SEARCHING FOR THE VALUES OF DAMPING ELEMENTS WITH REQUIRED FREQUENCY SPECTRUM

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Abstract: This paper concerns formulating and solving the problem of synthesis of vibrating discrete mechanical systems with two – terminal damper. In this paper a method of synthesis to determination of structure and inertial, elastical, damping parameters has been presented. Such task may be classified as a reverse problem dynamics of vibration subsystems.

1. INTRODUCTION

Vibration damping occupies a significant place among the extensive issues of machinery and apparatus dynamics. It is one of the factors of mechanical energy dissipation, inseparably connected with mechanical systems movement.
A proper selection of frequency and operation outside the resonance range solve the problem of vibration avoidance. Leaving the resonance zone is the basic condition of apparatus functioning, however, it does not eliminate the problem completely. In many machines a lot of free vibration frequencies appear. In these cases damping is of crucial importance, because it lowers vibration amplitude significantly.

Equally important phenomenon, in which damping plays a major role, is the phenomenon of transition through the resonance zone (Buchacz, 1995, 2004, 2006; Buchacz et al., 2005). A number of components are adapted to operate in supercritical zone i.e. at frequencies higher than resonance frequency. In such a case transition through the resonance zone occurs during machine movement. Then poorly damped components are becoming temporarily subjected to strong vibrations. When damping phenomenon limits vibration amplitude insufficiently, additional damping is introduced (Buchacz, 2004, 2006; Buchacz et al., 2005; Dymarek and Dzitkowski, 2005; Dzitkowski, 2004; Dzitkowski and i Dymarek, 2005). The problem of searching damping component values, existing in mechanical systems, is a typical task of parametric synthesis due to required dynamical properties (Bellert and Woźniacki, 1968; Berge, 1973; Buchacz, 1991, 1997, 1995; Dymarek, 2000, 2004; Dymarek and Dzitkowski, 2005; Dzitkowski, 2001, 2004; Dzitkowski and Dymarek, 2005).

In the study the methods for synthesis of discrete vibration systems (Bellert and Woźniacki, 1968; Berge, 1973; Buchacz, 1991, 1997, 1995; Dymarek, 2000, 2004; Dymarek and Dzitkowski, 2005; Dzitkowski, 2001, 2004; Dzitkowski and Dymarek, 2005) with damping was presented.

The synthesis was done, using well-known methods of discrete vibration systems synthesis and by introduction of an assumption concerning damping type element, existing in the sought after system, and taking rheological Voigt's-Kelvin's model and Rayleigh's damping assumption as a mass damping model (Dymarek, 2000; Dymarek i Dzitkowski, 2005).

2. THE SYNTHESIS OF MECHANICAL SYSTEM WITH DAMPING

In this section the algorithm for actions taken during discrete damping systems synthesis will be presented.

In order to start the synthesis of damping systems (Dymarek, 2000; Dymarek i Dzitkowski, 2005; Dzitkowski and Dymarek, 2005) we should specify the properties of the sought-after system and the damping model, in accordance with the presented course of action:

1. Specify the values of resonance and anti-resonance frequencies in case of free vibration, i.e.:

$$\begin{cases} \omega_{b1}, \omega_{b2}, \ldots, \omega_{bn}, \\ \omega_{z1}, \omega_{z2}, \ldots, \omega_{zn}. \end{cases} \tag{1}$$

2. Determine the value of dimensional vibration factor λ, in case of V-K model:

$$b_{ci} = \lambda\, c_i \quad , \tag{2}$$

$$0 < \lambda < \frac{2}{\omega_n} , \tag{3}$$

where: b_{ci} – damping, c_i – stiffness resulting from the synthesis carried out, λ=idem – dimensional vibration factor

3. Determine the parameter corresponding to damping in the system h_n in case of V-K model:

$$h_n = \frac{\lambda\, \omega_{bn}^2}{2} \quad , \tag{4}$$

or by providing damping decrement for the individual resonance frequencies and anti-resonance frequencies:

$$\delta_n \approx \frac{2\pi h_n}{\omega_n} . \tag{5}$$

4. Determine the value of dimensional vibration factor h in case of mass model of damping:

$$b_{mi} = 2h \cdot m_i \quad , \tag{6}$$

where: b_{mi} – damping, h – parameter corresponding to damping in the system, having frequency dimension, m_i – the value of inertia element determined as a result of synthesis.

The h parameter in the discussed case is constant, i.e.:

$$h = \text{idem} , \tag{7}$$

and its value should be chosen from the following bracket

$$0 < h < |\omega_{\min}| , \tag{8}$$

where: $|\omega_{\min}| \neq 0$ – the smallest value of resonance frequency, when the system restrained is synthesized or the smallest value which is equal to anti-resonance frequency when the half-defined system is synthesized.

5. Determine the value of dimensional vibration factor h in case of Rayleigh model of damping:

$$h_{Rn} = h_n + h \tag{9}$$

The examined characteristics are immobility $U(s)$ and mobility $V(s)$ built on the basis of assumed dynamic properties as described in 1-5.

The immobility function $U(s)$

$$U(s) = H \frac{d_l s^l + d_{l-1} s^{l-1} + \ldots + d_0}{c_k s^k + c_{k-1} s^{k-1} + \ldots + c_1 s} \tag{10}$$

and the mobility function $V(s)$

$$V(s) = H \frac{c_k s^k + c_{k-1} s^{k-1} + \ldots + c_1 s}{d_l s^l + d_{l-1} s^{l-1} + \ldots + d_0} , \tag{11}$$

where: l-odd or even numerator order with l-k=1, k- denominator order, H- any real positive number.

In order to determine elastic and inertial values, characteristic functions describing free vibrations in the form of immobility $U'(s)$ (12) and mobility $V'(s)$ (13) should be subject to the synthesis.

The immobility function $U'(s)$

$$U'(s) = H \frac{d_l s^l + d_{l-2} s^{l-2} + \ldots + d_0}{c_k s^k + c_{k-2} s^{k-2} + \ldots + c_1 s} , \tag{12}$$

The mobility function $V'(s)$

$$V'(s) = H \frac{c_k s^k + c_{k-2} s^{k-2} + \ldots + c_1 s}{d_l s^l + d_{l-2} s^{l-2} + \ldots + d_0} . \tag{13}$$

Then, the values of two-terminal damping type are to be determined on the basis of the relationship between (2) and (6), using inertial and elastic elements obtained in the first step of the synthesis

3. EXAMPLE

The assumed requirements of the desired system structure are:

$$\begin{cases} \omega_1 = 10 \, \frac{\text{rad}}{\text{s}}, \omega_3 = 20 \, \frac{\text{rad}}{\text{s}}, \omega_5 = 30 \, \frac{\text{rad}}{\text{s}} - \text{resonance frequencies}, \\ \omega_0 = 0 \, \frac{\text{rad}}{\text{s}}, \omega_2 = 15 \, \frac{\text{rad}}{\text{s}}, \omega_4 = 25 \, \frac{\text{rad}}{\text{s}} - \text{anti-resonance frequencies}. \end{cases}$$

In order to determine the analytical form of dynamical characteristics, the following assumptions should be taken.

Assumption 1

The characteristic sought after function is slowness $U(s)$.

Assumption 2

The requirements concerning damping for the two cases are assumed:

The mass model of damping:

$$b_{mi} = 2hm_i \, \frac{\text{Ns}}{\text{m}}, \quad h = 0.8 \, \frac{1}{\text{s}}. \tag{14}$$

The V-K model of damping:

$$b_{ci} = \lambda c_i \, \frac{\text{Ns}}{\text{m}}, \, h_n = \frac{\lambda \omega_n^2}{2} \, \frac{1}{\text{s}}, \quad \lambda = 0.01 \, \text{s}. \tag{15}$$

The Rayleigh model of damping:

$$b_{ci} \, \frac{\text{Ns}}{\text{m}}, \, b_{mi} \, \frac{\text{Ns}}{\text{m}}, \, h_{Rn} = h_n + h \, \frac{1}{s} \tag{16}$$

The dynamical characteristics takes the form as follows:

- when the assumption of (14) is met:

$$U(s) = H \frac{\left(s^2 + 2hs + \omega_1^2\right)\left(s^2 + 2hs + \omega_3^2\right)\left(s^2 + 2hs + \omega_5^2\right)}{s\left(s^2 + 2hs + \omega_2^2\right)\left(s^2 + 2hs + \omega_4^2\right)} , \tag{17}$$

- when the assumption of (15) is met:

$$U(s) = H \frac{\left(s^2 + 2h_1 s + \omega_1^2\right)\left(s^2 + 2h_3 s + \omega_3^2\right)\left(s^2 + 2h_5 s + \omega_5^2\right)}{s\left(s^2 + 2h_2 s + \omega_2^2\right)\left(s^2 + 2h_4 s + \omega_4^2\right)} , \tag{18}$$

- when the assumption of (16) is met:

$$U(s) = H \frac{\left(s^2 + 2h_{R1} s + \omega_1^2\right)\left(s^2 + 2h_{R3} s + \omega_3^2\right)\left(s^2 + 2h_{R5} s + \omega_5^2\right)}{s\left(s^2 + 2h_{R2} s + \omega_2^2\right)\left(s^2 + 2h_{R4} s + \omega_4^2\right)} . \tag{19}$$

What should be submitted when carrying out the synthesis of characteristics (17), (18) and (19) is immobility, which describes free vibration in the form of:

$$U'(s) = H\frac{\left(s^2+\omega_1^2\right)\left(s^2+\omega_3^2\right)\left(s^2+\omega_5^2\right)}{s\left(s^2+\omega_2^2\right)\left(s^2+\omega_4^2\right)}, \quad (20)$$

$$U'(s) = \frac{s^6+1400s^4+490000s^2+36000000}{s^5+850s^3+140625s}, \quad (21)$$

then, assumptions concerning damping (13), (14) and (15) should be taken into account.

As a result of synthesis carried out with the use of the method of characteristics function (21) distribution on continued fraction the values of inertial and elastic elements are obtained:

$$\begin{aligned} &m_1 = 1.000\text{kg},\ m_2 = 2.561\text{kg},\ m_3 = 6.061\text{kg},\\ &c_1 = 550.000\tfrac{\text{N}}{\text{m}},\ c_2 = 730.424\tfrac{\text{N}}{\text{m}},\ c_3 = 1390.822\tfrac{\text{N}}{\text{m}}. \end{aligned} \quad (22)$$

Assuming that damping should be proportional to two-terminal inertial type:

$$\begin{cases} h = 0.8\ \tfrac{1}{\text{s}},\\ b_{mi} = 2hm_i\ \tfrac{\text{Ns}}{\text{m}}, \end{cases} \quad (23)$$

the values elements of the system obtained (Fig. 1.) are as follows:

$$\begin{aligned} &m_1 = 1.000\text{kg},\ m_2 = 2.561\text{kg},\ m_3 = 6.061\text{kg},\\ &c_1 = 550.000\tfrac{\text{N}}{\text{m}},\ c_2 = 730.424\tfrac{\text{N}}{\text{m}},\ c_3 = 1390.822\tfrac{\text{N}}{\text{m}},\\ &b_{m1} = 1.600\tfrac{\text{Ns}}{\text{m}},\ b_{m2} = 4.097\tfrac{\text{Ns}}{\text{m}},\ b_{m3} = 9.697\tfrac{\text{Ns}}{\text{m}}. \end{aligned} \quad (24)$$

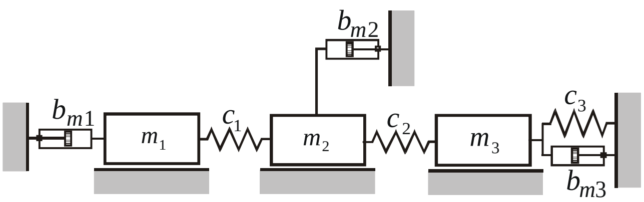

Fig. 1. Synthesised mechanical system with the mass model of damping

Assuming the V-K model of damping:

$$\begin{cases} h_n = \dfrac{\lambda\omega_n^2}{2}\ \dfrac{1}{\text{s}},\\ b_{ci} = \lambda c_i\ \dfrac{\text{Ns}}{\text{m}},\\ \lambda = 0.01\,\text{s}, \end{cases} \quad (25)$$

the values of damping elements are in the form of:

$$\begin{aligned} &b_{c1} = 5.500\tfrac{\text{Ns}}{\text{m}},\\ &b_{c2} = 7.304\tfrac{\text{Ns}}{\text{m}},\\ &b_{c3} = 13.908\tfrac{\text{Ns}}{\text{m}}. \end{aligned} \quad (26)$$

The structures of mechanicals systems with damping were created and its parameters are presented in Figs. 2, 3.

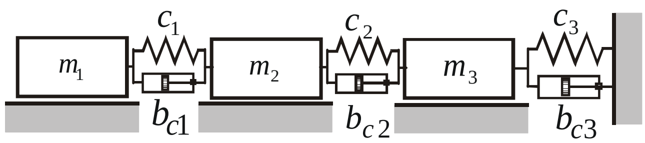

Fig. 2. Synthesised mechanical system with the K-V model of damping

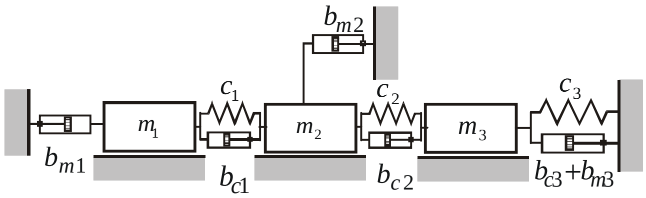

Fig. 3. Synthesised mechanical system with the Rayleigh's model of damping

where:

$$\begin{aligned} &m_1 = 1.000\text{kg},\ m_2 = 2.561\text{kg},\ m_3 = 6.061\text{kg},\\ &c_1 = 550.000\tfrac{\text{N}}{\text{m}},\ c_2 = 730.424\tfrac{\text{N}}{\text{m}},\ c_3 = 1390.822\tfrac{\text{N}}{\text{m}},\\ &b_{m1} = 1.600\tfrac{\text{Ns}}{\text{m}},\ b_{m2} = 4.097\tfrac{\text{Ns}}{\text{m}},\ b_{m3} = 9.697\tfrac{\text{Ns}}{\text{m}},\\ &b_{c1} = 5.500\tfrac{\text{Ns}}{\text{m}},\ b_{c2} = 7.304\tfrac{\text{Ns}}{\text{m}},\ b_{c3} = 13.908\tfrac{\text{Ns}}{\text{m}}. \end{aligned} \quad (27)$$

4. CONCLUSION

As a result of damped systems synthesis, when damping is proportional to stiffness and to values of mass, in case of synthesis of immobility, systems with cascade and branched structure are received. Received as a result of the synthesis inertial, elastic and damped parameters of the model are not the only ones which meet the assumed requirements referring to resonant frequencies – polars and zeros (Dymarek, 2000, 2004). Development of synthesis methods gives the possibilities of designing more and more complex mechanical systems with regard to requested dynamic features.

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