A BIOMECHANICAL MODEL OF OPERATED ACHILLES TENDON

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Abstract: The paper discusses a biomechanical model reflecting deformation and tear of a tendon part, namely, ellipric in cross-section strand formed by stochastically located collagen fibers whose strain to stress relation obeys the exponential law. Recovery of the tendon by a plastic reinforcement is shown to result in elevated rigidity and shorter limiting elongation of the strand proportionally to the reinforced portion length, the strength of the restored strand being preserved almost fully. The limiting deformation of the recovered strand made with incisions for adaptation of the ends increases the more the deeper are the incisions, their number and the total length of the areas being adapted. This, nevertheless, decreases essentially the breaking load during further loading.

1. STRUCTURE

The Achilles tendon consists of three or four parts each being linked to a certain group of muscles, namely, the internal and external gastrocnemius muscles and the soleus one (Sitnik, 2003; Medical Encyclopedia). The given work studies the model of only one part of the tendon called a strand.

It is anticipated that the bundles of collagen fibers are stochastically positioned over the strand cross-section (Fig. 1).

To distribute the sectional area of the fibers we have accepted Weibull's law with a mean value 0.2 mm^2 and variation factor ~0.4. The total sectional area of the fibers in a strand makes up $Ac \cong 40$ mm^2, the number of the fibers in a model strand is 250.

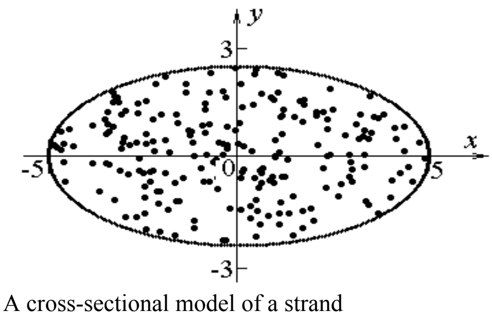

Fig. 1. A cross-sectional model of a strand

2. DEFORMABILITY AND STRENGTH OF THE FIBERS

The deformation law of the collagen fibers is accepted in the following form as an exponential function

$$\sigma(\varepsilon) = c_n \varepsilon^n, \qquad (1)$$

where c_n – elasticity modulus; n – power exponent (not obligatory an integer number).

The elasticity modulus and exponent in Eq. (1) are dependent on a patient's individual features. In further calculations we have taken that $c_n = 10$ Gpa.

We presumed that tear of the collagen fibers occurs after reaching some limiting strain without plastic flow

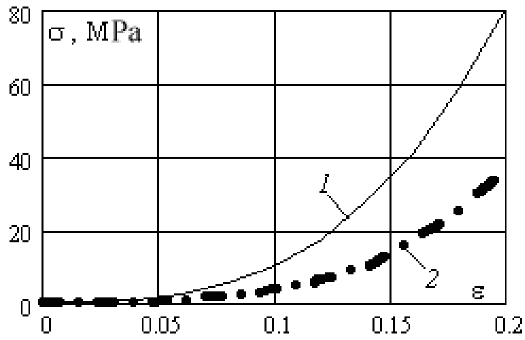

Fig. 2. Diagrams of fiber deformation at n = 3 (1) and n = 3.5 (2)

The limiting strain of the fibers was presented by a random value obeing a normal law of distribution with parameters $M\varepsilon$ and $s\varepsilon$. Along with above named, we have introduced a scale factor, which is a dependence of the limiting strain on sectional area A of the fibers in the form of an exponential function

$$k(A) = (Ac/A)^m, \qquad (2)$$

where Ac – mean sectional area of the fibers in a strand; m – an index accounting for the the degree of dependence of the limiting strain on dimensions.

The mean limiting strain value of a model bundle of fibers is $\varepsilon c = 0.208$, the variation factor is $V\varepsilon = 0.19$, which surpasses much the initial value $s\varepsilon/M\varepsilon = 0.125$ due to a size scatter of the fiber sections in a bundle.

At $M\varepsilon = 0.2$, $s\varepsilon = 0.025$ and $m = 0.2$, which is reflected in a histogram shown in Fig. 3.

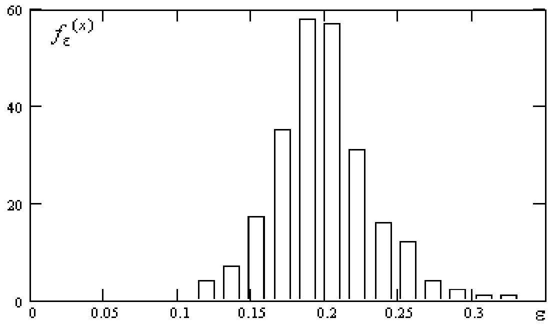

Fig. 3. Distribution histogram of limiting strains of collagen fibers

3. DEFORMABILITY AND STRENGTH OF A STRAND

The diagram for deformation of a strand (Fig. 4) was constructed by giving a sequential series of strains with account of the bearing capacity (stress multiplied by sectional area) of those fibers whose limiting strains exceed the given one.

$$F(\varepsilon) = \sum_{i`} \sigma_i`(\varepsilon) \cdot A_i` \quad (3)$$

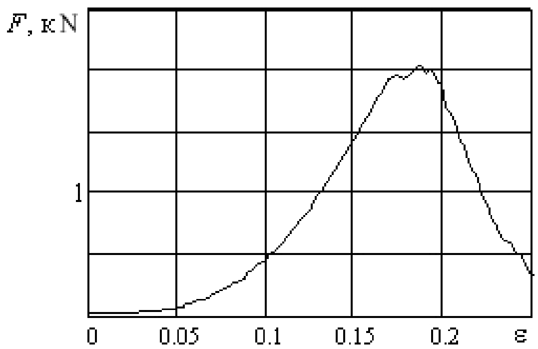

Fig. 4. Deformation diagram of a strand

The breaking force of 2 kN obtained by calculations is close to the values typical for the parts of Achilles tendon (Bergel, 1971). The deformation corresponding to a maximum load constitutes 18%, which also fits the range of observed during the experiment values. The descending branch indicates lowering of the bearing capacity of the strand after reaching some limiting load.

It is also noted in a number of works on pathogenesis and recovery of Achilles tendon that tear of a tendon is preceded by a gradual accumulation of degeneratyive dystrophic changes in the tissues and membrane of the tendon. This occurs most often during tedious and intensive physical exercises typical for sportsmen and ballet dancers (Aiuyb, 1997).

The degree of microdamage of the strand as a share of fibers breaking under loading *F* (Stavrov, 2008) was estimated by the formula

$$Q(F) = 1 - \frac{1}{As} \sum_{i`} A_i(F)`, \quad (4)$$

where $A_i(F)`$ – area of the fiber keeping its strength under load *F* on the strand; *As* – total (initial) cross-section area of strand fibers.

Proceeding from Eq. (4), we have constructed a diagram of fiber microdamage in a loaded strand (Fig. 5).

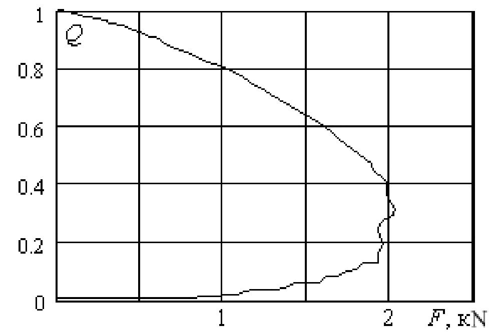

Fig. 5. Diagram of microdamage of the strand

The diagram visualizes that noticeable damages of the fibers are observed already under the loads twice as low as the limiting ones. This complies with available data on investigations of Achilles tendom failure.

The calculation results presented in Fig. 5 prove that the limiting load corresponds to rather large share of the damaged fibers reaching 30%. Apparently, under repeated loads approaching the limiting ones microdamages are to be observed systematically, while their healing will lead to variations in the mechanical properties of the tendon. This is why, estimation of the microdamage degree of the tendon fibers gives important data for prophylactics and curing of tendon tear.

The position of the foot effects the strained length of the strand and load distribution between the strands as parts of the tendon. The length of a random fiber in a strand was estimated roughly with account of deviations of the foot by the relation

$$L = lo - l`/\cos\phi + \Delta L \ , \quad (5)$$

where *lo* – length of a straight line part; *l`* – length in the zone of fixing to the muscles and calcanean tuber; φ – angle of deviation of the fixed fiber part; Δ*L* – length variation due to deviation of the foot from the normal position.

The length of the linear portion of the fibers was taken equal to 80-100 mm, the length in the zone of fixing was also 80-100 mm and distribution of angle φ was taken close to the normal law.

Variations in the fiber length at deviation of the foot from the horizontal position are not large and do not surpass 10^o, so far the corresponding changes in the fiber length are negligible. A larger deviation of the foot is in the vertical plane during the plantar and backside flexure.

In this case, $\Delta L \cong xo\ \Delta\theta$, where *x*o is distance over the contour line from the point of revolution of a subtalar articulation till the tendon; Δθ – plantar bend angle (counted from a normal position of the planta). By taking that xo = 40 mm, we obtain an increment of the fiber length as a function of the foot flexure.

The mean limiting deformation value of the fibers somewhat increases by means of increasing fiber length during backside foot flexure, whereas the limiting deformation distribution shifts to the side of higher values. The threshold load on the strand augments too (Fig. 6).

The character of these changes with increasing foot bend angle is analogous to that presented in Fig. 6, and its force is independent in the muscles stretching the Achilles tendon (Bergel, 1971). Evidently, above regularity reflects a natural adaptation of the tendon to a growing load in the course
of backside flexure of the foot.

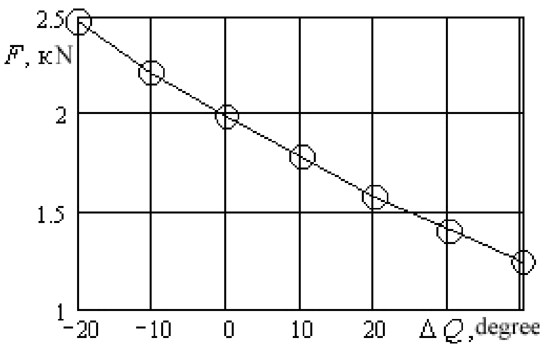

Fig. 6. Limiting load on strand versus foot bend angle

The level of microdamage corresponding to the fracture load on the strand during backside flexure of the foot cedes the one at plantar flexure (Fig. 7).

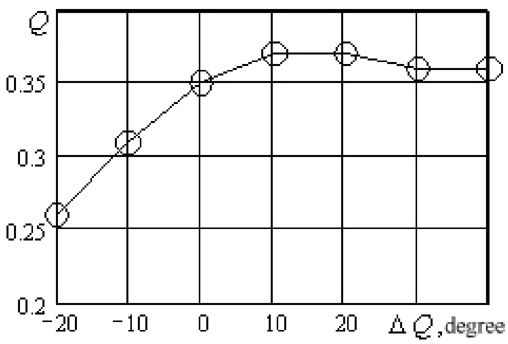

Fig. 7. Dependence of fiber microdamage under threshold load on strand upon plantar bend angle

Above dependence can be actually explained by the increasing mean value and the root mean square deviation of the fiber length.

4. PLASTIC REINFORCEMENT EFFECT

One of most accustomed methods of operative rehabilitation of the Achilles tendons tear is the plastic reinforcement of the initial suture with a patch of the gastronomies muscle, aponeurosis of the tendons or synthetic implants (Sitnik, 2003). Each of named cases leads to widening of the tendons cross-section area along with variation of its deformability in the zone of plastic reinforcement.

To estimate the effect of the plastic reinforcement on the tendon strength we have accepted that its reinforcement leads to variations in fiber deformability. The parameters of the law of deformation change too (1). The deformation diagram of the strand over the area of reinforcement is characterized by the relation

$$F``(\varepsilon) = As\, c_n``\varepsilon^{n``}, \qquad (6)$$

where As – nominal sectional area of the strand; $c_n``$ and $n``$ – stiffness parameters of the strand.

The length of the fibers on undamaged portions $L`$ reduces by a length of plastic reinforcement $L``$. Their limiting deformations change correspondingly.

Elongation of the recovered strand under load F and deformation of the fibers ε are found by the formula

$$\Delta L(\varepsilon) = \varepsilon\, L` + \left(\frac{F(\varepsilon)}{c_n``As}\right)^{1/n``} L``. \qquad (7)$$

The deformation diagram of the recovered strand has been constructed proceeding from the relationship between the load and elongation given by formula (7).

It was found out that as a result of plastic reinforcement the tendon acquires elevated rigidity at $c_n``$=20GPa and $n``$=2, the limiting load being kept howerver, at the previous level.

With lengthening of the reinforced portion (under invariable stiffness parameters) the limiting elongation, which corresponds to the breaking looad, reduces almost linearly (Fig. 8). The relative increment of the limiting elongation is not, nevertheless, large either.

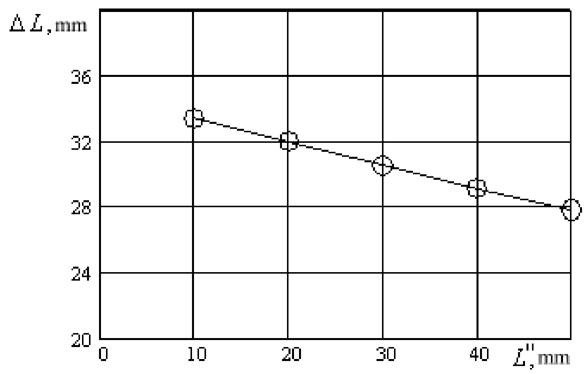

Fig. 8. Limiting elongation of recovered strand versus reinforcement portion length

Above described character of the plastic reinforcement effect is because the structure and properties of the undamaged portion remain invariable, while rigidity increases inversely proportional to the length of the reinforced area for the variant under consideration. Evidently, this effect will be different for a reinforcement portion with some other properties.

5. EFFECT OF INCISIONS

Incisions of the strand made for adapting the ends of the damaged tendon result in variations of its rigidity as well. Here, the incision depth *h* (and, respectively, angle *a*\`), width *la*\`\` (and angle β) and the number of incisions may vary (Fig. 9).

The incision depth influences the part of the fibers cut and the share of undamaged sectional area. The main parameters are interrelated by the following dependencies

$$\alpha`(h) = \arccos(1-h/a);\ x(\alpha`) = a\cdot\cos(\alpha`);$$
$$A(h) = \frac{ab}{2}(2\alpha`(h)\text{-}\sin(2\alpha`(h)), \qquad (8)$$

where *a* and *b* – ellipse semiaxes (Fig. 9).

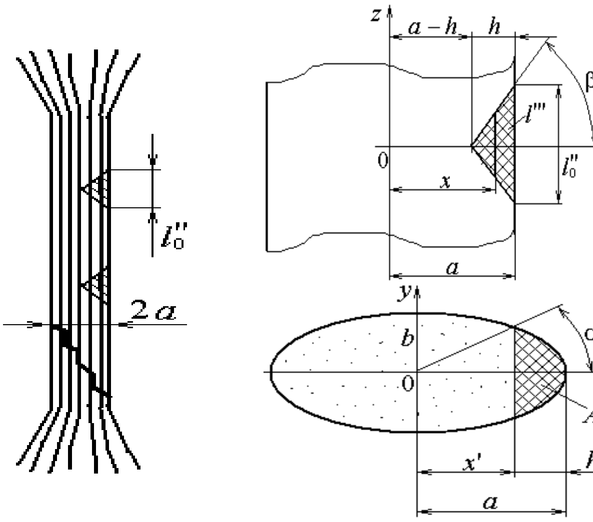

Fig. 9. A scheme of incisions on elliptical in incross-section strand during operation

The initial bearing capacity is preserved only by the fibers located on the sectional area $x < x(\alpha`)$. The limiting strain of the cut-off fibers (at $x > x(\alpha`)$) decreases inversly proportional to the recovered length

$$L``(h) = 2h\ k\ \mathrm{tg}\,\beta, \qquad (9)$$

where *k* – number of incisions of depth *h* and angle β.

So, elongation of the fiber is

$$\Delta L(\varepsilon) = \varepsilon\, L` + \varepsilon^{n/n``} L``(h) \qquad (10)$$

Distribution of the limiting strains of the fibers was determined with account of above relations. The distribution curve of the limiting strain of the fibers in the recovered strand shifts to the region of low values the more the deeper are the incisions and the more is their number.

Reduction of the level of limiting strains results in a lowered rigidity of the strand and decreased threshold force the recovered strand can withstand during further loading (Fig. 10).

Thus obtained regularities are because in part this model neglects the peculiar load transfer features in the zone of recovery. They can be accounted for if the respective information on mechanical properties of the recovered portions of the tendon is available.

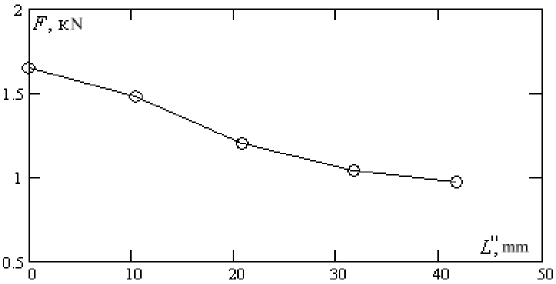

Fig. 10. Threshold load on recovered strand versus total length of incisions

6. CONCLUSIONS

A biomechanical model has been studied for the deformation and failure of one of the parts of a strand tendon of elliptical cross-section formed by stochastically located collagen fibers whose strains and stresses are found in an exponential dependence. The model accounts for the scatter of sectional areas and the length of collagen fibers. Named conditions promote almost linear strength reduction of the strand with increasing bend angle of the plantar, although the share of teared fibers under the maxumal load on the tendon varies insignificantly.

The model of the operated tendon in the form of sequentially linked undamaged and recovered portions has visualized that rehabilitation of the tendon by a plastic reinforcement leads to a raised rigidity and reduced limiting elongation almost proportional to the reinforcing portion length, in which conditions the recovered strand strength is preserved practically fully. The threshold deformation of the recovered strand in which incisions were made to adapt its ends grows and is the higher the deeper are the incisions and the more is their number and the total length of the adapted portions. This, however, leads to a considerable reduction of the breaking strength under further loading.

The present results can be used for selection of the optimal strategy of surgical rehabilitation and treatment of damaged Achilles tendons with allowance for anthropological features of the patient and the character of damage.

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