COMPARISON OF GRAPH-BASED METHODS OF KINEMATICAL ANALYSIS OF PLANETARY GEARS

Józef DREWNIAK[*], Stanisław ZAWIŚLAK[*]

[*]Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biała,
2 Willowa Street, 43-309 Bielsko-Biała, Poland

jdrewniak@ath.bielsko.pl, szawislak@ath.bielsko.pl

Abstract: In the paper three graph-based methods of planetary gear modeling are discussed. The following methods have been considered: Hsu's graph, contour graph and bond graphs-based methods. The theoretical ideas of the mentioned approaches were shortly revised and compared. Two of them were applied for analysis of an exemplary planetary gear. The consistency with traditional Willis method was checked. Advantages of the proposed approaches are highlighted.

1. INTRODUCTION

Design of gears especially planetary gears is a challenging engineering task. The computer or Artificial Intelligence (AI) aided approaches to design are more and more needed nowadays. In case of gears, graph-based methods for modeling of planetary gears seem to deliver a very efficient aid. The main advantages of graph-based models of planetary gears are as follows: the methods are algorithmic, they allow for comparison of results within a conceptual stage of design e.g. for kinematical analysis. They deliver algebraic structures to encode a planetary gear what allows in turn for evolutionary (Rao, 2000) or neuronal networks approaches to design (Zhang et al., 2010). They allow for creation of complete atlases of design solutions (functional schemes of planetary gears) which can not be done by means of any other tool (Tsai, 2001). The aim of the present paper is to present a rough comparison of three chosen graph based methods used for modeling of planetary gears and additionally an application of two of them for an analysis of an exemplary planetary gear. The practical engineering task e.g. kinematical analysis shows some details of modeling. Moreover, it allows for proving of usability of the proposed approach.

2. GRAPH-BASED MODELS OF PLANETARY GEARS

In the present paper, the graph-based methods of modeling of planetary gears were utilized for their kinematical analysis. The obtained results were compared with the traditional Willis method and the method of geometrical scheme of velocities. Three considered graph-based methods are as follows:

- Hsu graph-based (Drewniak and Zawiślak, 2009a, 2009b, 2010a; Zawiślak, 2008) which can be also called linear graph-based method; the original method were tailored by the authors to the analysis of some special types of planetary gears;
- contour graph-based (Marghitu, 2005; Drewniak and Zawiślak, 2010b; Zawiślak, 2010);
- bondgraph-based method (Cervantes et al., 2009, Wojnarowski et al., 2006).

Moreover, other tasks of planetary gear analysis (e.g. analysis of dynamics) can be performed by means of other graph-based approaches:

- vector network method (especially forces and moments of forces) (Lang 2005, Prahasto 1992);
- flow-graphs method (Nagaraj and Hariharan, 1973; Wojnarowski and Lidwin, 1975).

The ideas of three first mentioned methods are roughly compared in Tab. 1. The detailed description of the theoretical background for these methods can be found in cited references (Marghitu 2005, Tsai 2001) as well as in other works of the authors of the present paper. The application of these methods for an analysis of the exemplary planetary gear (Fig.1) is presented underneath.

3. EXEMPLARY PLANETARY GEAR

The exemplary planetary gear is presented in Fig.1 and Fig. 2. It has the following elements: main axis *I* with gear wheel *1*, planet carrier (arm) *h*, special rotating elements carried by the planet carrier (suns) i.e. on levels *b* and *d*. These elements have two toothings mounted on common axes i.e.: *2* and *3* as well as *4* and *5*, respectively - corresponding to axes *b* and *d*. Wheel *6* has an internal toothing.

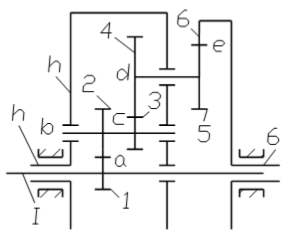

Fig. 1. Exemplary planetary gear; general layout

Other aspects of the gear are not taken into account in the considerations. Several modes of operation of the planetary gear are usually possible. If we brake a particular element (link) then the obtained ratio changes. In our case we consider the mode when link *6* is braked, e.g.: element *I* is considered as an input. An output is the arm *h* which external surface can be consider as a drum for a rope. Planet carrier *h* creates a housing form for majority of the gear elements. The numbers of teeth for the considered planetary gear are as follows: $z_1=18$; $z_2=36$; $z_3=24$; $z_4=36$; $z_5=18$; $z_6=90$ *or* -90 – in case of Willis-method notation. We assume that modules are: $m_1=1$; $m_2=1.5$ and $m_3=2mm$, respectively.

Additionally we consider two modes of working (operating) conditions:

(mode 1) - inputs: links *I* and *h*; output element *6*;
W – mobility (DOF – degree of freedom);
$W = 3n - 2p_5 - p_4 = 3\cdot 5 - 2\cdot 5 - 3 = 2$;

(mode 2) - input: link *I*, output: link *h* and element *6* is braked (is fixed);
$W = 3n - 2p_5 - p_4 = 3\cdot 4 - 2\cdot 4 - 3 = 1$;

where: n – number of movable links, p_5 , p_4 : numbers of full and half joints.

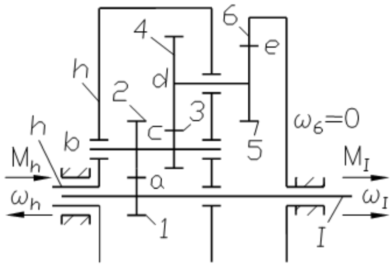

Fig. 2. Exemplary planetary gear; second operational mode

Tab. 1. Comparison of ideas of three graph based models of planetary gears

Task	General Comments	Graph-based methods		
		Hsu's graph	Contour graph	Bond graph
1	2	3	4	5
Abstracting Discretization Modeling	Rules of assignment	Relation between planetary gear elements, consideration of kinematical pairs: - meshing pair (gear pair), - rotating around common axis and - 'arm-planetary wheel' pair	Closed sequence of pairs of planetary gear elements e.g. starting and ending in fixed elements (e.g. bearings), relative rotational speeds of theses elements (links)are taken into account	General rules of assignment of bondgraphs to technical systems; in our case to mechanical system.
Generation of special graph structures: e.g: f-cycles, contours, … and cuts, paths etc.	Subgraphs used in modeling routines	f-cycles (Tsai, 2001) so called fundamental cycles in the meaning of their usage for representing some important features of mechanical system which is modeled via a graph with f-cycles	contours - (Marghitu, 2005) they can be recognized also as cycles but due to several books of Marghitu, the original notion is used	Assignment of physical properties and parameters to the set of nodes (bondgraph elements). Bonds (corresponding to the edges of common graph). Analysis of power flow paths.
	Method of making a representative set	Every meshing pair is taken into account plus additionally traditional meshing pairs (what is just considered)	All independent contours are taken into account. Every contour generates several equations e.g. sum of relative rotational velocities, sum of vector products: arm multiplied by an adequate rotational velocity	Bond-graph rules for creation of equations for particular types of nodes
Generation of codes Transfer of knowledge	Numerical/symbolical codes	Code of f-cycle: (a,b)c where: (a,b) meshing pair; a,b – geared wheels or a pinion or a ring with internal tooth, c – arm (planet carrier)	0→1→2→…→0; string of descriptions of consecutive gear elements; where: every of them passes rotary movement and power alongside a gear	Constitutional equations of bondgraph elements
Solution of the considered problem	Generation of Equations	E.g.: $\omega_a - \omega_c = \pm N_{ba} (\omega_b - \omega_c)$ where: + for internal and – for external meshing.	E.g.: $\Sigma\, \omega_{ij} = 0$ where: ω_{ij} - relative velocities	Generalized Kirchhoff laws
	Solution	Solution of system of algebraic equations	Turning of vector equations into scalar ones, e.g. due to the fact that all vectors act along the same direction. Solving the system of algebraic equations	Solution of DAE (differential and algebraic equations) system
Further possibilities	Different tasks: e.g.: synthesis, enumeration; which are possible especially due to graph-based methods	Enumeration according to functions, atlases of solutions, evolutionary design, neuronal network based analysis (graphs deliver a code of mechanical object)	Analysis of forces and moments; e.g. via contour method	Analysis of forces and moments, analysis of vibrations

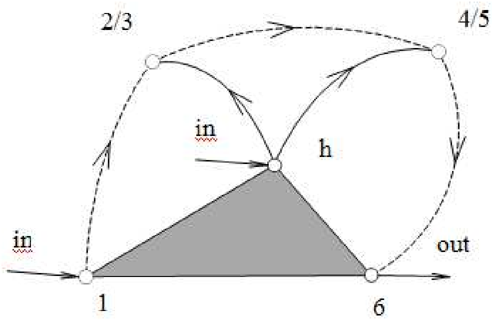

Fig. 3. Linear graph assigned to the exemplary gear

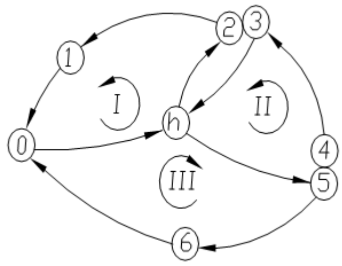

Fig. 4. Contour graph assigned to the exemplary gear

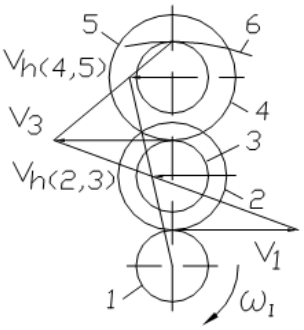

Fig. 5. Scheme of linear and rotational velocities

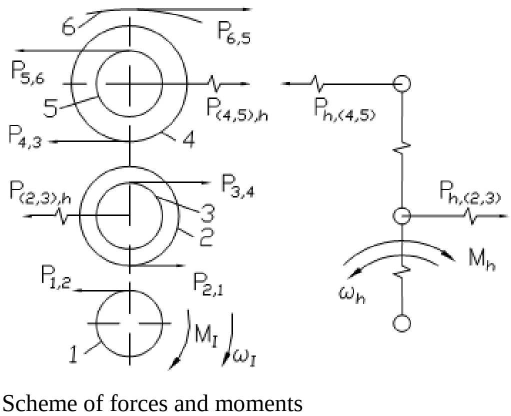

Fig. 6. Scheme of forces and moments

3.1. Linear-graph-based model of planetary gear

The linear graph assigned to the planetary gear is presented in Fig. 3. This type of graph was introduced by Hsu (enclosing a polygon instead of a clique) but the method was slightly updated (Zawiślak, 2008). The polygon represents the links rotating around the main axis of the planetary gear. Stripped line-edges represent pairs of links which are in mesh. Continuous line-edges represent pairs: planet wheel and planet carrier (arm). The notions 'in' and 'out' in Fig. 3 represent inputs and outputs, respectively. The paths from inputs to the output were introduced what was not considered by Hsu.

The graph based-method is performed in the following steps (Tab. 1): (a) abstraction i.e. process of generalization by reducing the information content – taking into consideration only gear kinematics and consideration of the moving chosen gear links; (b) discretization performed simultaneously with the step "a"; addionally – labeling the links; (c) distinguishing of relations among the elements (gear links); (d) assignment of a linear graph to a planetary gear – represented via its functional scheme; (e) transfer of knowledge from mechanics to graph theory; (f) generation of needed graph structures e.g. distinguishing of f-cycles; (g) generation of their codes; (h) generation of equations; (e) solving of equation system receiving – in the considered case – ratio or the output rotational velocity of the modeled planetary gear.

The structures related to a graph of the planetary gear are: so called f-cycles: *(1,2)h* and *(5,6)h*. Moreover we consider a pair *(3,4)* which represents two mating elements i.e. toothings *3* (on the link *2/3*) and toothings *4* (on the link *4/5*). For the last considered pair, the wheels have axes mounted on the arm which are not moved in mutual relation to themseves (axes fixed in relation to the arm *h*). However, they move always simultaneously with the arm. We consider the case when the gear works in the mode 1. Therefore, for the first mode of operation, the obtained system of equation is as follows: two equationns for two f-cycles plus one equation for traditional meshing of two geared wheels:

$$\begin{cases} \omega_1 - \omega_h = -N_{21}(\omega_2 - \omega_h) \\ \omega_5 - \omega_h = +N_{65}(\omega_6 - \omega_h) \\ \qquad \omega_5 = -N_{34}\omega_2 \end{cases} \quad (1)$$

where: ω_i ($i = 1,2, \ldots, 6$) and ω_h rotational velocities of the links; $N_{ij} = z_i/z_j$ local ratios; sign ± depends on external (-) or internal (+) meshing of a geared pair.

Taking into account ω_2 calculated from the first and ω_5 calculated from the second equation we have:

$$\omega_2 = \frac{\omega_h(N_{21}+1) - \omega_1}{N_{21}} \quad (2)$$

$$\omega_2(-N_{34}) = \omega_5 = +N_{65}\omega_6 + \omega_h(1 - N_{65}) \quad (3)$$

Taking into account the equations (2),(3) and after some transformations we can calcutate the wanted ω_6:

$$\omega_6 = \frac{\omega_1 - \omega_h\left[\left(1+\frac{z_2}{z_1}\right) + \frac{z_4}{z_3}\frac{z_2}{z_1}\left(1-\frac{z_6}{z_5}\right)\right]}{\frac{z_2}{z_1}\frac{z_4}{z_3}\frac{z_6}{z_5}} \quad (4)$$

The same formula was obtained by means of the Willis method.

3.2. Contour-graph-based model of planetary gear

Contour graph method consists in the following steps: (a) distinguishing of independent contours; (independence is considered here in the light of graph theory); (b) listing the codes of the contours and sets of indexes; (c) generation of system of equations; (d) solution of this system of equations. The similarities and differences between linear-graph and contour-graph based methods are listed in Tab. 1. The main advantage is: that they are algorithmic some procedures can be turned into computer programs. The contours for the planetary gear presented in Fig. 2 can be written upon the contour graph assigned to the considered planetary gear (Fig. 4). These contours are as follows:

$$\begin{aligned} &(\mathrm{I}): 0 \rightarrow h \rightarrow 2/3 \rightarrow 1 \rightarrow 0 \\ &(\mathrm{II}): h \rightarrow 2/3 \rightarrow 4/5 \rightarrow h \\ &(\mathrm{III}): 0 \rightarrow h \rightarrow 4/5 \rightarrow 6 \rightarrow 0 \end{aligned} \tag{5}$$

Every contour generates a system of equations concerning velocities, forces and moments (Marghitu, 2005). Here, we analyze kinematics, only, therefore we assign two equations to every contour. We consider the second mode of operation. The code of a contour generates a list of indexes – e.g. (I): *(h,0)*; *(2,h)*; *(1,2)*; *(0,1)*. Therefore the system of equations for the contour graph based method is as follows:

$$\begin{cases} \omega_h + \omega_{2h} + \omega_{12} - \omega_1 = 0 \\ \omega_{5h} + \omega_{34} + \omega_{h3} = 0 \\ \omega_{06} + \omega_h + \omega_{5h} + \omega_{65} = 0 \\ \omega_{2h}(r_1 + r_2) + \omega_{12} \cdot r_1 = 0 \\ \omega_{5h}(r_1 + \ldots + r_4) + \omega_{34}(r_1 + r_2 + r_3) + \omega_{h3}(r_1 + r_2) = 0 \\ \omega_{5h}(r_1 + \ldots + r_4) + \omega_{65}(r_1 + \ldots + r_5) = 0 \end{cases} \tag{6}$$

where the following formulas – coming off the method – are necessary to solve the system:

$$\begin{aligned} &\omega_{h3} = \omega_{h2} && \omega_{i0} = \omega_i \\ &\omega_6 = 0 && \omega_{ij} = -\omega_{ji} \\ &r_6 = r_1 + \ldots + r_5 && r_i = \frac{m_i z_i}{2} \end{aligned} \tag{7}$$

where: ω_{ij} – is a relative rotational velocity of the link *i* in relation to the link *j*.

Let's take into account ω_{i0} - so we consider a relative velocity in relation to the support (*0*) then we can omit this *0* and consider a general rotational velocity. The adequate notions were placed in the respective equations e.g.: the first in the system (6).

The last equation of the set (7) means that we consider the cylindrical geared wheels and we do not assume any corrections – therefore the distance between axes is equal to the sum of respective radiuses (pitch radiuses).

Originally, the equations of the system (6) enclose the vector quantities. However, cross products can be turned into scalar ones due to the physical and geometrical properties of planetary gear: rotational velocities as vectors act along the same axis and angles between radiuses and rotational velocities are equal to 90º. So, the transformed system is discussed for simplicity reasons. Solving the system (6) we eliminate all relative velocities which are not important for us, leaving only the general rotational velocities.

The solution of the system is as follows:

$$\frac{\omega_1}{\omega_h} = 1 - \frac{r_6 \cdot r_4 \cdot r_2}{r_5 \cdot r_3 \cdot r_1} = 1 - \frac{z_6 \cdot z_4 \cdot z_2}{z_5 \cdot z_3 \cdot z_1} \tag{8}$$

The equality of the expressions for radiuses and numbers of teeth holds due to the fact that there are the pairs of teeth being in mesh - in the numerator and denominator of the fraction so modules are pairwise reduced. So, it is interesting feature that the ratio does not depend on modules in consecutive geared pairs. Therefore we have finally:

$$\frac{\omega_1}{\omega_h} = 1 - \frac{90 \cdot 36 \cdot 36}{18 \cdot 24 \cdot 18} = -14 \tag{9}$$

The obtained result will be compared with the traditional methods of an analysis of planetary gears.

3.3. Willis formulas for the considered planetary gear

In case of the second mode of operation for the considered planetary gear, the Willis formulas are as follows:

$$\begin{aligned} i_{14}^h &= \frac{\omega_1 - \omega_h}{\omega_4 - \omega_h} = \left(-\frac{z_2}{z_1}\right) \cdot \left(-\frac{z_4}{z_3}\right) \\ i_{56}^h &= \frac{\omega_5 - \omega_h}{\omega_6 - \omega_h} = \left(-\frac{z_6}{z_5}\right) \end{aligned} \tag{10}$$

Additionally, we have:

$$\omega_6 = 0 \quad \omega_5 = \omega_6 \tag{11}$$

The first condition from (11) confirms that this is the second mode of operation. The next expression means that elements *5* and *6* are stiffly mounted on the same axis. The solution is as follows:

$$\left.\frac{\omega_1}{\omega_h}\right|_{\omega_6 = 0} = 1 + \frac{z_6 \cdot z_4 \cdot z_2}{z_5 \cdot z_3 \cdot z_1} = -14 \tag{12}$$

Traditionally in the Willis method, the number of teeth for an internal toothing is consider as negative. Therefore the formulas (8) and (12) are equivalent because it was assumed that $z_6 = -90$.

In the same manner the Willis formula was generated for the first mode of operation of the considered planetary gear.

4. GRAPHICAL-PHYSICAL ANALYSIS OF THE PLANETARY GEAR

Graphical physical analysis of planetary gear can also be performed. It allows for additional confirmation of the obtained results as well as for further insight into the working conditions of a gear.

Like it was mentioned above also this phase of considerations can be done via graph based approach (Prahasto, 1992) but it is beyond the scope of the presented paper.

The scheme of linear and rotational velocities is presented in Fig. 5 and the scheme of forces and moments in Fig. 6, respectively. Based upon the Fig. 5 the following formulas can be written:

$$v_1 = \omega_1 \cdot r_1 = \frac{\pi \cdot n_1}{30} \cdot \frac{d_1}{2} \qquad (13)$$

The value of ω_1 is the input velocity which is passed on the gear from the drive system (motor).

The next three relationships can be deduced upon the scheme presented in Fig. 5.

$$\frac{v_1 + v_{h,(2,3)}}{r_2} = \frac{v_3 - v_{h,(2,3)}}{r_3} \qquad (14)$$

$$\frac{v_3}{r_4 + r_5} = \frac{v_{h,(4,5)}}{r_5} \qquad (15)$$

$$\frac{v_{h,(2,3)}}{r_1 + r_2} = \frac{v_{h,(4,5)}}{r_1 + r_2 + r_3 + r_4 + r_5} \qquad (16)$$

Finally we can calculate the output rotational velocity:

$$\omega_h = -\frac{v_{h,(2,3)}}{r_1 + r_2} = \frac{v_{h,(4,5)}}{r_1 + r_2 + r_3 + r_4 + r_5} \qquad (17)$$

Solving the system of the above formulas the searched ratio of the gear can be calculated:

$$i^{6}_{1,h} = \left(\frac{\omega_1}{\omega_h} \right)_{\omega 6 = 0} = -14 \qquad (18)$$

The same result as previously was obtained.

Based upon the scheme of forces and moments further analyses of the planetary gear can be performed. Taking into account geometrical relationships as well as equilibrium conditions e.g.: $\boldsymbol{P}_{1,2} = -\boldsymbol{P}_{2,1}$ we can deduce that input and output rotational velocities have opposite directions and that the arm is a passive link.

5. CONCLUSIONS

Based upon the above presented considerations and references review, the following conclusions can be drawn:

1. the goals of modeling of planetary gears by means of graphs are versatile, for example:
   - derivation of calculation methods equivalent to traditional ones which however are algorithmic and allow for comparison of results;
   - performance of some tasks which could not be done in any other way e.g.: creating of complete atlases of design solutions;
   - creation of algebraic models for further theoretical analyses e.g.: isomorphism of structures representing gears (equivalence of kinematical schemes of relevant planetary gears);
2. there are many different methods of assignment ‘planetary gear’ ↔‘graph’;
3. some tasks can be done via several different graph-based methods, some of them are dedicated for particular purposes;
4. due to the fact that assignment of a graph generates inevitably an algebraic code of a planetary gear it – in consequence – allows for evolutionary, neuronal network and immune analysis of gears;
5. three graph methods were theoretically compared;
6. two methods were used for kinematical analysis of an exemplary gear;
7. compatible results were obtained for them and the traditional Willis method as well as the graphical-physical analysis;
8. the graph-based methods of modeling of planetary gears are just recently after a constant development due to a possibilities of artificial intelligence consequences of this modeling – e.g. possibility of algorithmic synthesis of gears.

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