NUMERICAL REVIEW OF FRICTION FORCES IN MICROBEARINGS WITH CURVILINEAR JOURNALS

Krzysztof WIERZCHOLSKI*

* Institute of Applied Mathematics, Pomeranian University Slupsk, 76-200 Słupsk, ul. Arciszewskiego 22, Poland

krzysztof.wierzcholski@wp.pl

Streszczenie: The research showed in this paper lead to presentation of hydrodynamic HDD micro-bearing with orthogonal curvilinear profile of journal for example for conical, hyperbolic and other cooperating micro-bearing surfaces. Pressure distributions and load carrying capacity values on the conical micro-bearing journals are calculated. The formulae describing the friction forces and friction coefficients for micro-bearing curvilinear journals are derived. Up to now, the dynamic behavior of HDD micro-bearings was considered mostly by Jang at al. (2005). Presented paper indicates that on the memory capacity have influence not only herringbone or spiral grooves but in many cases the shapes of micro-bearing journals and sleeves too.

1. INTRODUCTION

Present paper determines the hydrodynamic pressure distributions, load carrying capacity, friction forces and friction coefficients in slide micro-bearings gaps for a curvilinear orthogonal and particularly conical and hyperbolic journal shapes. Fig. 1 show the spindle system of a HDD and a coupled journal and thrust hydrodynamic bearing introduced by Bhushan (2007) and Jang at al. (2005, 2007).

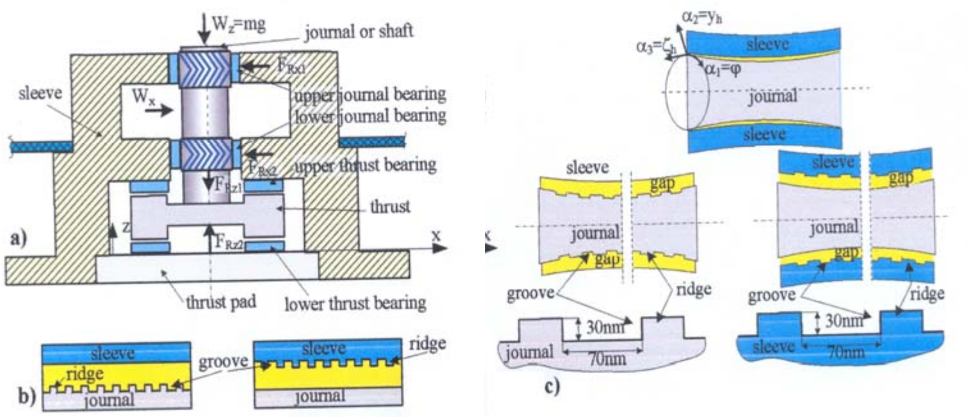

Fig. 1. Coupled curvilinear journal and thrust hydrodynamic HDD micro-bearing (20 000rpm): a) after G.H. Jang and J.W. Yoon (2002), b) classical ridges and grooves on the HDD journal or sleeve surface, c) height of ridge in hyperbolic journal and sleeve after Wierzcholski (2007)

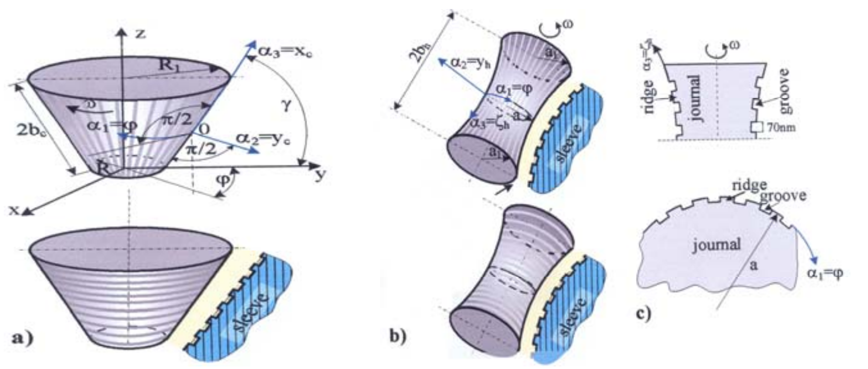

Fig. 2. The view of conical and hyperbolical micro-bearing journal surfaces: a) conical surface, b) hyperbolic surface with longitudinal and circumferential grooves, c) hyperbolic journal with grooves

The aim of the presented paper generalizes the recently calculation methods of the pressure and friction coefficients distributions in a thin layer of non-Newtonian, visco-elastic lubricant of slide micro-bearing gaps presented in: Jang (2005), Jang (2007), Jang (2002), Liu at al.(2007), Wierzcholski (2006, 2007),Yong (2005). The groove and ridge geometry located on the conical and hyperbolic surface are presented here. Fig. 2 shows that the grooves on the hyperbolic and conical journal surfaces can be situated in circumferential or longitudinal directions Jang at al. (2005). Groove location affects the dynamic performance of HDD spindle system.

The micro-bearing lubrication is characterized by the dynamic viscosity changes in thin gap- height direction.

2.PRESSURE DISTRIBUTIONS IN CURVILINEAR MICRO-BEARINGS GAPS

For the conical and hyperbolic micro-bearing we assume following conical and hyperbolic co-ordinates: $\alpha_1=\varphi$, $\alpha_2=y_c$, $\alpha_3=x_c$ and $\alpha_1=\varphi$, $\alpha_2=y_h$, $\alpha_3=\zeta_h$ respectively. Mentioned coordinates are presented in Fig. 2. For conical journal we assume:R_1–the largest radius of the conical shaft, R– the smallest radius of the conical journal, $2b_c$ the bearing length, γ–angle between cone generate line and the cross section plane of the journal (see Fig. 2). For hyperbolic journal we have: a_1 the largest radius of the hyperbolic journal, a the smallest radius of the hyperbolic journal, $2b_h$ the bearing length (see Fig. 2). From the system of conservation of momentum and continuity equation after thin boundary layer simplifications and boundary conditions in the curvilinear coordinates $(\alpha_1,\alpha_2,\alpha_3)$ we obtain the dimensional pressure function $p(\alpha_1,\alpha_3,t)$ satisfying the modified Reynolds equations in the following curvilinear form see Wierzcholski (2006, 2007), and Wierzcholski at al.(2007):

$$\frac{\partial}{\partial\alpha_1}\left[\frac{\partial E(p)}{\partial\alpha_1}E\left(\int_0^{\varepsilon_T}A_\eta d\alpha_2\right)\right]+\frac{h_1}{h_3}\frac{\partial}{\partial\alpha_3}\left[\frac{h_1}{h_3}\frac{\partial E(p)}{\partial\alpha_3}E\left(\int_0^{\varepsilon_T}A_\eta d\alpha_2\right)\right]= \omega h_1^2\frac{\partial}{\partial\alpha_1}\left[E\left(\int_0^{\varepsilon_T}A_s d d\alpha_2\right)-E(\varepsilon_T)\right]+h_1^2\frac{\partial E(\varepsilon_T)}{\partial t} \quad (1)$$

where: E denotes expectancy function, $\varepsilon_T(\alpha_1, \alpha_3,t)$ gap height. Flow is generated by journal rotation and the sleeve is motionless. Lubricant velocity components v_1,v_2,v_3 in α_1,α_2, α_3 directions, respectively, have the following form Wierzcholski (2007a):

$$v_1(\alpha_1,\alpha_2,\alpha_3,t)=\frac{1}{h_1}\frac{\partial p}{\partial\alpha_1}A\eta+(1-A_s)\omega h_1, \quad (2)$$

$$v_3(\alpha_1,\alpha_2,\alpha_3,t)=\frac{1}{h_3}\frac{\partial p}{\partial\alpha_1}A_\eta \quad (3)$$

$$v_2(\alpha_1,\alpha_2,\alpha_3,t)=-\int_0^{\alpha_2}\frac{1}{h_1}\frac{\partial v_1}{\partial\alpha_1}d\alpha_2-\int_0^{\alpha_2}\frac{1}{h_1h_3}\frac{\partial(h_1v_3)}{\partial\alpha_3}d\alpha_2, \quad (4)$$

and

$$A_s(\alpha_1,\alpha_2,\alpha_3,t)\equiv\frac{\int_0^{\alpha_2}\frac{1}{\eta}d\alpha_2}{\int_0^{\varepsilon_T}\frac{1}{\eta}d\alpha_2},\quad A_\eta(\alpha_1,\alpha_2,\alpha_3,t)\equiv \quad (5)$$

$$\int_0^{\alpha_2}\frac{\alpha_2}{\eta}d\alpha_2-A_s(\alpha_1,\alpha_2,\alpha_3,t)\int_0^{\varepsilon_T}\frac{\alpha_2}{\eta}d\alpha_2,$$

where $\eta=\eta(\alpha_1,\alpha_2,\alpha_3)$ – liquid dynamic viscosity, t time, $0\le\alpha_2\le\varepsilon_T$, $0\le\alpha_2<2\pi\theta_1$, $0\le\theta_1<1$ and $-b_c\le\alpha_3\le b_c$ for conical journal and $-b_h\le\alpha_3\le b_h$ for hyperbolic journal.

For the conical shapes of micro-bearing journals we have following coordinates: $\alpha_1=\varphi$, $\alpha_2=y_c$, $\alpha_3=x_c$, and Lame coefficients are as follows see Wierzcholski (2007):

$$h_1=R+x_c\cos\gamma,\ h_3=1, \quad (6a)$$

where γ angle between conical surface and the cross section plane of the journal.

For the hyperbolic shapes of micro-bearing journals we have following coordinates: $\alpha_1=\varphi$, $\alpha_2=y_p$, $\alpha_3=\zeta_p$, and Lame coefficients are as follows see Wierzcholski (2007):

$$h_1=a\cos^{-2}(\Lambda_{h1}\zeta_{h1}),$$
$$h_3=\sqrt{1+4(\Lambda_{h1}/L_{R1})^2\tan^2(\Lambda_{h1}\zeta_{h1})}\cos^{-2}(\Lambda_{h1}\zeta_{h1}), \quad (6b)$$

$$\Lambda_{h1}\equiv\sqrt{\frac{a_1-a}{a}},\quad L_{R1}\equiv\frac{b_h}{a},\quad \zeta_{h1}\equiv\frac{\zeta_h}{b_h} \quad (6c)$$

where R, a, a_1, b_h are defined before.

3. FRICTION FORCES IN CURVILINEAR MICRO-BEARING GAP

This section presents the friction forces calculation in curvilinear micro-bearing gaps.

The components of friction forces in curvilinear α_1, α_3 directions occurring in micro-bearing gaps have the following forms see Wierzcholski (2007):

$$F_{R1}=\iint_F\left(\eta\frac{\partial v_1}{\partial\alpha_2}\right)_{\alpha_2=\varepsilon_T}h_1h_3d\alpha_1d\alpha_3,$$
$$F_{R3}=\iint_F\left(\eta\frac{\partial v_3}{\partial\alpha_2}\right)_{\alpha_2=\varepsilon_T}h_1h_3d\alpha_1d\alpha_3 \quad (7)$$

where $\eta=\eta(\alpha_1,\alpha_2,\alpha_3)$ – liquid dynamic viscosity , t time, $0\le\alpha_2\le\varepsilon_T$, $0\le\alpha_2<2\pi\theta_1$, $0\le\theta_1<1$ and $-b_c\le\alpha_3\le b_c$ for conical journal and $-b_h\le\alpha_3\le b_h$ for hyperbolic journal, F – lubrication surface, and v_1,v_3 fluid velocity components (2), (3) in α_1, α_3 directions, respectively, and h_1, h_3 Lame coefficients (6a,b,c) in α_1, α_3 directions.

Putting formulae (2), (3) into equation (7) for curvilinear journal, then we obtain the friction components F_{R1}, F_{R3} in circumferential α_1, and longitudinal α_3 directions, respectively see Wierzcholski (2006):

$$F_{R1} \equiv F_{Rj} = \iint_F \left[\frac{\eta(\alpha_1,\alpha_2,\alpha_3)}{h_1} \frac{\partial p}{\partial \alpha_1} \frac{\partial A_\eta(\alpha_1,\alpha_2,\alpha_3)}{\partial \alpha_2} \right]_{\alpha_2=\varepsilon_T} \times h_1 h_3 d\alpha_1 d\alpha_3 +$$

$$-\iint_F \left[\omega h_1 \eta(\alpha_1,\alpha_2,\alpha_3) \frac{\partial A_s(\alpha_1,\alpha_2,\alpha_3)}{\partial \alpha_2} \right]_{\alpha_2=\varepsilon_T} \times h_1 h_3 d\alpha_1 d\alpha_3,$$

(8)

$$-\iint_F \left[\omega h_1 \eta(\alpha_1,\alpha_2,\alpha_3) \frac{\partial A_s(\alpha_1,\alpha_2,\alpha_3)}{\partial \alpha_2} \right]_{\alpha_2=\varepsilon_T} h_1 h_3 d\alpha_1 d\alpha_3,$$

$$F_{R3} = \iint_F \left[\frac{\eta(\alpha_1,\alpha_2,\alpha_3)}{h_3} \frac{\partial p}{\partial \alpha_3} \frac{\partial A_\eta(\alpha_1,\alpha_2,\alpha_3)}{\partial \alpha_2} \right]_{\alpha_2=\varepsilon_T} \times h_1 h_3 d\alpha_1 d\alpha_3 \quad (9)$$

where for conical journal $F_{R3} \equiv F_{Rx_c}$ and for hyperbolic journal we have $F_{R3} \equiv F_{R\zeta_h}$.

4. LOAD CARRYING CAPACITY CALCULATIONS

The load carrying capacities for conical and hyperbolic journal are calculated by see Wierzcholski (2007a) from the following formulae respectively:

$$C_{tot}^{(c)}(t) = \left\{ \left[\int_{-bc}^{+b_c} \left(\int_0^{\varphi_k} p(\varphi, x_c, t)(\sin\varphi) h_1 d\varphi \right) h_3 dx_c \right]^2 + \left[\int_{-b_c}^{+b_c} \left(\int_0^{\varphi_k} p(\varphi, x_c, t)(\cos\varphi) h_1 d\varphi \right) h_3 dx_c \right]^2 \right\} \quad (10)$$

$$C_{tot}^{(h)}(t) = \left\{ \left[\int_{-b_h}^{+b_h} \left(\int_0^{\varphi_k} p(\varphi, \zeta_h, t)(\sin\varphi) h_1 d\varphi \right) h_3 d\zeta_h \right]^2 + \left[\int_{-b_h}^{+b_h} \left(\int_0^{\varphi_k} p(\varphi, \zeta_h, t)(\cos\varphi) h_1 d\varphi \right) h_3 d\zeta_h \right]^2 \right\} \quad (11)$$

where symbol φ_k denotes the end coordinate of the film in circumferential direction.

Friction coefficients are calculated by Wierzcholski (2007a) as follows:

$$\mu_c = \frac{\left| \mathbf{e}_\varphi F_{R\varphi} + \mathbf{e}_{x_c} F_{Rx_c} \right|}{C_{tot}^{(c)}}, \qquad \mu_h = \frac{\left| \mathbf{e}_\varphi F_{R\varphi} + \mathbf{e}_{\zeta_h} F_{R\zeta_h} \right|}{C_{tot}^{(h)}}, \quad (12)$$

Where $\mathbf{e}_\varphi$, $\mathbf{e}_{x_c}, \mathbf{e}_{\zeta_h}$ are the unit vectors in conical and hyperbolic coordinates respectively.

5. NUMERICAL CALCULATIONS

We determine the pressure distributions and load carrying capacity values in HDD micro-bearing for conical journal in the lubrication region F, which is defined by the following inequalities: $0 \le \varphi \le \varphi_k$, $x_{c1}=x_c/b_c$, $-b_c \le x_c \le b_c$ where $2b_c$ – micro-bearing length. Numerical calculations are performed in Mathcad 14 Program by virtue of the equation (1), (8) by means of the finite difference method (see Fig. 3). If grooves length is situated in x_c and φ direction then gap height of the conical micro-bearing has the following form respectively:

$$\varepsilon_T(\varphi, x_c, t) = \varepsilon\left[(1+\lambda_c\cos\varphi)\sin^{-1}\gamma + \frac{4\varepsilon_{g1}}{\pi}\left(\sin\frac{\pi\varphi}{\varphi_T} + \frac{1}{3}\sin\frac{3\pi\varphi}{\varphi_T} + \ldots\right)\right] + \delta(\varphi, x_c, \varsigma), \quad (13)$$

$$\varepsilon_T(\varphi, x_c, t) = \varepsilon\left[(1+\lambda_c\cos\varphi)\sin^{-1}\gamma + \frac{4\varepsilon_{g1}}{\pi}\left(\cos\frac{\pi x_c}{x_T} - \frac{1}{3}\cos\frac{3\pi x_c}{x_T}\ldots\right)\right] + \delta(\varphi, x_c, \varsigma), \quad (14)$$

for $0 \le \varphi < 2\pi$, $-b_c \le x_c \le b_c$ where $\tilde{\lambda}_c$ eccentricity ratio in cylindrical micro-bearing, $\tilde{\varepsilon}$ radial clearance in cylindrical micro-bearing, $\varepsilon_{g1} \equiv \varepsilon_g/\varepsilon$, ε_g – ridge height. Symbols φ_T, x_T denote periods of grooves sequence about 65nm in φ and x_c directions respectively. Symbol δ denotes the dimensional random part of gap height changes resulting from vibrations, unsteady loading and surface roughness measured from the nominal mean level. The symbol ς describes the random variable, which characterizes roughness arrangement. We show in Fig. 3 the results of numerical calculations of pressure distributions without stochastic changes.

The grooves and ridges are now neglected. We assume the largest radius of the journal R=0.001 m, length/radius ratio $L_{c1}=b_c/R=1$, dynamic viscosity of the oil $\eta_o=0.03$ Pas, angular velocity $\omega=565.5\ s^{-1}$, characteristic dimensional value of hydrodynamic pressure $p_o=\omega\eta_o/\psi^2=16.96$ MPa, relative radial clearance $\psi=\varepsilon_T/R=0.001$, eccentricity ratio $\lambda_c=0.4$; $\lambda_c=0.2$. By virtue of good known [6] boundary Reynolds conditions the angular coordinate of the film end has the values: $\varphi_k=3.678$ radian; $\varphi_k=3.731$ radian respectively.

If eccentricity ratio increases from $\lambda_c=0.2$ to $\lambda_c=0.4$, then the maximum value of hydrodynamic pressure increases from 7.07 MPa to 18.87 MPa. The carrying capacity $C_y^{(c)}$ in y direction increases from 13.44N to 32.93 N and the $C_z^{(c)}$ total carrying capacity in z direction increases from 4.89 N to 11.98 N.

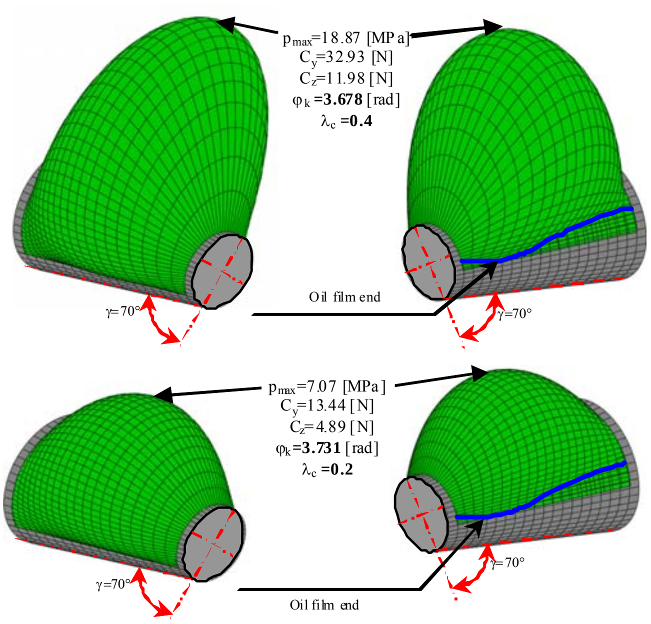

Fig. 3. The pressure distributions caused by the rotation in circumferential direction in conical micro-bearings. Left side presents the view from the film origin, right side shows the view from film end

5. CONCLUSIONS AND REMARKS

- We can simulate the increases of the capacity memory of fluid dynamic HDD micro-bearings not only by the herringbone or spiral groove indicated in papers by Jang *at al*. but also by the various conical and hyperbolic shapes of journal micro-bearings.
- If the sleeve surface is grooved than the peak pressures are higher than that which arise in micro-bearing with the grooved journal surface. These results were obtained for various journal shapes.
- The friction forces arising over micro-bearing surfaces of about 10 μm^2 during its lubrication attain value of about 0,4 nN. The numerical calculations and contactless AFM measurements methods are here very desirable compare in Wierzcholski (2007).

Nomenclature

a_1 – largest radius of the hyperbolic journal, m; b_h – half hyperbolic journal length, m; h_1,h_3 – Lame coefficients depended from the journal surface geometry and roughness; p – hydrodynamic pressure, Pa; F_R – friction force, N; F – lubrication surface, m^2; α_1,α_2, α_3 – curvilinear coordinates in circumference – gap height, and longitudinal directions; δ – dimensional random part of gap height changes, m; ε_T – total value of gap height, m; φ – circumference direction; ξ_h – dimensional longitudinal direction in hyperbolic coordinates, m; ς – random variable; η – fluid dynamic viscosity, Pas; μ – dimensionless friction coefficient; ω – angular velocity, s^{-1}; y_c – coordinate directed vertically to the conical surface, m;

REFERENCES

1. **Bhushan B**.(2007) Nano-tribology and nano-mechanics of MEMS/NEMS and BioMEMS, BioNEMS materials and devices, *Microelectronic Engineering*,, 84, 387-412.
2. **Jang G.H., Lee S.H., Kim H.W., Kim C.S**.(2005) Dynamic analysis of a HDD spindle system with FDBs due to the bearing width and asymmetric grooves of journal bearing, *Microsystems Technologies,* 11, 499-505.
3. **Jang G. H., Seo C.H., Ho Scong Lee** (2007) Finite element model analysis of an HDD considering the flexibility of spinning disc-spindle, head-suspension-actuator and supporting structure, *Microsystem Technologies*, 13, 837-847.
4. **Jang G. H., Park S.J., Kim C.S., Han J.H**. (2007) Investigation of the electromechanical variables of the spindle motor and the actuator of HDD due to positioning and the free fall, *Microsystem Technologies*, 13, 797-809.
5. **Jang G.H.,Yoon W**. (2002) Dynamic characteristics of a coupled journal and thrust hydrodynamic bearing in a HDD spindle system due to its groove location. *Microsystem Technologies* 8, Springer Verlag, 261-270
6. **Liu L.X., Spakovszky Z. S**.(2007) Effects of Bearing Stiffness Anisotropy on Hydrostatic Micro gas Journal Bearing Dynamic Behavior, *ASME, Journal of Engineering for Gas Turbines and Power*, Vol. 129, 177-184.
7. **Wierzcholski K**.: (2007) *Bio and slide bearings; their lubrication by non-Newtonian fluids and application in non-conventional systems,* Vol. I, Second Edition: Principles of Human Joint Lubrication. Monograph, Gdańsk University of Technology
8. **Wierzcholski K., Miszczak A.** (2007): *Flow on the bio cell surfaces as an element of the micro-bearing tribology*. Journal of Kones Powertrain and Transport, Vol. 14, No. 2, 553-560.
9. **Wierzcholski K**.(2006): Experimental measurements of friction forces on the tissue in probabilistic view, *Journal of Kones Powertrain and Transport,* Vol. 13, No. 3, 427-435.
10. **Yong-Bok Lee, Hyun-Duck Kwak, Chang-Ho-Kim, Nam-Soo Lee** (2005) Numerical prediction of slip flow effect on gas-lubricated journal bearing for MEMS/MST- based micro-rotating machinery*, Tribology International,* 38, 89-96.

Acknowledgement

This research project has been financially supported by a Marie Curie Transfer of Knowledge Fellowship of the European Community's Sixth Framework Program under contract number MTKD-CT-2004-2008-517226