

## FINITE ELEMENT ANALYSIS OF TEMPERATURE DISTRIBUTION IN AXISYMMETRIC MODEL OF DISC BRAKE

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**Abstract:** A transient thermal analysis is developed to examine temperature expansion in the disc and pad volume under simulated operation conditions of single braking process. This complex problem of frictional heating has been studied using finite element method (FEM). The Galerkin algorithm was used to discretize the parabolic heat transfer equation for the disc and pad. FE model of disc/pad system heating with respect to constant thermo-physical properties of materials and coefficient of friction was performed. The frictional heating phenomena with special reference to contact conditions was investigated. An axisymmetric model was used due to the proportional relation between the intensity of heat flux perpendicular to the contact surfaces and the rate of heat transfer. The time related temperature distributions in axial and radial directions are presented. Evolution of the angular velocity and the contact pressure during braking was assumed to be nonlinear. Presented transient finite element analysis facilitates to determine temperature expansion in special conditions of thermal contact in axisymmetric model.

### 1. INTRODUCTION

The automotive application of the disc brakes is susceptible to relatively high and stability of the friction coefficient. However its value affects the temperature rise, which is firmly intensive during repetitive braking process or emergency, high-speed stops. It is essential to predict the impact of the real geometrical set of the disc brake system to facilitate evaluation of the heat expansion with special operation conditions. High temperature exceeding permitted values may cause premature wear, brake fade, thermal judder or thermal cracks in the rotor material.

The calculation of the heat generated between two bodies in sliding contact such as disc brake systems necessitates appropriate model including contact conditions to obtain reliable outcomes. Various techniques have been employed for the computation of sliding surface temperatures. Analytical methods of heat conduction problem are limited of the half-space or the half-plane. More accurate for finite object, transform technique have been used, but numerous mathematical difficulties implies simplifications in geometry. The finite element method among numerical techniques is held as the most suitable for thermal problem investigation recently.

Talati and Jalalifar (2008, 2009) proposed two models of frictional heating in automotive disc brake system: namely macroscopic and microscopic model. In the macroscopic model the first law of thermodynamics has been taken into account and for microscopic model various characteristics such as duration of braking, material properties, dimensions and geometry of the brake system have been studied. Both disc and pad volume have been investigated to evaluate temperature distributions. The conduction heat

transfer was investigated using finite element method (Talati and Jalalifar, 2008). The same authors solved heat conduction problem analytically using Green's function approach (2009).

For simulation cause of the experimental results, the finite element method is proposed as a relevant numerical simulation of disc/pad interface temperature estimation by Qi and Day (2007). Special effort is employed in the real and apparent area of contact between two sliding bodies due to topography of the friction surface. Authors attempt to determine range of the affection on its performance including temperature growth and wear presence.

Choi and Lee (2004) deal with the thermoelastic behaviour of disc brakes. Contact problems in disc/pad interface including heat transfer and elastic problem have been studied. In addition, the influence of the material properties were analysed. Based on numerical results, the carbon-carbon materials with expected excellent characteristics were compared.

Heat transfer problem in the mine winder disc brake using FE modelling technique has been developed by Ścieszka and Żolnierz (2007). Temperature distributions including thermoelastic instability phenomena were analysed. Wide variety of the parameters used in the computations were adopted from examinations comprising infrared mapping. The numerical simulation was confirmed in the experimental investigations.

In this paper the finite element method for temperature assessment due to frictional heating in an axisymmetric arrangement of the disc brake model is developed. Irresistible advantages of this numerical technique are reported by Grześ (2009). Perfect contact conditions of thermal behavior of disc/pad zone have been established.

## 2. STATEMENT OF THE PROBLEM

Considering physical substance of the friction systems, the energy conversion should be noticeable as a dominant. The large amount of the thermal energy are transferred into kinetic energy to decelerate vehicle being in motion. In the disc brake systems two major parts may be distinguished: rotating axisymmetric disc and immovable non-axisymmetric pad (Fig. 1). While braking process occurs total heat is dissipated by conduction from disc/pad interface to adjacent components of brake assembly and hub and by convection to atmosphere in accordance to Newton's law. In common the radiation is neglected due to relatively low temperature and short time of the braking process.

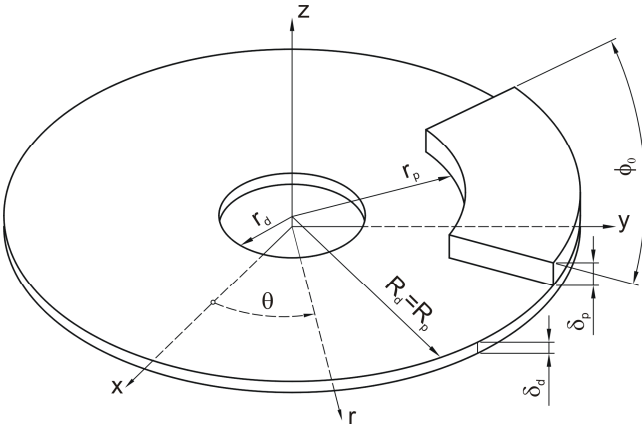


Fig. 1. The schematic assignment of disc brake system

The procedure of the temperature distribution assessment utilizing finite element method, adapted in an axisymmetric model is an efficient method which has already been reported in the area of frictional heating problem (Choi and Lee, 2004; Grześ, 2009; Ramachandra Rao et al., 1989). Grześ (2009) analyzed two types of the disc brake assembly related to different boundary conditions including evolution of contact pressure and velocity of the vehicle for validation purposes of the developed numerical technique.

In this paper temperature distributions in the disc and the pad volume have been studied. Material properties are assumed to be isotropic and independent of the temperature. The real surface of contact between a brake disc and pad in operation is equal to the apparent surface in the sliding contact. Perfect contact conditions for simplification purposes were assumed.

Single disc with pad presence has been analyzed with its simplification to symmetrical problem. Hence one side of the disc has been insulated in the FE model. Furthermore adiabatic boundary conditions are prescribed on the boundary of the inner radii of the disc and on the piston side of the pad.

Excluding both thermally insulated boundaries and the area of sliding contact where the intensity of heat flux has been established, on all remaining surfaces to consider realistic heat conditions, the exchange of thermal energy by convection to atmosphere has been implied.

It is assumed that the pressure varies with time (Chichinadze et al., 1979)

$$p(t) = p_0 \left( 1 - e^{-\frac{t}{t_m}} \right), \quad 0 \leq t \leq t_s, \quad (1)$$

where:  $p_0$  is the nominal pressure,  $t_m$  is the growing time,  $t_s$  is the braking time.

The angular velocity corresponding to pressure (1) equals (Yevtushenko et al., 1999)

$$\omega(t) = \omega_0 \left[ 1 - \frac{t}{t_s^0} + \frac{t_m}{t_s^0} \left( 1 - e^{-\frac{t}{t_m}} \right) \right], \quad 0 \leq t \leq t_s, \quad (2)$$

where:  $\omega_0$  is the initial angular velocity,  $t_s^0$  is the time of braking with constant deceleration.

## 3. MATHEMATICAL MODEL

To evaluate the contact temperature conditions, both analytical and numerical techniques have been developed. The starting point for the analysis of the temperature field in the disc and pad volume is the parabolic heat conduction equation given in the cylindrical coordinate system which is centered in the axis of disc and  $z$  points to its thickness (Nowacki, 1962)

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_d} \frac{\partial T}{\partial t}, \quad r_d \leq r \leq R_d, \quad 0 < z < \delta_d, \quad t > 0, \quad (3)$$

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k_p} \frac{\partial T}{\partial t}, \quad r_p \leq r \leq R_p, \quad \delta_d < z < \delta, \quad t > 0$$

where  $k_{d,p}$  is the thermal diffusivity,  $\delta_{d,p}$  is the thickness,  $r_{d,p}$  and  $R_{d,p}$  are the internal and external radius of the disc and pad respectively,  $\delta = \delta_d + \delta_p$ . The subscripts  $p$  and  $d$  imply the pad and the disc respectively. The substantiation of the axisymmetric arrangement of the problem has already been reported as a relevant foundation (Grześ, 2009).

Two-dimensional model of disc brake is presented in Fig. 3. The boundary and initial conditions for the disc and pad are given as follows:

$$K_d \frac{\partial T}{\partial z} \Big|_{z=\delta_d} = \begin{cases} h[T_a - T(r, \delta_d, t)], & r_d \leq r \leq r_p, \quad t \geq 0, \\ q_d(r, \delta_d, t), & r_p \leq r \leq R_d, \quad 0 \leq t \leq t_s, \end{cases} \quad (4)$$

$$K_p \frac{\partial T}{\partial z} \Big|_{z=\delta_d} = -q_p(r, \delta_d, t), \quad r_p \leq r \leq R_p, \quad 0 \leq t \leq t_s, \quad (5)$$

$$K_d \frac{\partial T}{\partial r} \Big|_{r=R_d} = h[T_a - T(R_d, z, t)], \quad 0 \leq z \leq \delta_d, \quad t \geq 0, \quad (6)$$

$$K_p \frac{\partial T}{\partial r} \Big|_{r=R_p} = -h[T_a - T(R_p, z, t)], \quad \delta_d \leq z \leq \delta, \quad t \geq 0, \quad (7)$$

$$K_p \frac{\partial T}{\partial r} \Big|_{r=R_p} = h[T_a - T(R_p, z, t)], \quad \delta_d \leq z \leq \delta, \quad t \geq 0, \quad (8)$$

$$\left. \frac{\partial T}{\partial r} \right|_{r=r_d} = 0, \quad 0 \leq z \leq \delta_d, \quad t \geq 0, \quad (9)$$

$$\left. \frac{\partial T}{\partial z} \right|_{z=0} = 0, \quad r_d \leq r \leq R_d, \quad t \geq 0, \quad (10)$$

$$\left. \frac{\partial T}{\partial z} \right|_{z=\delta} = 0, \quad r_p \leq r \leq R_p, \quad t \geq 0, \quad (11)$$

$$T(r, z, 0) = T_0, \quad r_d \leq r \leq R_d, \quad 0 \leq z \leq \delta_d, \quad (12)$$

$$T(r, z, 0) = T_0, \quad r_p \leq r \leq R_p, \quad \delta_d \leq z \leq \delta, \quad (13)$$

where (Ling F. F., 1973)

$$q_d(r, z, t) \Big|_{z=\delta_d} = \frac{\phi_0}{2\pi} \gamma f p(t) r \omega(t), \quad r_p \leq r \leq R_p, \quad 0 \leq t \leq t_s, \quad (14)$$

$$q_p(r, z, t) \Big|_{z=\delta_d} = (1 - \gamma) f p(t) r \omega(t), \quad r_p \leq r \leq R_p, \quad 0 \leq t \leq t_s, \quad (15)$$

$f$  is the friction coefficient,  $p$  is the contact pressure,  $\omega$  is the angular velocity,  $t$  is the time,  $r$  is the radial coordinate,  $z$  is the axial coordinate.

The above cases are two-dimensional problem for transient analysis. The boundary and initial conditions are specified for disc and pad volume respectively.

#### 4. FE FORMULATION

Understanding of overall formulation is crucial for the solution of the considering thermal problem. In Fig. 2 the interface conditions of contact model are shown. In order to simulate perfect contact during braking process, two terms at the subsequent pair of nodes on the contact surfaces have been imposed

- 1) the equality of the temperature at any instant of time

$$T_{pi}(0+, t) = T_{dj}(0-, t) \quad (16)$$

- 2) and the following heat balance condition at each of the contact surfaces given by

$$q_{pi}(0+, t) + q_{dj}(0-, t) = q(t) \quad (17)$$

The object of this section is to develop approximate time-stepping procedures for axisymmetric transient governing equations. The detailed description of the two-dimensional discretization was presented by Grześ (2009).

Using Galerkin's approach the following matrix form of the Eq. (3) is formulated (Lewis et al., 2004)

$$[C] \left\{ \frac{dT}{dt} \right\} + [K][T] = \{R\} \quad (18)$$

where  $[C]$  is the heat capacity matrix,  $[K]$  is the heat conductivity matrix, and  $\{R\}$  is the thermal force matrix.

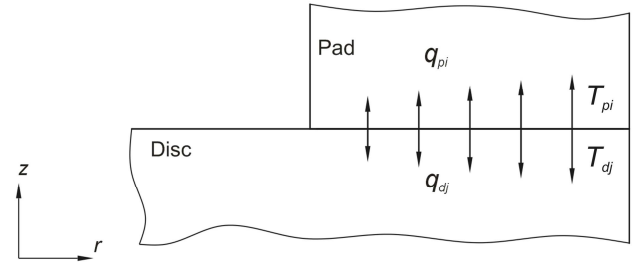


Fig. 2. Contact model for the thermal problem

In order to solve the ordinary differential equation (18) the direct integration method was used. Based on the assumption that temperature  $\{T\}_t$  and  $\{T\}_{t+\Delta t}$  at time  $t$  and  $t+\Delta t$  respectively, the following relation is specified

$$\{T\}_{t+\Delta t} = \{T\}_t + \left[ (1 - \beta) \left\{ \frac{dT}{dt} \right\}_t + \beta \left\{ \frac{dT}{dt} \right\}_{t+\Delta t} \right] \Delta t \quad (19)$$

Substituting Eq. (19) to Eq. (18) we obtain the following implicit algebraic equation

$$([C] + \beta \Delta t [K]) \{T\}_{t+\Delta t} = ([C] - (1 - \beta) [K] \Delta t) \{T\}_t + (1 - \beta) \Delta t \{R\}_t + \beta \Delta t \{R\}_{t+\Delta t} \quad (20)$$

where  $\beta$  is the factor which ranges from 0.5 to 1 and is given to determine an integration accuracy and stable scheme.

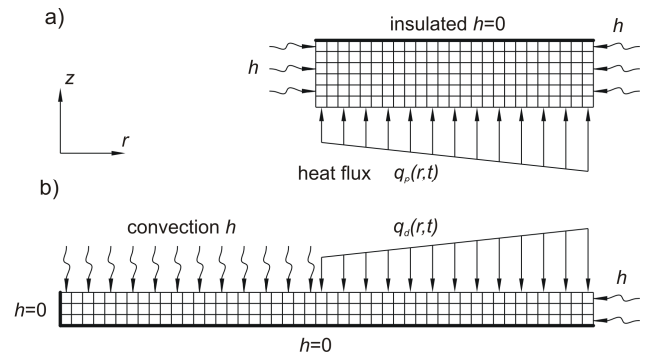


Fig. 3. FE model with boundary conditions for the transient analysis: a) pad, b) disc

The finite element formulation of the disc brake with boundary conditions is shown in Fig. 3. Disc and pad components described below were analyzed using the MD Patran/MD Nastran software package (Reference Manual MD Nastran, 2008; Reference Manual MD Patran, 2008). In the thermal analysis of disc brake an appropriate finite element division is indispensable. In this study eight-node quadratic elements were used for the finite element analysis. The disc brake FE model consists of 576 elements and 1933 nodes of disc and 663 elements and 2118 nodes of pad. High order of elements ensure appropriate numerical accuracy. For the purpose of providing perfect contact conditions between each pair of nodes in the disc/pad interface, 103 Multipoint Constraints (MPC) were used.

To avoid inaccurate or unstable results, a proper initial time step associated with spatial mesh size is essential (Reference Manual MD Nastran, 2008).

$$\Delta t = \Delta x^2 \frac{\rho_d c_d}{10K_d} \quad (21)$$

where  $\rho$  is the density,  $c$  is the specific heat and  $K$  is the thermal conductivity,  $\Delta t$  is the time step,  $\Delta x$  is the mesh size (smallest element dimension). In this paper fixed  $\Delta t = 0.005s$  time step was used.

### 5. RESULTS AND DISCUSSION

In this paper thermal FE models of the disc brake with pad presence have been investigated. Proposed FE modeling technique (Grześ, 2009) was confronted with the analytical solution (Talati and Jalalifar, 2009) and FEA (Gao and Lin, 2002). The simulation includes conductive and convective terms of the real brake exert. Temperature distributions were predicted for the operation conditions given in Tab. 2. Material properties adopted in the analysis for FE model are specified in Tab. 1. The transient solution was performed for the pressure  $p$  and angular velocity of the disc  $\omega$  evolution shown in Fig. 4.

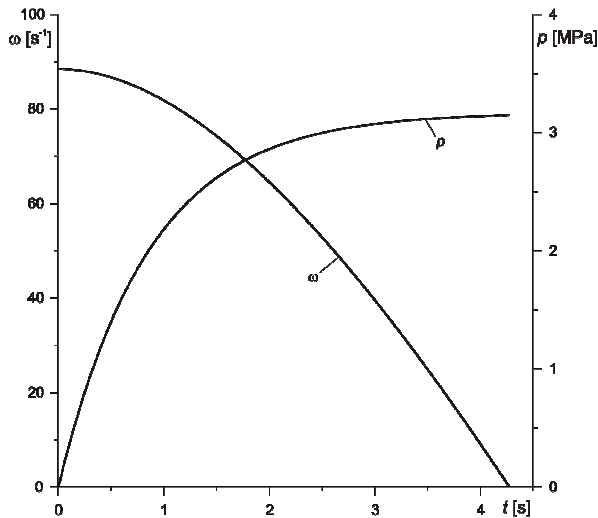


Fig. 4. Evolution of the pressure  $p$  and angular velocity  $\omega$  during braking

The temperature distributions in arbitrarily specified instants of braking time are presented in Fig. 5. The equilibrium of the temperatures on contact surfaces at the  $z$  position of 0.006mm is noticeable in the solution. The intensity of heat flux entering into the disc and pad respectively excites growth of the temperature from the contact zone, which in subsequent measures extends particularly into the disc volume. Temperature variations through braking duration in  $z$  coordinate are relatively smooth in the disc area and rapid in the pad zone. The dissimilarity of heat dissipation between disc and pad volume in axial coordinate at the each step of the analysis are fundamentally dictated by the properties of materials adopted in this study. At the

time of  $t=4s$  temperature distribution of the disc is approximately equal at any position in radial direction. Temperature field of the pad is constant upwards of  $r=0.010m$  at any moment of presented results. The highest value of the temperature obtained in the analysis occurred at radius of  $r=0.127m$  at the contact surface. The results are believed to be physically justifiable values.

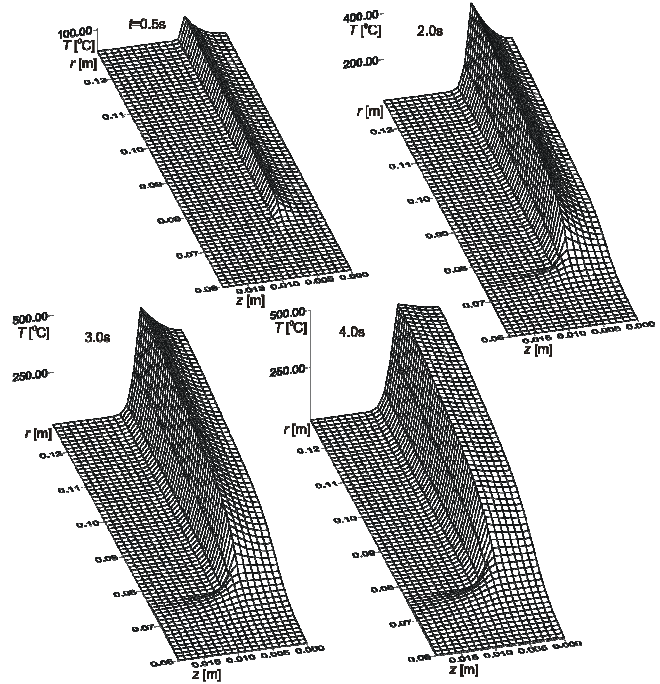


Fig. 5. The temperature distribution in the disc and pad volume at specified instants of braking time

Axial temperature distributions at the radius of 0.127m of disc brake are presented in Fig. 6. Within the analysis time of  $t_s=4.274s$  heat was generated due to friction behavior at the disc/pad interface. Therefore temperature increase is noticeable. Over the final time step of braking ( $t=3.5-4.274s$ ) the interface temperatures can be seen to decrease slightly. This effect corresponds intermediately to the intensity of heat flux, which rises with time until the value of velocity and pressure product attains highest, critical value at the particular, radial position. The temperature expansion is significantly affected by the thermo-physical properties of materials submitted to the thermal load. As it can be seen the differences in axial directions are sufficiently high, particularly in pad zone. The gradient of temperatures in subsequent periods of time during single braking action is an issue of transformation of large amounts of the kinetic energy into heat energy in relatively short time. In addition, temperature of the disc and pad are affected by external convective conditions and decreases due to Newton’s law of cooling. This phenomenon may be intensified when the vehicle is still moving and cooling is forced by the air flow.

Comparison of the radial temperature values at the contact surfaces including free surface of the disc during braking process is illustrated in Fig. 7. Maximum temperature rise up to 496.6°C at 0.127m of radial position and 3.375s

of time However, it can be seen that at the external location of the radius in the range of 0.001m, the temperature varies slightly. The impact of the intensity of heat flux entering the disc and pad respectively is noticeable above the radial location of 0.077m. Presented isotherms validate the adiabatic boundary condition at the inner radius of the disc where the temperature value is constant during braking process.

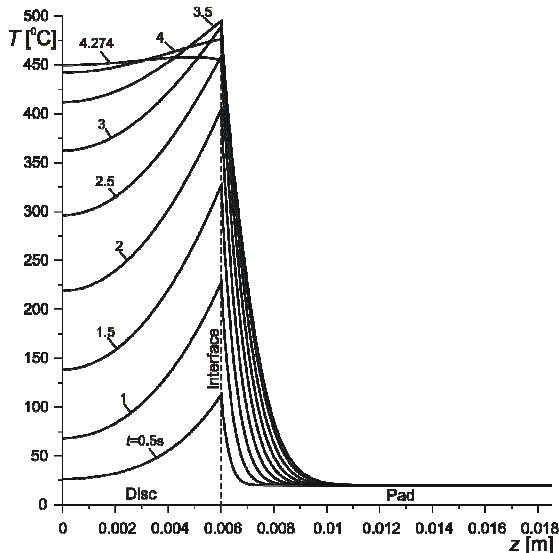


Fig. 6. Axial temperatures profiles at radius of 0.127m

axial positions are presented. The symmetry in axial coordinate  $z$  has been assumed. Profiles from  $z=0m$  which indicates central location of the real disc to its maximum thickness of  $z=0.006m$  are evaluated. At the initial period of braking process maximum temperature appears at the disc/pad interface ( $z=0.006m$ ). Tendency to convergence of temperature at different axial positions at the end of braking process is noticeable. It is connected with alignment of temperatures in disc area in subsequent stage of the process when the intensity of heat flux descents.

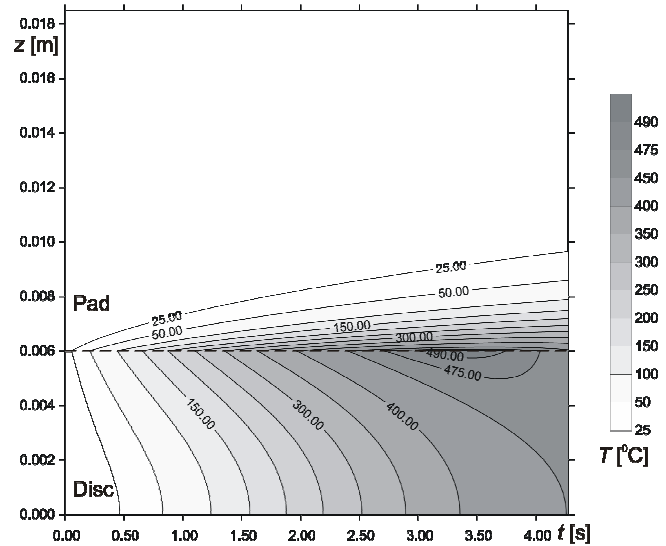


Fig. 8. Axial isotherms at radius of 0.127m

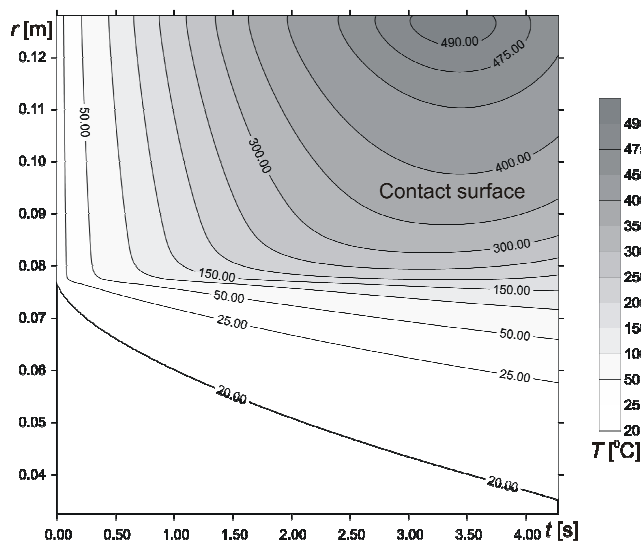


Fig. 7. Radial isotherms at the disc/pad interface

In Fig. 8 axial temperature evolution in the period of single braking process is shown. Fundamental differences of the temperature expansion between two considered zones of the disc and pad are noticeable. Temperature of the disc in axial coordinate rises relatively rapidly in the entire thickness at considered radius of 0.127m, while majority of pad area remains unheated. The isotherm of the highest value of temperature of 490°C obtained in the analysis outlines slight area near to the contact position of  $z$  direction.

In Fig. 9 disc temperature at  $r=0.127m$  and at different

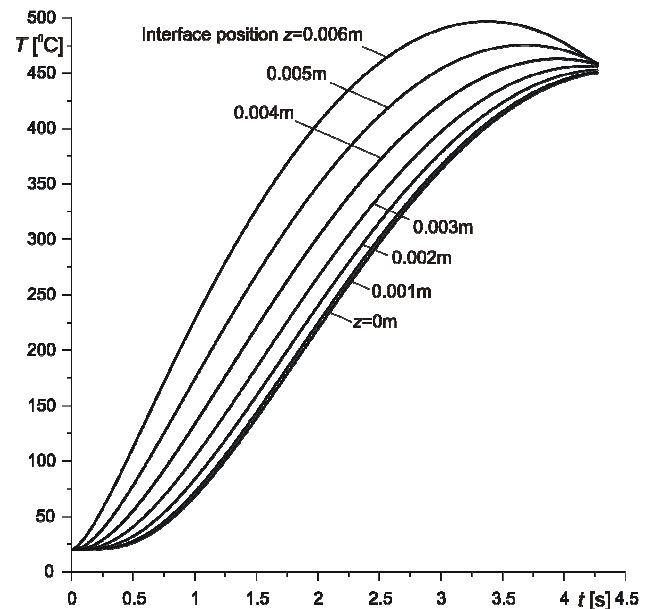


Fig. 9. Evolution of the disc temperature at different axial distances and at radial position of 0.127m

In Fig. 10 the pad temperature distributions at  $r=0.127m$  in different axial positions are shown. As it can be seen the pad temperature at the contact surface ( $z=0.006m$ ) has the highest value obtained during simulation process and gradual decay of the temperature should be noticed at different axial positions up to  $z=0.010m$  (0.004m of pad



thickness), where temperature remains approximately constant.

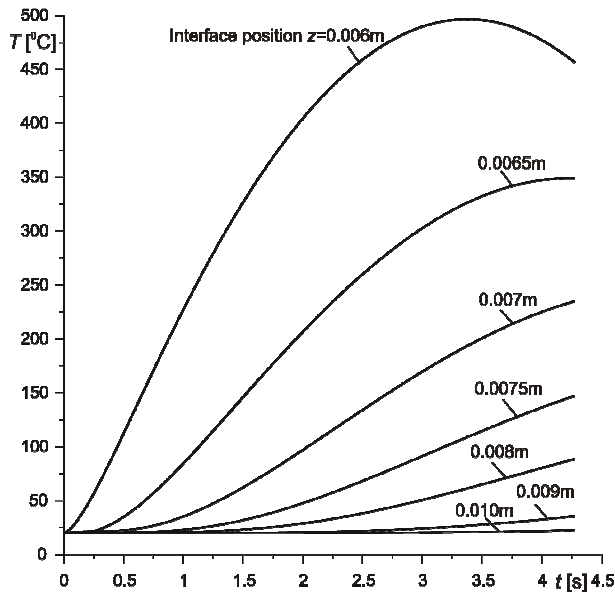


Fig. 10. Evolution of the pad temperature at different axial distances and at radial position of 0.127m

Tab. 1. Material properties [2]

Thermo-physical properties	Disc	Pad
Thermal conductivity, $K_{d,p}$ [W/mK]	48.46	1.212
Specific heat, $c_{d,p}$ [J/kgK]	419	1465
Density, $\rho_{d,p}$ [kg/m <sup>3</sup> ]	7228	2595

Tab. 2. Operation conditions [2]

Operation conditions	Disc	Pad
Inner radius, $r_{d,p}$ [mm]	32.5	77
Outer radius, $R_{d,p}$ [mm]	128	
Cover angle of pad, $\phi$	64.5	
Thickness $\delta_{d,p}$ [mm]	6	12.5
Initial velocity $\omega$ [s <sup>-1</sup> ]	88.46	
Time of braking, $t_s$ [s]	4.274	
Pressure $p_0$ [MPa]	3.17	
Coefficient of friction $f$	0.5	
Heat transfer coefficient $h$ [W/m <sup>2</sup> K]	100	
Initial temperature $T_0$ [°C]	20	
Ambient temperature $T_a$ [°C]	20	
Time step $\Delta t$ [s]	0.005	

## 6. CONCLUSION

In this paper transient thermal analysis of disc brake in the single brake application was developed. The frictional heating phenomena using the finite element method of the thermal contact problem was performed. This study provides an evaluation of temperature distribution during braking action including both the disc and pad presence in an axisymmetric arrangement.

The outcomes obtained in the analysis show inequality in the temperature distribution of axial positions at the subsequent values of radial locations considering the disc and pad areas due to the different thermal properties of materials.

The influence of the convective heat transfer terms has been found relatively insignificant in the temperature distributions of considering behavior of single, emergency braking.

In view of the disc geometry aspect the results shows negligibly low temperature variations in the area of the disc beneath internal radius of the pad.

However imposed terms of perfect contact of disc/pad interface specify special, idealized conditions neglecting wear and debris (third body), the behavior of considering phenomena characterizes nature of the heat expansion and facilitates predicting the magnitude of the temperature rise during braking process.

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