the system with switches

The switching occur between them in the points (4.3).

In this case both general models are asymptotically stable and the matrices satisfy the condition (3.10). By Theorem 3.3 the positive switched system with (4.3) and (4.2) is asymptotically stable (Fig. 2).

The presented considerations can be easily extended to the positive switched linear systems consisting of q ($q > 2$) autonomous general models.

5. CONCLUDING REMARKS

The positive switched 2D linear systems described by the general models have been addressed. Necessary and sufficient conditions have been established for the asymptotic stability of the positive switched systems for any switching.

The considerations for positive switched 2D linear systems described by the Fornasini-Marchesini models and the Roesser 2D model are particular cases of the positive switch 2D linear systems described by the general model.

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APPENDIX

Consider the positive asymptotically stable general model (2.1) for the positive constant input $u_{i,j} = u$.

Definition A.1. A state x_e satisfying the equation

$$x_e = \overline{A}x_e + \overline{B}u,\quad \overline{A} = \overline{A}_0 + \overline{A}_1 + \overline{A}_2,\quad \overline{B} = \overline{B}_0 + \overline{B}_1 + \overline{B}_2 \quad (A.1)$$

is called the equilibrium point of the positive asymptotically stable general model (2.1) for $u > 0$.

Theorem A.1. The equilibrium point of the positive general model (2.1) is given by

$$x_e = [I - \overline{A}]^{-1}\overline{B}u\,. \quad (A.2)$$

Proof. If the system is asymptotically stable then the matrix $[I - \overline{A}]$ is invertible and from (A.1) we obtain (A.2).

From (A.1) it follows that for positive general model x_e is strictly positive if Bu is strictly positive vector.

In particular case from (A.2) for $\overline{B}u = 1_n$, $1_n = [1 \quad \dots \quad 1]^T \in \Re_+^n$ we obtain strictly positive vector

$$x_e = [I - \overline{A}]^{-1}1_n > 0\,. \quad (A.3)$$