

REQUIREMENTS FOR AIRBORNE GRAVIMETRY SYSTEM

Elena BEZVESILNAYA*, Yuryj PODCHASHINSKY**, Alexandr DOBROZHANSKY**

*Department of Instrument-making, National Technical University of Ukraine "KPI",
Peremogy str., 37, Kiev, 03056, Ukraine

**Department of Informational-computing Technology, Zhitomir State Technological University,
Chernyachovsky str., 103, Zhitomyr, 10005, Ukraine

bezvesilna@mail.ru, ju-p@ztu.edu.ua, victoriya_rh@rambler.ru

Abstract: In the article the requirements to system for airborne gravimetry are formulated. A system for airborne gravimetry must consist of five functional subsystems for 1) specific force measurement, 2) geometric stabilization, 3) terrestrial navigation, 4) altimetry, and 5) computation. The general error of measurements of gravitational anomalies does not exceed 10 mgal. The accuracy requirement to main bodies of the block diagram of system for airborne gravimetry are determined based on analysis of errors of measurements.

1. INTRODUCTION

A knowledge of gravity anomalies on a global scale is all that is required to determine the shape of the geoid and to map deflections of the vertical through use of the formulas of Stokes and Vening Meinesz. The computations involved require a relatively dense and accurate gravity net near the computation point, and less dense and accurate measurements over the rest of the world. The techniques of airborne gravimetry seem particularly suited to the task of supplying coarse data on remote regions of the world for use in conjunction with more accurate local nets established by surface measurements.

2. STRUCTURE OF SYSTEM FOR AIRBORNE GRAVIMETRY

A system for airborne gravimetry must consist of five functional subsystems for 1) specific force measurement, 2) geometric stabilization, 3) terrestrial navigation, 4) altimetry, and 5) computation (Bezvesilnaya, 2001; 2007). In determining the accuracy required of such a system, or in evaluating the utility of a given system, we must recall that the only use for global gravity data is the computation of geoid heights and deflections of the vertical. Overall system accuracy should then be evaluated in terms of the resulting accuracy in these computations. Although measurement accuracies on the order of ± 1 to 3 mgal may ultimately be required, significant improvement in the existing gravity net would result from measurements accurate to 10 mgal.

Even with a high speed data gathering system such as an airborne gravimeter, we cannot hope to obtain an infinitely dense measurement net over the entire surface of the earth. The ultimate accuracy attainable in geoid heights and deflections of the vertical is limited by the

density of the gravity net if the net is ideal (i. e. zero measurement error). A method for computing the error in geoid heights and deflections of the vertical due to a given measurement spacing is given in (Bezvesilnaya, 2007), and results are calculated for ideal measurement profiles spaced 2° , 4° , 10° and 20° apart. The question which must then be answered is, "What measurement accuracy is necessary along these profiles in order that no significant additional errors occur above those due to non-continuous gravity data coverage?" The general steps in obtaining the answer to this question are as follows:

- The desired accuracy in computing geoid heights and deflections of the vertical must be specified by geodesists.
- Based on the existing worldwide gravity net and the economics of airborne system operation, the spacing of measurement profiles must be specified.
- Assuming perfect data in the form specified in b) to be available, an optimum interpolation scheme is used to predict anomalies at points between the measurement profiles, and the average error in these interpolated values is evaluated. The measurement errors along the profiles should then be small relative to this average interpolation error.

The average interpolation error in establishing a gravity anomaly profile between two data points:

$$\overline{\varepsilon^2} = C_0 \left[1 - \frac{\sigma}{d} + 2 \left(e^{2d/\sigma} - 1 \right)^{-1} \right], \quad (1)$$

where C – mean squared gravity anomalies over the whole earth, d – the distance between the two data points, σ – correlation length for gravity anomalies,

The procedures and reasoning outlined above will, in most cases, be of only academic interest to the design engineer, since he will most often be asked to provide the lowest overall system uncertainty possible within the state-of-the-art.

In attempting to determine surface gravity anomalies from measurements at altitude, two effects must be considered. First, gravity anomalies are attenuated at altitude, and the downward extension of airborne anomaly measurements will result in a surface map on which small amplitude variations have been filtered. Second, gravity measurements at altitude do not represent surface point anomalies, but give the weighted average of gravity anomalies over an area whose diameter is about 18 times the flight altitude. If the gravimeter output is further averaged in time, this area will be extended along the ground track of the aircraft.

Gravity anomalies are caused by mass anomalies in the earth's crust. Consider a hypothetical gravity anomaly Δg_0 , measured at the earth's surface, due to a mass excess at a depth d beneath the surface. The magnitude of this anomaly is proportional to d^2 . If we now move a height h above the surface and again measure the gravity anomaly Δg_h , the result will be proportional to $(d + h)^2$. Thus the gravity anomaly measured at altitude is attenuated with respect to surface value by a factor of

$$\frac{\Delta g_h}{\Delta g_0} = \frac{d^2}{(d + h)^2} = \frac{1}{\left(1 + \frac{h}{d}\right)^2}. \quad (2)$$

It should be noted that this attenuation is not corrected by the free air correction, which only takes account of the variation of reference gravity with altitude. The influence of this effect on an airborne gravimeter may be thought of in terms of an all-pass filter with variable attenuation connected to the output. This attenuation cannot be accurately compensated since the depth compensation of the Airy-Heiskanen isostatic system (~40 km), the attenuation factor becomes 0.64. In making this comparison, an attenuation correction was applied based on the known surface data. Such a correction would, of course, not be available

in an operational system. While downward continuation techniques can be used to reduce the anomalies measured at altitude to a sea level datum, they cannot recover the detail in the gravity field lost through attenuation into the noise level of the system. This then places a fundamental limitation on the amplitude resolution obtainable with airborne gravimetry systems.

The spatial resolution of an airborne gravimetry system will also be limited due to the fact that anomalies measured at altitude are influenced by anomalies over a sizable area on the ground. The gravity anomaly measured at an altitude h may then be thought of as a spatial average of surface anomalies over a circular area centered directly beneath the aircraft. In order to gain some insight as to the area involved, we may analyze a simplified model.

3. THE GRAVIMETER STABILIZATION SUBSYSTEM

In order to carry out a gravity survey from a moving vehicle, some means of stabilizing the gravimeter along a reference direction is required. Since it is ultimately necessary to deduce the specific force in the direction

of the local geographic vertical, the direct instrumentation of the vertical provides the most desirable measurement environment. Instrumentation of the vertical on a moving base requires however, a rather complex subsystem using high-grade inertial navigation data. The drawbacks of complexity are reduced somewhat by the fact that such a stabilization system can also serve as the heart of a geographic inertial navigator.

As an alternate to stabilization along the vertical, the gravimeter may be allowed to track the apparent vertical, provided the proper compensation term is added to the gravimeter output. Stabilization along the apparent vertical also places a greater load on any gravimeter output-filtering scheme due to the presence of components of short term horizontal acceleration in the gravimeter output.

An airborne gravimetry system may be thought of as the instrumentation of a single dynamic equation, relating the outputs of the required subsystem to the indicated gravity anomaly. As this equation shows, the indicated gravity anomalies are obtained by compensating the output of a specific force sensor (gravimeter) which is stabilized along a vertical or apparent vertical axis. Four types of compensation term appear: 1) vertical accelerations of the aircraft, 2) Coriolis and centrifugal force corrections, sometimes called Eotvos corrections, 3) free air gravity reduction terms, and 4) the computed reference value of gravity at sea level. If an apparent vertical stabilization system is used, the Browne correction must also be applied. All but the first of these compensation terms can be easily computed from the outputs of the previously specified subsystems. The first term, aircraft vertical acceleration, is more difficult to deal with, because it cannot be measured directly due to the indistinguishability of gravitational and inertial accelerations. There remains the possibility of double differentiation of altitude data, separation by filtering and combinations of these techniques.

The sensitivities of the other, more readily computed compensation terms, to errors in the navigation and altimeter subsystem outputs. Compensation error due a given velocity measurement error varies with, both aircraft heading and latitude, the minimum sensitivity for any latitude occurring on a due west heading.

For a given specific force sensor uncertainty, the minimum system uncertainty results when the sensor is physically stabilized along the z axis (vertical axis) of an instrumented local geographic coordinate frame. Errors in the z axis alignment of such a frame result in 1.20 mgal error for each arc minute of misalignment due to projection of horizontal Coriolis forces along the measurement axis, and a smaller second order error which reaches 0.4 mgal at 3 arc minutes vertically error.

4. COMPENSATION OF VERTICAL ACCELERATIONS

In ground-based gravimetry, the term gravimeter has been applied to devices used to measure the difference between gravity at some reference point and at the measurement point. Although the term is still applied to these sensors when used in moving-base systems, they no longer indicate gravity changes alone, but the net specific force

acting on a test mass. They are thus more correctly referred to as specific force sensors.

Several sensors developed for land or sea use, such as the LaCoste-Romberg, the Askania-Graf, and the Worden, have been modified for airborne use. These devices all have been successfully tested in an airborne environment, but they do have some disadvantages, primarily in the areas of data readout and dynamic range.

There exists a large class of specific force sensors developed for use as accelerometers in guidance and navigation system. Several of these sensors seem particularly well suited to use in airborne gravimetry systems. One of the more promising devices, the pendulous integrating gyro accelerometer or PIGA, is currently being readied for flight tests by the MIT Experimental Astronomy Laboratory under Air Force Cambridge Research Laboratory sponsorship.

It is probably neither economically nor technically feasible to choose a single navigation technique such as Doppler, inertial, etc. That can fully meet the requirements of an airborne gravimetry system. The rather specialized requirements for continuous accurate position and velocity output, together with the requirement for global capabilities, indicate the choice of a hybrid navigation system making use of both onboard measurements and navigation aids such as satellites. Specifically, a Doppler-inertial navigator used in conjunction with position fixes from a satellite navigation system would seem best suited. Such a system should

be capable of indicating velocity to 0.5 knot or better and position to 0.5 mile or better for long duration flights at 500 knots.

An examination of the currently available sources of altitude data shows that a direct and continuous determination of sea-level altitude to the accuracy required by an airborne gravimetry system is not possible using any single source

of information. Radar altimeter appears capable of supplying data on sea-level altitude to a sufficient accuracy, but only when over regular terrain or water of known elevation. Errors in determining atmospheric parameters cause prohibitively large errors in pressure altitude measurements relative to a sea-level datum. Pressure measurements can, however, be used to compute with adequate precision,

the altitude deviations from a nearby isobaric surface. The hypsometer would seem to be the best-suited instrument for this measurement.

Combination of air-mass velocity measurements with ground velocity and heading information from the navigation system can, through use of Henry's correction, yield information on the slope of the isobaric surface being flown. Additional data on the height of this isobaric surface can be provided by periodic radar measurements, and by measurements made at surface weather stations.

Data from various sources can be combined in a manner assigned to minimize the mean-squared error in the resulting estimate. This estimate of isobaric surface height, together with the output of a hypsometer, can provide the required altitude data for gravimeter compensation.

The nature of the signal processing and filtering problem is, in most cases, such that post-flight data processing

is possible. This allows the design of a filter free of the usual realizability constraints.

The noise present in the gravimeter output before filtering is mostly due to aerodynamic, wind, and turbulence loading of the airframe. These interfering forces result in aircraft accelerative that are partially counteracted by the autopilot system. The characteristics of the airframe – autopilot system will, in general, change with time, thus the truly optimum, filter should be adaptive in nature.

5. CONCLUSIONS

A system for airborne gravimetry must consist of five functional subsystems for 1) specific force measurement, 2) geometric stabilization, 3) terrestrial navigation, 4) altimetry, and 5) computation. In determining the accuracy required of such a system, or in evaluating the utility of a given system, we must recall that the only use for global gravity data is the computation of geoid heights and deflections of the vertical.

From the results of the error analysis, we see that an airborne gravimetry system capable of measurement accuracy of the order of 10 mgal, must be capable of nominal subsystem accuracy as follows (Bezvesilnaya, 2007):

- velocity over the ellipsoid 0.5 knot;
- latitude 1.0 mile;
- verticality 3.0 arc minutes;
- sea-level altitude 30 feet;
- specific force measurement 2 mgal.

For a system capable of accuracy on the order of 3 mgal, these subsystem requirements become:

- velocity
- no heading restriction 0.18 knot;
- no westerly headings 0.4 knot;
- latitude 0.5 mile;
- verticality 1 arc;
- sea-level altitude 10 feet;
- specific force measurement 1 mgal.

REFERENCES

1. **Безвесільна О.М.** (2001), *Вимірювання прискорень: Підручник*. – К.: Либідь.
2. **Безвесільна О.М.** (2007), *Авіаційні гравіметричні системи та гравіметри: Монографія*. – Житомир: Житомирський державний технологічний університет.