ESTIMATION OF DIAGNOSTICS OF FIBERS FAILURE IN COMPOSITE MATERIALS BY THE METHOD OF ACOUSTIC EMISSION

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Abstract: The model of acoustic emission caused by formation of penny-shaped cracks in fiber composite materials taking into account stress relaxation in breaking fibers is proposed. It is found that the maximal values of components of displacement vector are directly proportional to the total area of defect, which is formed, and inverse proportional to the relaxation time.

1. INTRODUCTION

Construction of new high-strength fibers with low specific weight induced many researchers to put the efforts at the development of new criteria for evaluation of fracture of composite materials (CM) (Skalsky, 1997). Fiber composites unlike homogeneous structural materials consist of two or more phases of different shape. Main of them are a high-strength, brittle phase forming thin fibers and low-strength more plastic phase, a matrix, which fills a space between fibers and tightly bounds with them. Choosing the orientation of fibers and combining their types with various materials of matrix allows manufacturing CM with the best strength and elastic properties.

In contemporary engineering, investigating fracture processes a method of acoustic emission (AE) (Lipovetskii and Bondarenko, 1983) is frequently used, because it is considered as one of most perspective in this area (Kishi, 1975; Frydman et al., 1975; Nadolinnyy, 1984). The moment of fracture of CM fiber is accompanied by sharp growth of amplitudes of AE signals. Usually strain-gauge testing and visual examination of the damaged CM elements do not allow identifying this process, while the AE method enables to detect early stages of fibers fracture without any outward indication of damages in CM.

2. MODEL OF AE DIAGNOSTICS OF PENNY-SHAPED CRACK FORMATION IN FIBER COMPOSITES

Let us consider the element of a fiber composite (Fig. 1a) with forming the mode I penny-shaped crack under load applied in the direction of the fiber orientation (Fig. 1b).

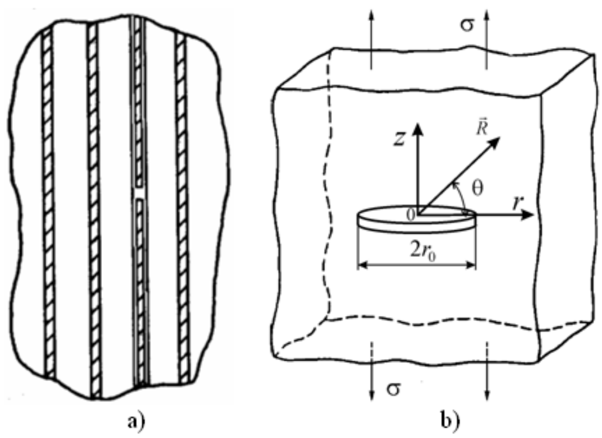

Fig. 1. CM element with a penny-shaped crack in a fiber (a), scheme of this crack (b)

The dynamic displacement field in an elastic body can be found from the equation of motion

$$(\lambda+\mu)\,\mathrm{graddiv}\vec{u}(x,y,z,t)+\mu\Delta\vec{u}(x,y,z,t)-\rho\,\partial^2\vec{u}(x,y,z,t)/\partial t^2=0\,, \tag{1}$$

where $u(x, y, z, t)$ is the displacement vector, λ and μ are the Lame constants, ρ is the density of material, Δ is the Laplace operator. Boundary conditions, which correspond to formation of the penny-shaped crack, in the cylindrical coordinate system $Or\varphi z$ with the center O coinciding with the center of the crack and the Oz axis perpendicular to the crack plane can be written as follows (Andreykiv et al., 1987):

$$\begin{aligned}&\sigma_{zz}(r,0,t)=-\sigma_0 f(t)H(t), \quad r<r_0\,,\\&u_z(r,0,t)=0, \quad r\ge r_0\,,\\&\tau_{rz}(r,0,t)=0, \quad 0<r<\infty\,.\end{aligned} \tag{2}$$

Here σ_{zz} and τ_{rz} are the components of stress tensor,

$H(t)$ is the Heavyside function, r_0 is the radius of a penny-shaped crack (a fiber radius), σ_0 is the critical stress, at which a fiber brakes, the function $f(t)$ describes time relaxation of stress at the crack surfaces due to relaxation processes (with characteristic time τ_r) in plastic matrix. Initial conditions are to be zero

$$\vec{u}(x,y,z,t)=\dot{\vec{u}}(x,y,z,t)=0\,. \qquad (3)$$

Solution of the formulated dynamic problem (1) – (3) we find as follows.

Firstly we consider an auxiliary problem, which differs from the formulated above by that $f(t)\equiv 1$ (instantaneous formation of a penny-shaped crack). The solution for displacements of this auxiliary problem we found previously (Andreykiv, 1987; Andreykiv and Lysak, 1989). Its asymptotes in the spherical coordinate system $OR\theta\varphi$ have the form:

$$u_i(\overline{R},\theta,T)=AB_i(\theta)\left(1/\overline{R}\right)\times$$
$$\times\left\{\int_0^1 2qb(q)m(q)J_0(m\tau_i)M_i(k)dq\right\}H(\tau_i^{(\min)})+O(R^{-2}), \qquad (4)$$

where $i=R$, θ; $T=c_1t/r_0$; $\tau_i=T-e_i\overline{R}$; $\tau_i^{(\min)}=T-e_i\overline{R}^{(\min)}$; $\overline{R}=R/r_0$; $\overline{R}^{(\min)}$ is the least distance from the crack region to the view point; $e=c_2/c_1$; $e_1=e$, $e_2=1$; c_1 and c_2 are the velocities of longitudinal and transverse waves, respectively; $M_1(k)=K(k)$; $M_2(k)=2E(k)-K(k)$; $k=q/2R\cos\theta$; $b(q)$ and $m(q)$ are the approximation functions written in Andreykiv (1987), Andreykiv and Lysak (1989); $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind, respectively; $J_0(.)$ is the Bessel function of the first kind of zero order; functions $B_1(\theta)=e/\pi(1-2e^2\cos^2\theta)$ and $B_2(\theta)=1/\pi\sin 2\theta$ determine angular dependence of radiation for modeling a crack by the system of three mutually perpendicular dipoles (Wadley and Scruby, 1983).

By using the method of Laplace integral transforms we obtained the solution of dynamic problem (1) - (3) for general case $f(t)\neq 1$ in the form

$$\vec{u}_k(\vec{x},t)=\left\{\int_0^t f'(t-\tau)u_k^{(0)}(\vec{x},\tau)d\tau+f(0)u_k^{(0)}(\vec{x},\tau)\right\}\times$$
$$\times H\left(t-e_iR^{(\min)}\right), \qquad (5)$$

$k=L,T$,

where the indexes L and T correspond to the components of longitudinal and transverse waves, respectively; $\vec{u}^{(0)}(\vec{x},s)$ is the solution of the auxiliary problem on instantaneous ($f(t)\equiv 1$) penny-shaped crack formation.

Computation of the displacement vector components in the spherical coordinate system we conducted for $\nu=0{,}28$, $c_2/c_1=0{,}535$, where ν is the Poisson's ratio. The function
of stress relaxation at the surfaces of the crack was chosen in the form proposed in Kaplitskii et al. (1984):

$$f(t)=1-\exp(-t/\tau_r), \qquad (6)$$

where characteristic relaxation time $\tau_r\geq r_0/c_1$. In the case, when f(t) has the form of equation (6), the dependence (5) takes the form:

$$u_i(\overline{R},\theta,T)=AB_i(\theta)\left(\frac{1}{\overline{R}T_r}\right)\int_{\tau_i^{(\min)}}^{T}\exp\left(-\frac{T-\tau_i}{T_r}\right)\times$$
$$\times\left\{\int_0^1 2qb(q)m(q)J_0(m\tau_i)M_i(k)dq\right\}H(\tau_i^{(\min)})+O\left(\overline{R}^{-2}\right), \qquad (7)$$

where $i=R$, θ; $T_r=c_1\tau_r/r_0$.

The example of computation of the dimensionless components $u_R(R, \theta, T)/A$ of the displacement vector against the dimensionless time T for dimensionless distance between the crack center and the view point, according to equation (7), is shown in Fig. 2, for $\overline{R}=20$ (a) and $\overline{R}=500$ (b), respectively, the angle between direction to the view point and the crack plain $\theta=30^{\circ}$. The curves 1 correspond to the case of instantaneous penny-shaped formation ($f(t)\equiv 1$), and the curves 2 are calculated for $T_r=1$.

It is obvious from Fig.2 that accounting the stress relaxation at surfaces of the crack results in decreasing of maximal values of the displacement vector component and increasing of the width of first maximum of oscillation. In general, the obtained dependences, taking into account the stress relaxation, have more smooth character in comparison with the case of instantaneous penny-shaped crack formation.

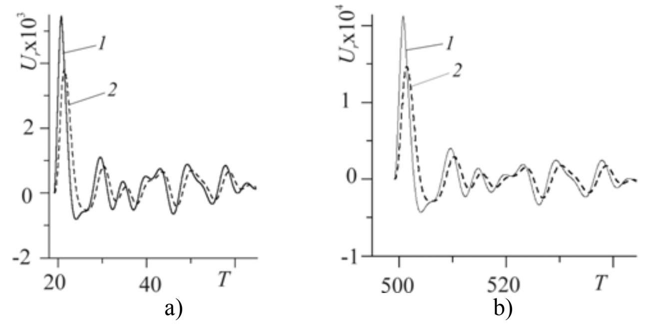

Fig. 2. Dependence of dimensionless component $u_R(R, \theta, T)/A$ of the displacement vector on the dimensionless time T for $\theta=30^{\circ}$ $\overline{R}=20$ (a) and $\overline{R}=500$ (b): *1* – with stress relaxation, *2* – without stress relaxation

The conducted calculations have shown that this tendency remains valid, namely, the maximal values of components of displacement vector continue decreasing and the width of the first maximum grows with increasing of relaxation time T_r.

Analysis of calculations results, conducted according to dependence (7) has shown that for the maximal values of components $u_R(R, \theta, T)$ and $u_\theta(R, \theta, T)$ of displacement vector, which correspond to propagation of longitudinal and transverse waves, respectively, the following approximation formulas characterizing the angular distributing of radiation of elastic waves for formation of penny-shaped crack ($R>>r_0$) are valid:

For the longitudinal wave

$$\Phi_L^{(\tau_r)} = \frac{1-2e^2\cos^2\theta}{\sqrt{1+\chi_1\cos^2\theta}}, \quad (8)$$

for the transverse wave

$$\Phi_T^{(\tau_r)} = \frac{|\sin 2\theta|}{\sqrt{1+\chi_2\cos^2\theta}}. \quad (9)$$

Here parameters χ_1 and χ_2 are equal 0,21 and 2,18, respectively. Their numerical values are obtained with the least-squares method by comparison of formulas (8) and (9) with the maximal values of components of the displacement vector calculated from dependence (7). The dependences (8) and (9) have similar form to those ones obtained previously for the case of instantaneous formation of mode I penny-shaped crack (Andreykiv et al., 2001). However, numerical values of the parameters of approximation χ_1 and χ_2 an in this case (with the accounting stress relaxation) differ from the obtained in Skalsky (1997), χ_1=0,68 and χ_2=2,69. Note that in general different relaxation times T_r correspond to different values of approximation parameters χ_1 and χ_2. Their numerical values can be found in each case by the least-squares method comparing the exact values obtained from the dependence (7) with the approximation formulas (8) and (9).

By using the dependences (8) and (9) and analyzing the component of the dynamic displacement field obtained from the equation (7) the following approximation formulas for estimation of maximal values of components $u_R(R, \theta, T)$ and $u_\theta(R, \theta, T)$ of the displacement vector are obtained:

$$u_{\max}\big|_{c_1} = \frac{\alpha_i\sigma_0\Phi_i^{(\tau_r)}(\theta)r_0^2}{T_r\rho c_1^2 R}. \quad (10)$$

where α_i are the numerical factors for longitudinal (i=1) and transverse (i=2) waves, respectively; α_1=0,37, and α_2=0,63.

Approximation dependence (10) differs from obtained previously (Andreykiv et al., 2001) for the case of instantaneous formation of penny-shaped crack by the relaxation time T_r in the denominator, by other numerical values of α_i and by the functions $\Phi_i^{(\tau_r)}(\theta)$, which determine the angular distributing of radiation, depend on the parameter T_r. In general, the shape of these functions is smoother in comparison with the case of instantaneous formation of a penny-shaped crack. The multiplier r_0^2 proportional to the crack area in the numerator of dependence (10) is relevant for both cases (instantaneous crack formation and crack formation with accounting the relaxation time). This circumstance allows considering that the product $u_{\max}|_{c1}R$ does not depends on distance between the view point and the crack and is proportional to the area S of the formed defect.

3. FORMATION OF CRACK SYSTEM

Now we consider formation of two cracks in CM. Let in certain moment of time considering as initial two penny-shaped cracks have formed in this body as a result of local loss of strength. They have the identical radiuses r_0 and located in parallel planes, which are perpendicular to direction of applied load. Suppose that the distance between their centers, $2d$, be sufficiently large so that one can neglect
an interference of the stress states caused by these cracks in static case. In such case we can consider these cracks as independent. Therefore the displacement field caused by formation of two penny-shaped cracks can be obtained by superposition of two displacement fields caused by isolated cracks described by equation (7). Then the obtained solution of the problem is valid until the wave emitted
by one crack arrive the view point reflecting by other crack.

Let us introduce the system of Cartesian coordinates $Oxyz$. The center of this system is located arbitrarily and the plane xOy is parallel to the planes of cracks location (see Fig. 3). For each crack we introduce also the system of local Cartesian coordinates $O^{(i)}x^{(i)}y^{(i)}z^{(i)}$, where i=1, 2 for the left and right crack, respectively. The axes $O^{(i)}z^{(i)}$ are parallel to Oz. Lets locations of the centers $O^{(i)}$ of these coordinate systems are defined by the vectors $\vec{R}_0^{(i)}$ and location of the view point is defined by the vectors $\vec{R}^{(i)}$.

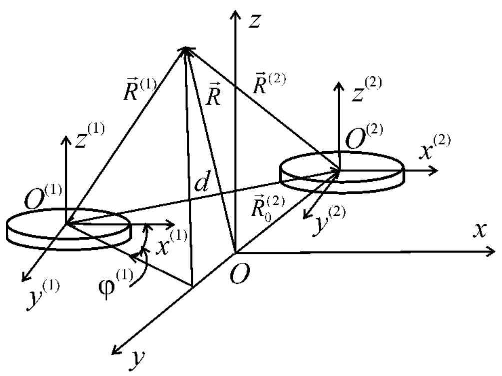

Fig. 3. Arrangement of two penny-shaped cracks

In this case $\vec{R}^{(i)} = \vec{R} - \vec{R}_0^{(i)}$ and components $u_x^{(i)}$ and $u_y^{(i)}$ of the displacement vector in local Cartesian coordinate system takes the form

$$u_x^{(i)} = u_r^{(i)}\cos\varphi_i, \quad u_y^{(i)} = u_r^{(i)}\sin\varphi_i, \quad (11)$$

where $\phi_i = \arccos\left(x^{(i)}/r^{(i)}\right)$, $r^{(i)} = \sqrt{x_i^2 + y_i^2}$.

The components $u_r^{(i)}$ and $u_z^{(i)}$ are determined by the dependence (7) written in the cylinder coordinate system. The components u_x, u_y, u_z of the displacement vector of the total field we represent as follows:

$$u_x = u_x^{(1)} + u_x^{(2)}, \; u_y = u_y^{(1)} + u_y^{(2)}, \; u_z = u_z^{(1)} + u_z^{(2)}. \quad (12)$$

Numerical calculations we conducted for the system of two penny-shaped cracks located in one plane, the distance between crack centers $2d$=10r_0.

Spectral characteristics of these displacements were calculated by the method of the fast Fourier transform (Ahmed and Rao, 1975). They are more oscillating in comparison with the case of formation of one crack.

The directivity of emission for a longitudinal wave we investigated calculating maximum of the module of the displacement vector depending on the angles of view φ and θ for $R>>d$. As a result we found that the angular distribution of emission is not axisymmetric as it was observed in the case of formation of one penny-shaped crack. It is clear that the diagram of directivity is most deformed in the plane perpendicular to the line, which connects the centers of defects, i.e. at $\varphi=\pi/2$ and least deformed at $\varphi=0^o$ in comparison with the case of one crack. The mentioned circumstance might substantially complicate determination of space orientation and dimensions of defects. Note that in this case it is also not easy to find their location. It is conditioned by the fact that one sensor used for the crack location can detect firstly the signal emitted by one defect, but at the same time other sensor can detect firstly the signal emitted by other defect.

In general, in the case of formation (not simultaneous) of the system of L cracks under limitations, which are discussed above, the components of the displacement vector can be written in the form:

$$u_i(R,\theta,T)=\sum_{l=1}^{L}u_i^l\left(R^{(l)},\theta^{(l)},T-T_0^{(l)}\right), \tag{13}$$

where $T_0^{(l)}$ is the moment of l-th crack formation, $R(l)$, $\theta^{(l)}$ are the components of the local spherical coordinate system. Its center coincides with the center of l-th crack.

Substitution of equation (7) into expression (13) with taking into account equations (8) – (10) yields the dependence that connects the peak values of the AE signals, which we consider to be proportional to the maximal values of the components of the displacement vector, and the sum of formed cracks areas.

$$\sum_i A_i R_i = \beta / T_r \sum_i S_i \ , \tag{14}$$

where R_i are the distances between the centers of the cracks and the view point, β is the proportionality factor that has to be determined experimentally.

The formula (14) can be simplified assuming that the linear dimension Δ of the region Λ, where the cracks form, is such that the condition $\Delta<<R_i$ is held. In this case all $R_i \approx R_s$, where R_s is the distance from the center of region Λ to the view point. Then dependence (14) then takes the form

$$\sum_i A_i = b / T_r \sum_i S_i \ , \tag{15}$$

where b is the proportionality factor to be determined experimentally.

4. CONCLUSIONS

Linear dependence between the maximum values of the components of displacement vector and the total area of the cracks, which are formed, and inverse proportional dependence of these maximal values and the relaxation time are found.

Angular distributing of the AE amplitudes during simultaneous formation of two penny-shaped cracks is not axisymmetric as it was observed in the case of one crack formation. In comparison with the case of one crack formation, the diagram of directivity differs mostly in the plane perpendicular to the line connecting the centers of defects, thereby substantially complicating determination of spatial orientation and dimension of the defects.

In general case of a system of cracks formation the sum of AE amplitudes is proportional to the sum of crack areas.

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