DETERMINING THE UNCERTAINTY OF THE OBJECT COORDINATE SYSTEM POSITION IN COORDINATE MEASUREMENTS OF FREE-FORM SURFACES

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Abstract: Coordinate measurements are a source of digital data in the form of coordinates of measurement points with a discrete distribution on the measured surface. Geometric deviations of free-form surfaces are determined at each point as normal deviations of these points from the nominal surface (a CAD model). The calculations are preceded by fitting the measurement data to the CAD model. The relations between the object coordinate system and the coordinate system of the machine are described by the transformation parameters. This paper presents the idea of the process of data fitting with the use of the least square algorithm method as well as the way of determining the uncertainty on the assumption that transformation parameters are subject to a multivariate normal probability distribution. The theoretical issues were verified by experiments carried out on a free-form surface obtained in the milling process and characterised by random geometric deviations.

1. INTRODUCTION

Computer-aided measurement techniques tend to dominate in measuring geometric dimensions connected with machine parts. These techniques involve determining the coordinate values of measurement points situated (using a touch or non-touch system) on the object surface. As the result of the measurement, a set of discrete data is obtained. From the point of view of CAD/CAM techniques, the most important feature of coordinate measurements is providing data concerning the object in the digital form.

A typical machine part geometry is described with simple geometric shapes: straight lines, planes, circles, cylinders, etc. In coordinate measurements, macroinstructions built in software are used; on the basis of the coordinates of measurement points, first geometric associated features, and later their dimensions and shape and location deviations, are determined. The accuracy inspection is reduced to comparing the determined dimensions with the data contained in construction drawings.

Growing demands concerning product functionality, ergonomics, and aesthetics, force creating machine parts composed of 3D curvilinear surfaces. Such parts are shaped by surfaces which cannot be described with simple mathematical equations. The accuracy inspection involves digitalising the measured object (coordinate measuring with the use of the scanning method) and later comparing the ob-tained measurement points coordinates to the CAD design (model). At each measurement point, geometric deviations, or the distances of these points from their projections on the nominal surface, are determined. The processing accuracy inspection results may be presented in the form of a three-dimensional plot or a deviation map.

The majority of problems in the coordinate measurement technique theory results from the discrete character of measurement data. These problems might be divided into two categories:

– different calculation algorithms produce different measurement results for the same set of data;
– different sampling strategies (number and location of measurement points) provide different measurement results for the same surface regardless of applying the same calculation method.

The latter problem category is connected to the fact of measuring a finite number of discrete points on the measured surface described actually with an infinite number of points. Since geometric deviations are different at each point, measurement results depend on the number and location of these points. For the same reason, the number and location of points influence determining geometric features which form the basis of the object coordinate system (Feng et al., 2007; Dhanish and Mathew, 2006; Yau, 1998; Rajamohan et al., 2007). The surface geometric deviations variability is therefore the source of uncertainty in determining the object coordinate system. Consequently, the values of measurement points coordinates determined in this system (and thus the values of geometric deviations) are also characterised by this uncertainty.

Before determining geometric deviations of regular surfaces it is necessary to determine an associated feature from the obtained data. In measuring such surfaces composed of typical geometric features (circles, cylinders, cones, etc.), one of the four methods of determining associated features might be applied (Ratajczyk, 2005). However, it is not possible to determine nominal shapes of curves and free-form surfaces out of measurement data. Processing and measuring these types of surfaces are performed on numerical control devices, using the information on nominal shapes, included in the imported CAD model, to create controlling programmes. For the above mentioned reasons, software of coordinate measurement machines best-fits obtained data to the nominal surface (CAD model), and the least square method is the most often used method here (Yau and Menq, 1996). The idea of this process is described in Chapter 2.

This paper presents the idea of determining the limits of the uncertainty of the coordinate system location of a object determined in the process of fitting data to the nominal surface with the least square method. The experiments were performed on a free-form surface characterised by random geometric deviations.

The experiments were carried out with the use of a MISTRAL STANDARD 070705 coordinate measuring machine equipped with a Renishaw TP200 touch trigger probe with a stylus of 20mm in length, with a ball tip of 2mm in diameter, MPE_E=2,5+L/250.

2. IDEA OF FITTING MEASUREMENT DATA

An ideal (nominal) shape of a surface part might be described with the *N(p)* shape function, where *p* is the set of parameters describing the surface. After the object has been made, its real shape might be described as follows:

$$M(p) = N(p) + \varepsilon(p) \tag{1}$$

where: *M(p)* – the real shape of a surface part, *ε(p)* – geometric deviations.

In coordinate measurements, the coordinates of measurement points are determined on the real surface in the machine coordinate system. The determined coordinates of the *i*-th point on the *M(p)* surface might be described as follows:

$$X_i = T(t)M_i(p) + e_i \tag{2}$$

where: *T(t)* – transformation matrix between the object coordinate system and the machine coordinate system, *t* – transformation, rotation and translation parameters, e_i – measurement error.

If the measurement errors are small when compared to the geometric deviations of the measured object surface, the geometric deviation at each measurement point might be calculated from the following dependence (3):

$$\varepsilon_i(t) = X_i - T(t)N_i(p) \tag{3}$$

where: $\varepsilon_i(t)$ – geometric deviations in the machine coordinate system, $N_i(p)$ – the X_i measurement point projection on the *N(p)* nominal surface in the machine coordinate system.

As it was already mentioned, in measurements performed in the CAD environment, best-fit algorithms of coordinate measuring machines software carry out the operation of fitting the measurement data to the nominal surface (CAD model), or:

$$\varepsilon_i(p) = T^{-1}(t)X_i - N_i(p) \tag{4}$$

where: $T^{1}(t)X_i$ – measurement point coordinates in the object system, $N_i(p)$ – the $T^{-1}(t)X_i$ transformed point projection on the nominal surface, $\varepsilon_i(p)$ – geometric deviation at the measurement point, determined in the object coordinate system.

Before determining geometric deviations it is necessary to establish the transformation matrix which is a function of a three-dimensional rotation and translation (Kiciak, 2000; Yau and Menq, 1996). When applying the least square method to data fitting, the following function *F* should be minimised:

$$F = \sum_{i=1}^{m} \varepsilon_i(p)^2 = \sum_{i=1}^{m} \left| T^{-1}(t)X_i - N_i(p) \right|^2 \tag{5}$$

where: *m* – the number of measurement points.

The fitting effect depends on each of the points selected to establish the transformation matrix. Because of the presence of geometric deviations at each point, different numbers and locations of points result in different fitting effects and thus different locations of the object coordinate system, which means they influence the relations between the object coordinate system and the machine coordinate system (the transformation matrix). Consequently, different values of geometric deviations at each measurement point are obtained for different sampling strategies. This is illustrated in Fig. 1 which shows the outlines of geometric deviations of the milled free-form surface for three different sets of data used to perform the process of fitting the measurement data to the CAD model. As the result of surface scanning, coordinates of 1500 measurement points were obtained. From the scanned data set, three sets of points of different numbers and locations were selected. After having performed the process of fitting these points to the CAD model, the surface geometric deviations were determined. The differences in the deviations values and their distribution contours on the surface are clearly visible.

Minimising the *F* function, the *T(t)* transformation matrix between the object coordinate systems and the machine coordinate systems is determined according to the dependence (5). This is a 4 x 4 matrix in the form (Yau and Menq, 1996; Kiciak, 2000):

$$T(t) = \begin{bmatrix} & R & & \vec{P} \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

where: *T(t)* – transformation matrix between the object and the machine coordinate systems, *t* – transformation, rotation and translation parameters vector, *R* – 3 x 3 rotation matrix, $R(t\alpha, t\beta, t\gamma)$, $t\alpha$, $t\beta$, $t\gamma$ axis rotation angles, $\vec{P}$ – translation vector between the origins, $\vec{P} = [tx, ty, tz]^T$.

A transformation matrix (6) is a combination of rotation and translation, and in general case it has six degrees of freedom, and the transformation parameters set (vector) can be described as $t=[tx,ty,tz,t\alpha,t\beta,t\gamma]$. For 3D axis-symmetrical surfaces the number of parameters is smaller. For example, for a cylinder and a cone it amounts to 5 (three translation vector components and two rotation vector components). In the case of a 2D surface there are two translation components and one rotation angle. In the specific case of a 2D circle (an axis-symmetrical shape), the transformation parameters vector has two components (these of translation).

a)

b)

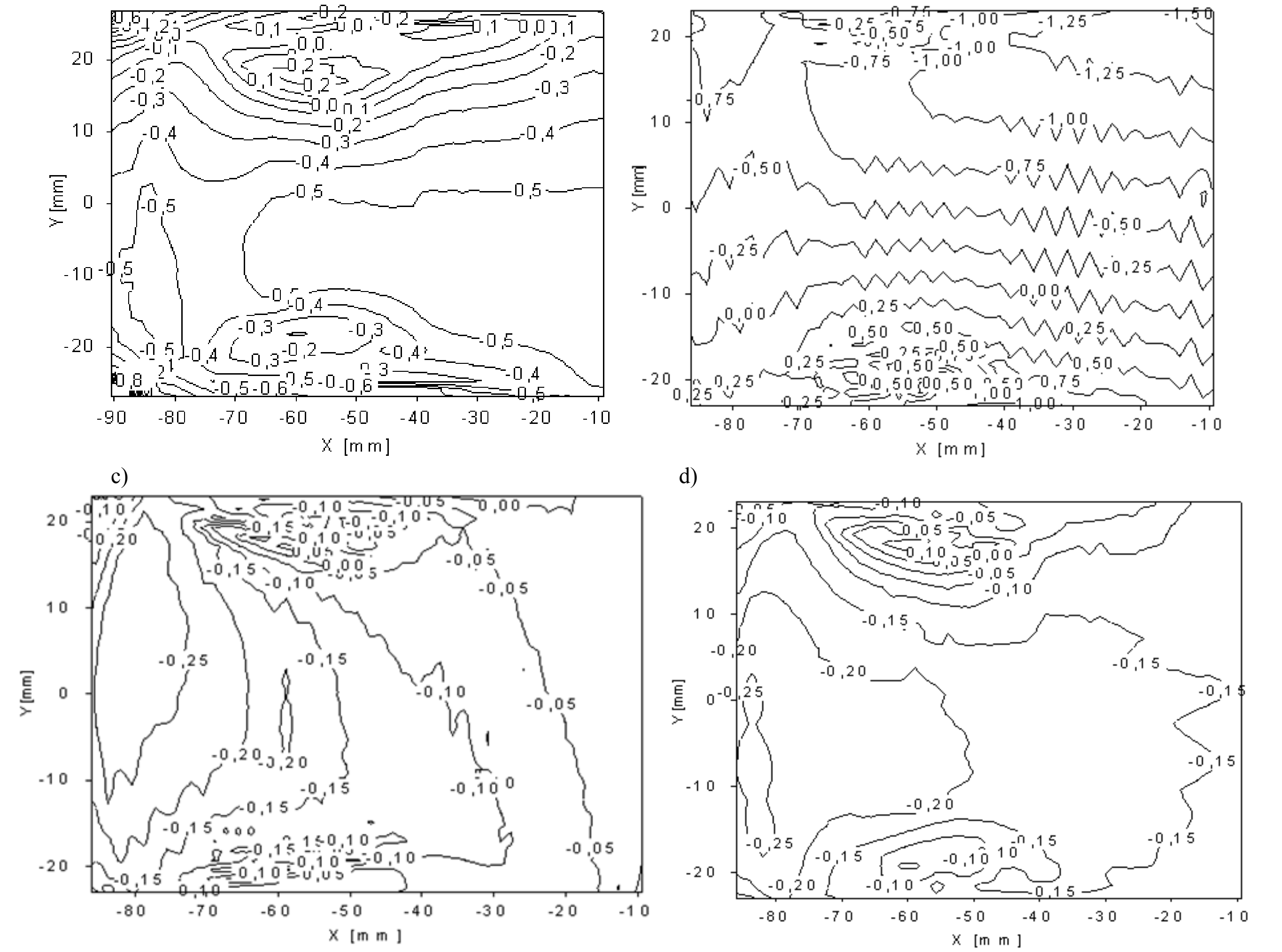

Fig. 1. Contour graphs of geometric deviations: a) before best fitting, b) after best fitting of 15 points, c) after best fitting 108 points, d) after best fitting 1500 points

3. TRANSFORMATION PARAMETERS DISTRIBUTION

The location and orientation of the object coordinate system in relation to the machine coordinate system is described with the *T(t)* transformation matrix. The object coordinate system location is obtained after applying the procedure of fitting the scanned measurement data to the nominal surface. Different sampling strategies result in scattering of the object coordinate system location and orientation, and in variability of transformation matrix parameters, or fitting uncertainty. Fitting uncertainty is therefore inseparably connected with the values and distribution of the object processing errors as well as with the number of measurement points.

Surface geometric deviations are attributed to many factors. Different sources of errors in the production process leave traces on the surface, and deviations are the cumulative effect of the influence of these sources. Geometric deviations may be divided into three components: shape deviations, waviness, and roughness. Components connected with shape deviations and waviness are strongly correlated and are usually deterministic in character. Surface roughness means irregularities of great frequency; in the context of the distance between measurement points it might be assumed that they are random in character. The share of random phenomena on a surface depends on the machining type. The literature shows that after precision milling, values of random geometric deviations of the surface are greater than these of deterministic deviations.

If random surface geometric deviations have normal distribution, for a big enough number of measurement points assumed as the transformation base, it can be assumed that the transformation parameters are random variables of normal distribution. In a border situation, for an infinite number of measurement points, the expected values of transformation parameters describing the location of the coordinate system of the specific measured surface will be obtained. Consequently, the distributions of transformation parameters deviations from the expected values are also normal (Chapter 5).

In general case, the multivariate normal distribution of many variables has the following form (Kotulski and Szczepiński, 2004):

$$f(x_1,...,x_n) = \frac{1}{\sqrt{(2\pi)^n \det[\lambda]}} \exp\left[-0,5(x-\mu)^T \lambda^{-1}(x-\mu)\right] \quad (7)$$

where: λ – n x n covariance matrix, $x=[x_1 ,..., x_n]$ – the independent random variables vector of normal distributions, $\mu=[\mu_1,..., \mu_n]^T$ – the expected values vector.

For the case of analysing the joint distribution $f(\Delta t)$ of the vector of transformation parameters deviations centred around the expected values (μ = 0), the above dependence (7) can be illustrated as follows:

$$f(\Delta t) = \frac{1}{\sqrt{(2\pi)^n \det[\lambda]}} \exp\left[-0{,}5(\Delta t)^T \lambda^{-1}(\Delta t)\right] \quad (8)$$

where: λ – 6 x 6 covariance matrix, $\Delta t= [dx,dy,dz,ax,ay,az]$ – the vector of transformation parameters deviations from their expected values.

Variability of the parameters deviations vector is connected with equal probability (probability concentration) surfaces described by equation (9):

$$(\Delta t)^T \lambda^{-1}(\Delta t) = \eta^2 \quad (9)$$

where: η – the constant dependent on the assumed probability.

These surfaces have the shapes of hyperellipsoids whose centres are determined by the expected values vector.
The directions of the hyperellipsoids axes determine eigen (unit) vectors of the covariance matrix, and the squared lengths of the semi-axes – the corresponding eigen values of the covariance matrix.

The eigen vectors and values of a covariance matrix might be obtained by decomposing this matrix (10) (matrix properties allow for this).

$$\lambda = U \Lambda U^T \quad (10)$$

where: U – matrix whose columns are the covariance matrix eigen vectors, Λ – diagonal matrix of the covariance matrix eigen values.

The hyperellipsoid size is dependent on the assumed probability, and the constant η value is determined from the chi-square distribution, in this case for six degrees of freedom (Kotulski and Szczepiński, 2004).

The aim of the procedure is to determine the fitting uncertainty, or the scatter limits of the Δt transformation parameters deviations vector from the expected values of these parameters vector for a specific probability. The limits are in the shape of hyperellipsoids contours whose centres are located in the point determined by the expected values vector; the object coordinate system origin (transformation parameter vector) will be found in the space limited by them with the assumed probability.

4. MEASURED SURFACE CHARACTERISTICS

The experiments were performed on a free-form surface obtained in a three-stage milling process. In the last stage (profiling), the following parameters were applied: a ball-end mill of 6 mm in diameter, rotational speed equal to 7500 rev/min, working feed 300 mm/min and zig-zag cutting path in the XY plane

The surface was subsequently scanned with the UV method, 2500 (50 rows and 50 columns) uniformly distributed measurement points were scanned from the surface (Fig. 2), and the process of fitting the data to the nominal surface was then carried out in which the least square method was applied and all the measurement points were used.

Geometric deviations of a free-form surface, or normal deviations of measurement points from the nominal surface, might be calculated after previously determining the deviations components in the x,y,z directions (Werner A., Poniatowska M., 2006). Coordinate measuring machines software automatically performs such calculations for each measurement point in the UV scanning option.

The first stage consisted of making a detailed characteristics of the measured surface which meant determining the values and character of the obtained deviations ε. The surface was characterised by deviations whose statistical parameters shows Tab. 1 and map is illustrated in Fig. 3, and the standard deviation of the geometric deviations from the nominal surface amounted to 0.0047 mm. Fig. 4 shows the geometric deviations probability distribution. It can be assumed that the values of geometric deviations undergoes a normal probability distribution.

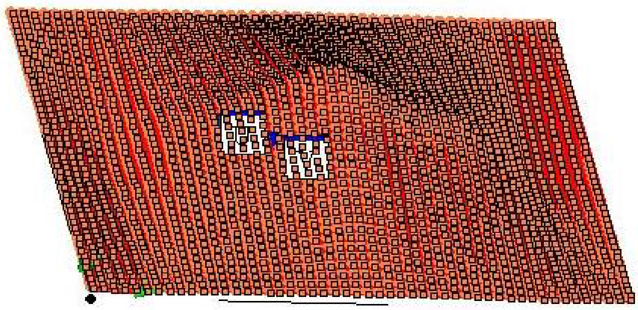

Fig. 2. Measurement points distribution on CAD model (CMM software)

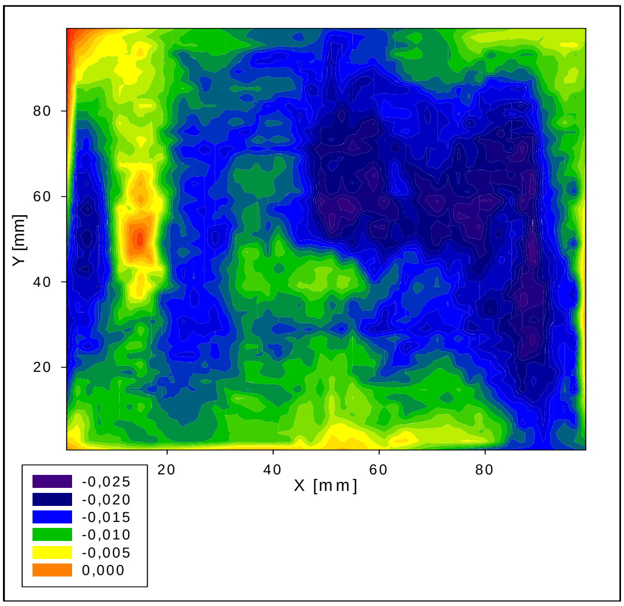

Fig. 3. Map of geometric deviations

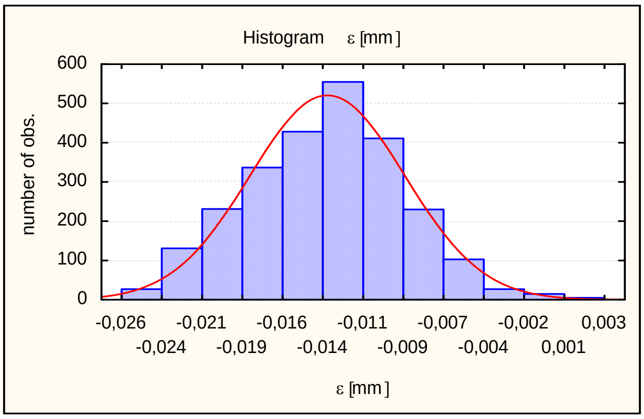

Fig. 4. Geometric deviations probability distribution

Tab. 1. Statistical parameters of ε sample (in mm)

	geometric dev. ε	component x	component y	component z
mean	-0.0137	-0.0006	-0.0001	-0.0141
std. dev.	0.0047	0.0092	0.0055	0.0057
min.	-0.0263	-0.0316	-0.0167	-0.0108
max	0.0034	0.0299	0.0227	0.0330

5. DETERMINING FITTING UNCERTAINTY

In the next stage, groups of 50 measurement points were randomly selected out of the scanned 2500 points fifty times in order to perform the fitting. 50 sets of transformation parameters deviations from their expected values, or the values obtained in the process of fitting on the basis of all the scanned points, were obtained. Standard deviations of parameters are presented in Tab. 2. The normalities of the transformation parameters deviations (*dx, dy, dz, ax, ay, az*) distributions were checked graphically. Probabi-lity distribution of all transformation parameters were quasi-normal. An example distribution for the parameter deviation *dx* is shown in Fig. 5. As the result, the joint distribution
of the vector of transformation parameters deviations was also normal.

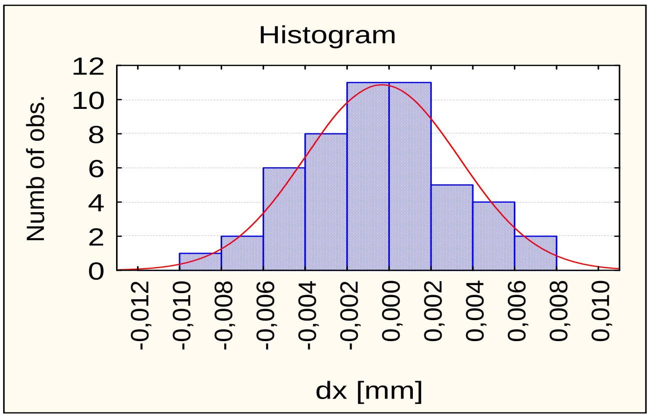

Fig. 5. Probability distribution of the transformation parameter deviation *dx*

Tab. 2. Standard deviations of transformation parameters

	parameter *dx* [mm]	parameter *dy* [mm]	parameter *dz* [mm]	parameter *ax*[deg]	parameter *ay* [deg]	parameter *az* [deg]
std. dev.	0.0036	0.0011	0.0025	0.003	0.002	0.002

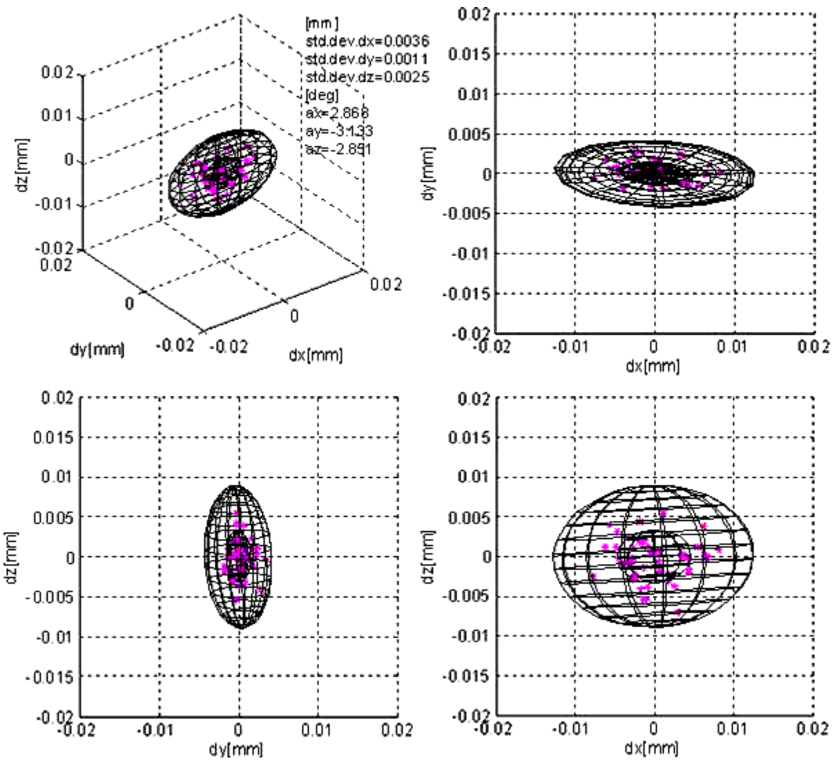

Fig. 6. Uncertainty contours and theirs projections on the coordinate system main planes

Assuming the P=0.95 ($\eta^2=\chi^2_{0.95}$) (6)=12.59 probability for the upper limit of the possible scatter range of the coordinate transformation and P=0.05 ($\eta^2=\chi^2_{0.05}$) (6)=1.63 for the lower limit from the (9) dependence, the equal probability hyperellipsoids limiting the (uncertainty) space were established. The computations and graphical illustration (Fig. 6) of the results were performed in the Matlab programme. The asterisks represent the transformation vector deviations scatter. It can be observed that the deviations of the transformation vector from their expected value, obtained in the experiment, are in the space within the uncertainty contours.

The origin of object coordinate system was located in the space limited by the obtained uncertainty contour with the probability P=0.95. Uncertainty of the object coordinate system was transferred to the uncertainty of each point determined in this system (and obviously, each geometric deviation). The contour dimensions were approx. 0.0252x0.0078x0.0176 mm. A symptom of the fact that the *X*-axis of the hyperellipsoid was greater size from two others (semi-axis is approx. 0.0126 mm) was caused by the greatest scatter of the component *x* of geometric deviations ε (Tab. 1).

6. CONCLUSION

In coordinate measurements, before determining the geometric deviations of a 3D surface, the process of fitting the measurement data to the nominal surface (CAD model) is performed. The transformation (rotation and translation) parameters describing the relation between the object coordinate system and the machine coordinate system are determined that way. The fitting effect is dependent on the number and location of the measurement points because of the occurrence of geometric deviations in producing particular surfaces in technology processes.

This paper presents the idea of fitting the measurement data to the CAD model with the use of the least square method, as well as the idea of determining the uncertainty contours at the assumption that the six transformation parameters are subject to a multivariate normal probability distribution. These equal probability contours are in a shape of hyperellipsoids determined from the multivariate normal distribution of six transformation parameters for the assumed confidence level.

The theoretical issues were verified by the experiments carried out on a free-form surface obtained in the milling process and characterised by random geometric deviations. Experimental values of the transformation parameters vector are located in the space limited by theoretically determined uncertainty contours which were determined and presented graphically. Scatters of translation parameters *dx, dy, dz* were approx. 3 times smaller then scatters of *x, y, z* components of geometric deviations. Dimensions of uncertainty hyperellipsoid centered around the expected values were 0.0252x0.0078x0.0176 mm.

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