SELF-REGULAR STRESS INTEGRAL EQUATIONS METHOD FOR AXISYMMETRIC ELASTICITY

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Abstract: The stress hypersingular integral equations of axisymmetric elasticity are considered. The singular and hypersingular integrals are regularized using the imposition of auxiliary polynomial solution, and self-regular integral equations are obtained for bounded and unbounded domains. The stress-BEM formulation is considered basing on the proposed equations. Considered numerical examples show high efficiency of the proposed approach. New problem for inclusion in finite cylinder is considered.

1. INTRODUCTION

The pioneering work concerning regularization of hypersingular stress integral equations for the axisymmetric elasticity was that by de Lacerda and Wrobel (2001). The further researches and a new more convenient solution strategy of the hypersingular equations with their previous regularization was presented by Mukherjee (2002). The last paper also presents practically full review of the major works concerning the boundary element method (BEM) and the integral equations for the axisymmetric elasticity.

The main aim of the above mentioned papers was the application of hypersingular integral equation as a basic one for the numerical scheme of BEM, therefore its regularization is not complete and still there are singular integrals, which principal values are to be evaluated using special techniques. Therefore, the regularization approaches of Mukherjee (2002); de Lacerda and Wrobel (2001) are actually unsuitable for calculation of stresses in the whole domain continuously up to the boundary, because of the boundary layer effect, which arise due to the numerical integration of nearly-singular integrals (Cruse, 1969). For this purpose, it is necessary to provide full regularization of both singular and hypersingular integrals.

2. FORMULATION AND SOLUTION OF THE PROBLEM

Consider the linear elastic isotropic solid B bounded by the surface ∂B. Assume that B is axially symmetric and symmetrically loaded with the respect to the axis of symmetry Oz. The integral equation for determination of stresses in an internal source point $\boldsymbol{\xi} \in B, \boldsymbol{\xi} \notin \partial B$ according to (de Lacerda and Wrobel, 2001) can be written as:

$$\sigma_{ij}(\boldsymbol{\xi}) = \\ = \int_{\Gamma} \alpha(\mathbf{x}) \left[D_{ijk}(\boldsymbol{\xi},\mathbf{x}) t_k(\mathbf{x}) - S_{ijk}(\boldsymbol{\xi},\mathbf{x}) u_k(\mathbf{x}) \right] d\Gamma(\mathbf{x}), \quad (1)$$

where $i, j, k = r, z$; $\boldsymbol{\sigma}$, $\mathbf{t}$, $\mathbf{u}$ are the stress tensor, traction and displacement vectors, respectively; $\alpha(\mathbf{x})=2\pi r(\mathbf{x})$; Γ is a curve formed by the intersection of the boundary ∂B with an axial plane; $r(\mathbf{x})$ is the distance between point x and axis Oz. Kernels D_{ijk} and S_{ijk} are discussed and explicitly written in (de Lacerda and Wrobel, 2001).

When the source point $\boldsymbol{\xi} \in B$ limits some boundary point $\mathbf{x} \in \partial B$, that is when $\|\mathbf{x} - \boldsymbol{\xi}\| \to 0$, the kernel function $\mathbf{D}$ becomes singular of a type $O(1/\|\mathbf{x} - \boldsymbol{\xi}\|)$, and a kernel function $\mathbf{S}$ hypersingular of a type $O(1/\|\mathbf{x} - \boldsymbol{\xi}\|)^2$ (de Lacerda and Wrobel, 2001). Thus, when calculating stresses or deformations in a point, which is placed close enough to the boundary, the integrand in the equation (1) will intensively change in the neighborhood of the point $\mathbf{x}=\mathbf{y}$, where $\mathbf{y} \in \Gamma$ is the nearest to $\boldsymbol{\xi}$ boundary point. The analytical calculation of integral (1) is not affected by this behavior of the integrand, so the correct result is obtained (the value of stress tensor). Nevertheless, in BEM the integral (1) is calculated numerically and the intensive variation of the integrand essentially reduces the accuracy of numerical integration. Thus, the boundary layer effect is observed: the error of stress or deformation calculations is intolerable in the points that are very close to the boundary. To eliminate the boundary layer effect, as it was shown for the 2D elastic problems in (Richardson and Cruse, 1999), it is necessary to use the self-regular boundary integral equations, so the full regularization is to be provided.

According to Mukherjee (2000, 2002) when $\boldsymbol{\xi} \to \mathbf{y} \in \Gamma$ the equation (1) can be rewritten as:

$$\sigma_{ij}(\mathbf{y})+A_{ijk}u_k(\mathbf{y})-C_{ijkp}u_{k,p}(\mathbf{y})= \\ =\int_\Gamma \alpha(\mathbf{x})D_{ijk}(\mathbf{y},\mathbf{x})\left[\sigma_{kp}(\mathbf{x})-\sigma_{kp}(\mathbf{y})\right]n_p(\mathbf{x})d\Gamma(\mathbf{x})- \\ -\int_\Gamma \alpha(\mathbf{x})S_{ijk}(\mathbf{y},\mathbf{x})\left[u_k(\mathbf{x})-u_k(\mathbf{y})- \right. \\ \left. -u_{k,p}(\mathbf{y})\left(x_p-y_p\right)\right]d\Gamma(\mathbf{x}). \tag{2}$$

Here

$$A_{ijk}=\lim_{\xi\to\mathbf{y}}\int_\Gamma \alpha(\mathbf{x})S_{ijk}(\xi,\mathbf{x})d\Gamma(\mathbf{x}), \\ C_{ijkp}=\lim_{\xi\to\mathbf{y}}\int_\Gamma \alpha(\mathbf{x})\left[E_{mlkp}D_{ijm}(\xi,\mathbf{x})n_l(\mathbf{x})- \right. \\ \left. -S_{ijk}(\xi,\mathbf{x})\left(x_p-\xi_p\right)\right]d\Gamma(\mathbf{x}) \tag{3}$$

are hypersingular and singular integrals respectively (according to the definition of (Mukherjee, 2000; Lin'kov 1999); the components of tensor **E** are defined by the expression $\sigma_{ij}=E_{ijkm}u_{k,m}$. Integrals in the right hand side of (2), according to Mukherjee (2000) and Lin'kov (1999), are regular. The presented in (Mukherjee, 2002; de Lacerda and Wrobel, 2001) approaches are engaged in calculation of integrals (3).

To withdraw the evaluation of singular and hypersingular integrals another regularization approach for equation (1) should be used. If such auxiliary solution of axisymmetric elasticity can be found that its superposition with (1) gives zero values of $u_k(\mathbf{y})$ and $u_{k,p}(\mathbf{y}$ in boundary point **y**, then in expression (2) the terms with tensors A_{ijk} and C_{ijkp} vanishes. Moreover, when the source point ξ is located close to $\mathbf{y}\in\Gamma$, such representation, according to (Richardson and Cruse, 1999), will eliminate the boundary layer effect.

The auxiliary solution $\mathbf{u}^*(\xi, \mathbf{y})$ must satisfy the partial differential equations of the problem (e.g. see Timošenko, 1972)

$$\frac{1}{1-2\nu}\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{u_r}{r}\right)+\frac{\partial^2 u_r}{\partial z^2}+\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)=0; \\ \frac{1}{1-2\nu}\frac{\partial}{\partial z}\left(\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}+\frac{u_r}{r}\right)+\frac{\partial^2 u_z}{\partial z^2}+\frac{\partial^2 u_z}{\partial r^2}+\frac{1}{r}\frac{\partial u_z}{\partial r}=0, \tag{4}$$

and the displacements u^*_i along with their partial derivatives $u^*_{i,k}$ in a point **y** are to be equal to the corresponding values of the considered problem (1). The simplest way to obtain this auxiliary solution is to use the polynomial one. It is easy to verify by direct substitution, that the displacement field with the following structure

$$u_r^*=C_1r+C_2rz+C_3r^3, \\ u_z^*=C_4+C_5z+C_6\left[z^2(1-2\nu)-r^2(1-\nu)\right]-\frac{0.5C_2r^2}{1-2\nu}- \\ -\frac{1}{1-2\nu}\left[8C_3z\left(r^2\left(1-3\nu+2\nu^2\right)-\frac{1}{3}z^2(1-2\nu)^2\right)\right] \tag{5}$$

satisfies the partial differential equations (4) and consequently can be used as an elementary solution for imposition. Here ν is a Poisson ratio.

The factors C_k can be determined using the mentioned above conditions:

$$u_i^*(\mathbf{y})=u_i(\mathbf{y});\ u_{i,j}^*(\mathbf{y})=u_{i,j}(\mathbf{y}). \tag{6}$$

By substitution of equations (5) in (6) and solution of the resulting system of linear algebraic equations, the explicit expressions for the factors C_k can be obtained:

$$C_1=\left(3u_r-2u_{r,z}z-u_{r,r}r\right)/(2r), \\ C_2=u_{r,z}/r,\ C_3=\left(u_{r,r}r-u_r\right)/\left(2r^3\right), \\ C_4=u_z-u_{z,z}z-\left(u_{r,z}z^2+\left(\alpha z^2+\beta r^2\right)u_{z,r}\right)/(2r\beta)+ \\ +4z\left(\alpha z^2+3\beta r^2\right)\left(u_r-u_{r,r}r\right)/\left(3r^3\right), \\ C_5=u_{z,z}+z\left(u_{r,z}+\alpha u_{z,r}\right)/(r\beta)- \\ -4\left(\alpha z^2+\beta r^2\right)\left(u_r-u_{r,r}r\right)/r^3, \\ C_6=4z\left(u_r-u_{r,r}r\right)/r^3-\left(u_{r,z}+\alpha u_{z,r}\right)/(2r\alpha\beta), \tag{7}$$

where $r=r(\mathbf{y})$, $z=z(\mathbf{y})$, $u_i=u_i(\mathbf{y})$, $u_{i,j}=u_{i,j}(\mathbf{y})$, $\alpha=1\text{-}2\nu$, $\beta=1\text{-}\nu$.

It should be mentioned that conditions (6) are insufficient for determination of C_k, when the regularization point **y** is placed on the axis of symmetry Oz. In this case, taking into account that for r=0 nonzero are only u_z, $u_{z,z}$, $u_{r,r}$, the factors C_k can be defined as follows:

$$C_2=C_3=C_6=0;\ C_1=u_{r,r}(\mathbf{y}); \\ C_4=u_z(\mathbf{y})-z(\mathbf{y})u_{z,z}(\mathbf{y});\ C_5=u_{z,z}(\mathbf{y}). \tag{8}$$

This choice satisfies conditions (6) with the reference that the regularization point **y** is placed on the symmetry axis Oz.

In terms of (1), the stresses, which are induced by the elastic displacements (5), equal

$$\sigma_{ij}^*(\xi)= \\ =\int_\Gamma \alpha(\mathbf{x})\left[D_{ijk}(\xi,\mathbf{x})t_k^*(\mathbf{x})-S_{ijk}(\xi,\mathbf{x})u_k^*(\mathbf{x})\right]d\Gamma(\mathbf{x}). \tag{9}$$

Subtracting equation (9) from (1), the following stress integral equation is obtained:

$$\sigma_{ij}(\xi)=\sigma_{ij}^*(\xi)+ \\ +\int_\Gamma \alpha(\mathbf{x})D_{ijk}(\xi,\mathbf{x})\left[t_k(\mathbf{x})-t_k^*(\mathbf{x})\right]d\Gamma(\mathbf{x})- \\ -\int_\Gamma \alpha(\mathbf{x})S_{ijk}(\xi,\mathbf{x})\left[u_k(\mathbf{x})-u_k^*(\mathbf{x})\right]d\Gamma(\mathbf{x}). \tag{10}$$

Considering (6) and the Hook's law it follows that $t_k(\mathbf{y})-t_k^*(\mathbf{y})=0$. Also from (6) it directly follows that $u_k(\mathbf{y})-u_k^*(\mathbf{y})=0$ and $u_{k,p}(\mathbf{y})-u_{k,p}^*(\mathbf{y})=0$. Thus the representation (10), according to (Mukherjee, 2000; Lin'kov, 1999), completely regularize singular and hypersingular integrals which arise in (1) when the source point $\xi\in B$ limits point **y** on boundary ∂B. Besides, the self-regular integral equation (10) makes it possible to eliminate the boundary layer effect and to calculate stresses extremely close to the boundary of a solid and even on it. Equation (10) can be also used as the basic integral equation for the numerical scheme of BEM to solve the axisymmetric elastic problems, including those of fracture mechanics. Full regularization (10) permits to avoid thus calculation of the principal value integrals that is necessary to do using those BEM schemes of (Mukherjee, 2002; de Lacerda and Wrobel, 2001).

As for the infinite medium, the integral representation of stress tensor components (10) cannot be applied and should be slightly modified. Assume that a solid is bounded with the surface $\Gamma_\Sigma = \Gamma_R \bigcup \Gamma$, where Γ is a boundary of voids and Γ_R is a sphere of a radius R. Integration of (10) over the surface Γ_Σ and the limiting procedure when $R \to \infty$ gives

$$\begin{aligned}\sigma_{ij}(\boldsymbol{\xi}) &= \sigma_{ij}^{\text{hom}}(\boldsymbol{\xi}) + \\ &+ \int_\Gamma \alpha(\mathbf{x}) D_{ijk}(\boldsymbol{\xi},\mathbf{x})\left[t_k(\mathbf{x}) - t_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x}) - \\ &- \int_\Gamma \alpha(\mathbf{x}) S_{ijk}(\boldsymbol{\xi},\mathbf{x})\left[u_k(\mathbf{x}) - u_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x}),\end{aligned} \tag{11}$$

where $\sigma_{ij}^{\text{hom}}(\boldsymbol{\xi})$ is a homogenous solution of the problem for the infinite medium without voids. That is the problem is reduced to the analysis of the perturbation influence of voids, which are bounded domains.

3. STRESS BEM FORMULATION

Integral equations (10) and (11) due to the applied regularization technique are continuous to the boundary. Though the limit procedure of $\boldsymbol{\xi} \to \mathbf{y} \in \Gamma$ can be done by simple substitution. Taking into account that for the regularization point $\mathbf{y}$ according to (6) and Hook's law $\boldsymbol{\sigma}(\mathbf{y})=\boldsymbol{\sigma}^*(\mathbf{y})$ the following integral equations are obtained from (10) for bounded domains

$$\begin{aligned}0 &= \int_\Gamma \alpha(\mathbf{x}) D_{ijk}(\mathbf{y},\mathbf{x})\left[t_k(\mathbf{x}) - t_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x}) - \\ &- \int_\Gamma \alpha(\mathbf{x}) S_{ijk}(\mathbf{y},\mathbf{x})\left[u_k(\mathbf{x}) - u_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x})\end{aligned} \tag{12}$$

and from (11) for unbounded domains

$$\begin{aligned}\sigma_{ij}(\mathbf{y}) - \sigma_{ij}^{\text{hom}}(\mathbf{y}) &= \\ &= \int_\Gamma \alpha(\mathbf{x}) D_{ijk}(\mathbf{y},\mathbf{x})\left[t_k(\mathbf{x}) - t_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x}) - \\ &- \int_\Gamma \alpha(\mathbf{x}) S_{ijk}(\mathbf{y},\mathbf{x})\left[u_k(\mathbf{x}) - u_k^*(\mathbf{x})\right] d\Gamma(\mathbf{x}).\end{aligned} \tag{13}$$

For the solution of equations (12) or (13) the standard BEM procedure (see Richardson and Cruse, 1999) is applied. Unknown derivatives of displacements $u_{i,j}$ are obtained using Hook's law from the following system of equations

$$\begin{cases} \dfrac{2G\nu n_1(\eta)}{1-2\nu} u_{2,2} + \dfrac{2G(1-\nu)}{1-2\nu} n_1(\eta) u_{1,1} + \\ \qquad + G n_2(\eta)\left(u_{1,2} + u_{2,1}\right) = t_1 - \dfrac{2G\nu n_1(\eta)}{1-2\nu}\dfrac{u_1}{r}, \\ \dfrac{2G\nu n_2(\eta)}{1-2\nu} u_{1,1} + \dfrac{2G(1-\nu)}{1-2\nu} n_2(\eta) u_{2,2} + \\ \qquad + G n_1(\eta)\left(u_{1,2} + u_{2,1}\right) = t_2 - \dfrac{2G\nu n_2(\eta)}{1-2\nu}\dfrac{u_1}{r}, \\ -u_{1,1} n_2(\eta) J(\eta) + u_{1,2} n_1(\eta) J(\eta) = u_{1,\eta}, \\ -u_{2,1} n_2(\eta) J(\eta) + u_{2,2} n_1(\eta) J(\eta) = u_{2,\eta}, \end{cases} \tag{14}$$

where G is shear modulus; $n_i(\eta)$ are the components of unit normal vector to the boundary element; η is a boundary element parameter; $J(\eta)$ is a Jacobian of the considered boundary element. Derivatives $u_{i,\eta}$ are obtained directly from the used approximation, e.g. if displacements on the element are given as

$$u_i = \sum_{p=1}^{n} N^p(\eta) u_i^p,$$

then $u_{i,\eta}$ equals

$$u_{i,\eta} = \frac{du_i}{d\eta} = \sum_{p=1}^{n} \frac{d}{d\eta} N^p(\eta) u_i^p,$$

where $N^p(\eta)$ are base functions.

All the rest of BEM procedure is standard (see Richardson and Cruse, 1999).

4. NUMERICAL EXAMPLES

To demonstrate the efficiency of the proposed regularization technique for calculation of the stress field let us consider numerical examples for the unbounded and bounded domains.

As an example of the unbounded domain consider the perturbation of the stress field by the spherical cavity of the radius R in the infinite elastic isotropic medium that is loaded on infinity by the homogeneous stresses q acting along axis Oz. According to (Barber, 2004) the maximum stresses on the boundary of the cavity for such loading equal

$$\sigma_{zz} = \frac{27-15\nu}{2(7-5\nu)} q .$$

For the BEM model of this problem 5 quadratic isoparametric boundary elements are used. The boundary nodes are uniformly distributed. The relative error of σ_{zz} determination by the equation (11) on the boundary of the cavity at a point (r=R, z=0) is less, than 0.2 %. At the same time the ordinary stress equation (1) gives a considerable error even far enough from the boundary of a cavity. Fig. 1 shows the values of stress tensor component $\sigma_{zz}(\boldsymbol{\xi})$ on the axis Or, when x=$r(\boldsymbol{\xi})$ comes close to R.

It can be seen from Fig. 1, that ordinary stress equation (1) gives good results only for the points that are far enough from the boundary. When the source point approaches the boundary, due to the calculation errors, the solution begins to oscillate, and the received values can differ from the true one even in ten times. The regularized equation (11) gives good values in the whole domain. The continuous curve on the plot practically coincides with the analytical solution of the problem (Barber, 2004) (it is not possible to distinguish these results on the plot).

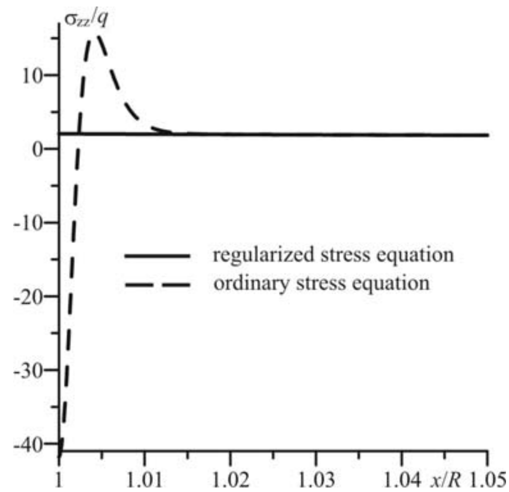

Fig. 1. Determination of stress field near the spherical cavity using
the regularized and ordinary stress integral equations

As an example of a problem with the bounded domain, consider the Lame problem. Its solution can be found, for example, in (Timošenko, 1972). For numerical solution the following parameters are used: the ration of internal and external pressure is p_1/p_2=0,5; the ratio of internal and external radius is R_1/R_2=0,6. For the axisymmetric BEM model the length of a pipe was equal 4 R_2. The maximum relative error of the hoop stresses $\sigma_{\theta\theta}$ determination using the self-regular equation (10) is less than 0.8 %. Corresponding plots of change of these stresses with the thickness of the pipe received by equations (1), (10) and analytical solution (Timošenko, 1972) are shown on Fig. 2.

It can be noticed (Fig. 2), that the hoop stress $\sigma_{\theta\theta}$ received using the ordinary stress equation (1) more than in 100 times differs from the true one. The curves received by the formula (10) and analytical solution of the problem practically coincide.

Now consider a new problem for a finite cylinder with an ellipsoidal elastic inclusion. It is well known that if the ellipsoidal inclusion is bonded into infinite medium the stress field in it is constant (Eshelby, 1957). It is interesting to obtain the influence of cylinder size on the stress field in inclusion and to obtain the size, for which this field is nearly constant.

Consider the cylinder of a radius R and heights $2R$. In the center of cylinder the ellipsoidal inclusion of length a and heights b is placed (a/b=10). The Poisson ratios of cylinder and inclusion are equal 0.3. The ratio of Young's modulus of inclusion and cylinder is denoted by k. The relative size of inclusion is $\lambda = a/R$. The scheme
of the problem and stress deviations

$$\delta\sigma_{ij} = \left[\sigma_{ij}(\lambda) - \sigma_{ij}(0)\right] / \sigma_{ij}(0) \cdot 100\%$$

are plotted in Fig. 3. Hear $\sigma_{ij}(0)$ are the stresses for inclusion in infinite medium (Eshelby, 1957).

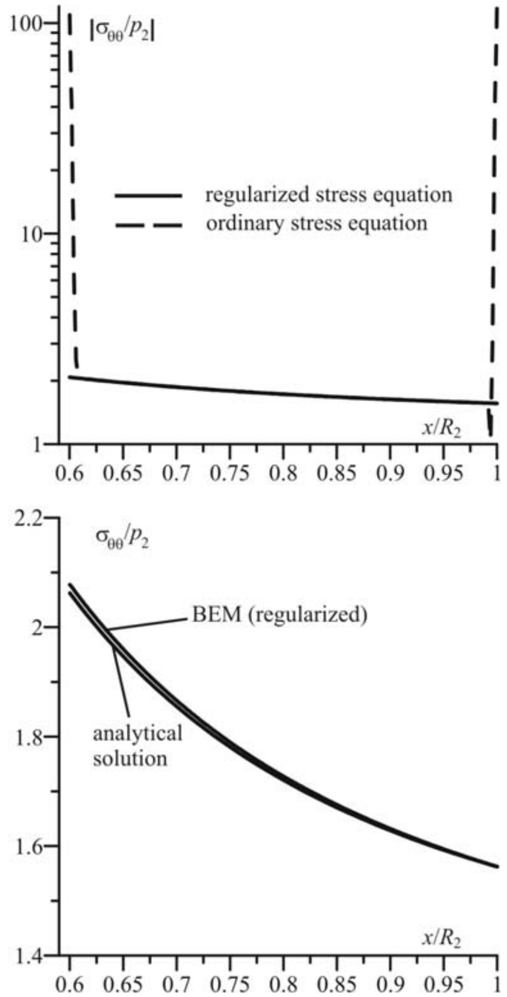

Fig. 2. The BEM and analytical solution of the Lame problem

Stress deviations at the center O of inclusion are denoted by continuous curves, at the point A by dashed curves, and at point B by dash-dot. It can be seen that for the inclusion in cylinder stresses at its center are little greater than those on the surface. The influence of size is greater for soft inclusions ($k < 1$). Fig. 3 shows that with an error of 10 % the solution (Eshelby, 1957) for the infinite medium can be applied to the problem of finite cylinder if the relative size of inclusion is less than 0.4. If the error should be less than 5 % then $\lambda < 0.2$, less than 1 % – $\lambda < 0.1$.

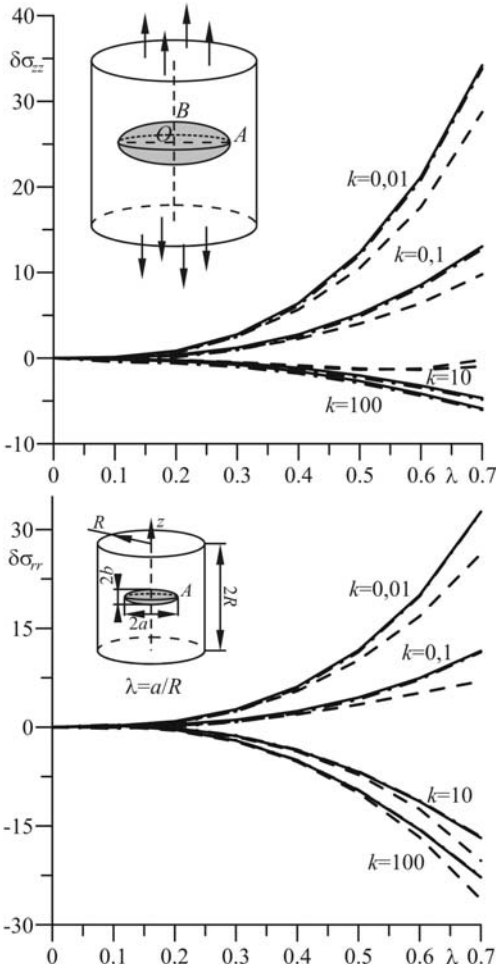

Fig. 3. Stress deviation in inclusion inside the cylinder

5. CONCLUSION

The ordinary hypersingular integral equations of axisymmetric elasticity actually do not suite when the stress field is determined close enough to the boundary of a solid. Due to the numerical integration of nearly-singular integrals, which are crucial for the accuracy, the computational error of ordinary equations is intolerable and the received values are in tens to hundreds times greater, than the true ones. So, instead of ordinary one the self-regular integral equations are to be used. Using the polynomial solution of the partial differential equations of the problem both singular and hypersingular integrals can be regularized and self-regular integral equations are received for both bounded and unbounded domains. These equations are utilized in the new self-regular stress BEM formulation. The numerical procedure of this BEM has much in common with one for 2D elasticity. The peculiarities of the new BEM are discussed separately. Presented numerical examples show high efficiency of the proposed integral equations. New results are obtained for the problem of ellipsoidal inclusion in cylinder.

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The work was supported by the State Committee for Scientific Research of Poland under grant No. 7 T07C 006 12.