CONTROL OF THE CRACK TRAJECTORY BY THE ELECTROMAGNETIC FIELD IN MEDIA WITH EMBEDDED ACTUATORS

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Abstract: The variation principle is applied for defining a crack in the solid body. Crack propagation in non-homogeneous media has been considered. It is shown that electromagnetic fields in the material are essentially affecting the trajectory. The crack trajectory stability has been studied as function of fracture energy, phase portraits of the trajectory in different media have been built, and various attractor types have been revealed. Different crack morphologies from single straight and oscillating crack propagation to straight double crack propagation were theoretically founded.

1. INTRODUCTION

The ability to determine the direction of crack growth as a function of medium properties or its deformed state is useful. This ability is the rule rather than the exception when the technology of material fabrication can control the praise nature of the material micrstructure. These provide the means to suppress or enhance crack propagation. In other words, the direction of crack growth can be predetermined if the loadings are known.

The aim of the present paper is to find the conditions of the material properties for crack propagation in the wave-guide mode. Studied are the crack path and its stability in inhomogeneous media by application of variational principle. The stability and stochastization of the solution of the crack trajectory equation are investigated.

Consider the medium that is stretched along the y-axis from infinity. We believe that the crack propagate in such a way that the energy involved in fracture process is minimal along the trajectory of the crack. The variational equation for specific energy (i.e. energy per unit length) has the general form Miklashevich and Chigarev (2002), Miklaschevich (2005).

$$\delta\int_L \gamma + w\left(x, y\right)dl = 0 \tag{1}$$

where γ is the specific surface energy and w(x, y) the specific potential deformation energy while dl stands for the differential length, δ the variation symbol and L the integration path. For the brevity we can introduce the functional of specific energy γ+ w(x, y)=F(x, y, y', ŷ) in equation 1.

The Euler's equation for the problem (1) which follows from condition of minimum of functional for functional of elastics energy is given in standard way by Miklashevich (2005). Because we investigate the stability of trajectory not stability of the process of fracture we obtain

$$\begin{aligned}&\frac{d}{dx}\left(\frac{\partial}{\partial y'}\left[\sqrt{1+y'^2}\right]\left(F\left(x,y,y'\right)\right)\right)\\&-\frac{\partial}{\partial y}\left[\sqrt{1+y'^2}F\left(x,y,y'\right)\right]=0\end{aligned}. \tag{2}$$

Now assume that the boundary conditions are chosen in such a way that they maintain the stress state unchanged during the crack propagation („dead stress"). For the reduced case media properties relative smooth changing F ≠ F(x, y, y')=F(x, y), equation 2 can be written in the form Miklashevich and Chigarev (2002).

$$y''-y' f_1\left(x,y\right)\left(1+y'^2\right)+f_2\left(x,y\right)\left(1+y'^2\right)^2=0 \tag{3}$$

For the sufficiently smooth crack term y'^2 to vanish, there results the crack trajectory equation:

$$y''-y' f_1\left(x,y\right)+f_2\left(x,y\right)=0\,, \tag{4}$$

where the notations

$$\begin{aligned}&f_1\left(x,y\right)=\frac{\partial \ln Q\left(x,y\right)}{\partial x},\quad f_2\left(x,y\right)=\\&\frac{\partial \ln Q\left(x,y\right)}{\partial y},\ Q\left(x,y\right)=F^{-1}\end{aligned} \tag{5}$$

have been adopted.

2. CRACK PROPAGATION THROUGH STRUCTURE BOUNDARY

Crack propagation across structural non-homogeneities represents a significant practical interest, for example, for composites and structured materials. The theory of crack propagation through the structure boundary in statement of "ideal mechanics of fracture" started from well known articles of Dundurs. In the elastic formulation, an idealized problem of interaction of the crack with non-homogeneity (a borders or a dislocation) can be solved, for example, by the method of complex potentials of Muskhelishvili. Moreover, all the real materials have internal structural borders (for example, grain structure, walls of dislocations, etc.) and "ideal" theory may be corrected by the more physical way. In the variational statement, the problems of accounting borders are connected with discontinuity of function F(x,y) in borders 0, Li, i=1,...k, etc. These causes a necessity to consider, at varying the fracture energy, piecewise smooth functionals. Equation for the energy functionals, in case of crack passage across i borders should be rewritten in the form of:

$$\delta F = \delta F\Big|_{A}^{\varepsilon-0} + \delta F\Big|_{0-\varepsilon}^{0+\varepsilon} + \delta F\Big|_{0+\varepsilon}^{L_i-\varepsilon} + \delta F\Big|_{L_i-\varepsilon}^{L_i+\varepsilon} + \ldots \delta F\Big|_{L_K+\varepsilon}^{B} \qquad (6)$$

where A, B starting and end point of the crack, respectively and lim ε=0. Term $\delta F\Big|_{L_i-\varepsilon}^{L_i+\varepsilon}$ in (6) is the additional energy, connected with a L_i sharp body boundary.

In real materials, a transition from one material to another takes place in a narrow zone of contact of grains (layers). The width of this transition zone depends on technological and other factors and defines the integration interval in the second term of expression (6). Only for an ideal material, the width of the transition zone from one material to another is zero. According to our problem statement, the crack approaches the border "almost normally", and the fracture energy, connected with the existence of the border, is the energy required for separation of the material along the layer. In the statement of the problem of ideal fracture:

$$\delta F\Big|_{1} = \int_{-\infty}^{+\infty} dU\left(1\right)\mathbf{D}\left(x-1\right)dx$$

where D(x) is the delta-function, dU(1) is the bond energy of the border number one (Miklashevich, 2002).

However, real materials do not have any zero transition zones. Then, with account of the real width of contact zone 2ε, the delta-function can be approximated by a smooth function:

$$\frac{1}{\varepsilon\sqrt{\pi}}\exp\left(\frac{-x^2}{\varepsilon^2}\right)\xrightarrow[\varepsilon\to\varepsilon]{}\mathbf{D}\left(x\right).$$

In this case it is not possible to have a crack propagating in materials with stabilized trajectory (Miklashevich, 2005, 2008). Unfortunately, because of the complexity of the coefficient, we managed to obtain a precise analytical solution of equation (4) only for the limited case of special form of the coefficients in equation for linear media (Miklashevich, 2008).

3. CRACK BEHAVIOUR IN PIECE-WISE STRUCTURES

The equation (4) was analyzed in case of ideal piece-wise composite materials (Miklashevich and Chigarev, 2002; Miklaschevich, 2005. For the piece-wise material the media properties can be taken in the form

$$f^0_{2,y} = -\omega^2,\ f^0_{2,y^2} \equiv 0,\ f^0_{2,y^3} = -\alpha\omega^2,\ f_1 = 0.$$

This corresponds to representation of ln Q(y) in the form as Miklashevich and Chigarev (2002):

$$\ln Q\left(x,y\right) = \ln Q\left(y\right) + \nu\left(x,y\right),$$
$$\ln Q\left(y\right) = -\omega^2 y^2 - \alpha\omega^2 y^4.$$

In this case we can find the condition of stochastization of the crack trajectory (Miklashevich and Chigarev, 2002). Consider the stability of equation (4) when the medium's properties change smoothly along the x axes. Let the media properties expressed by Eq.(5) be in the form

$$\left(\ln Q\left(x,y\right)\right)_{,x} = \delta + \gamma\cos\omega\quad x\,. \qquad (7)$$

The above representation corresponds to body fracture energy in form

$$Q\left(x\right) = C_1\exp\frac{-\left(\delta x\omega - \gamma\sin\left(\omega x\right)\right)}{\omega},$$

where C1 is the integration constant. The behaviour of the non-homogeneity is in general exponential (the slope degree of the exponent is regulated by index δ), Fig. 1. A deviation of the function from smoothness (scatter of the properties
of the composite by layers) is regulated by parameter γ (See Fig. 1).

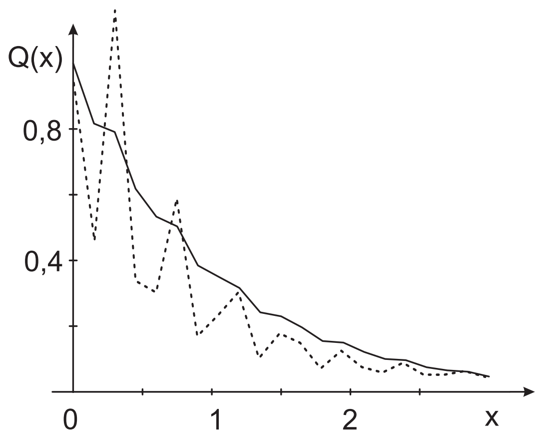

Fig.1. Dependence of fracture energy from coordinate x. Parameters ω = 15, δ = 1. Solid curve γ = 1, dotted curve γ = 15

Substitute the Eq. (7) into (4) we obtain the well known equation of Duffing type (Lichtenberg and Liberman, 1983) in form

$$\ddot{y} - y + y^3 + \varepsilon\delta\dot{y} = \delta + \gamma\cos\omega\ \ x$$

The crack bechaviour in last case presented in Fig. 2, Fig. 3. and whole is stable.

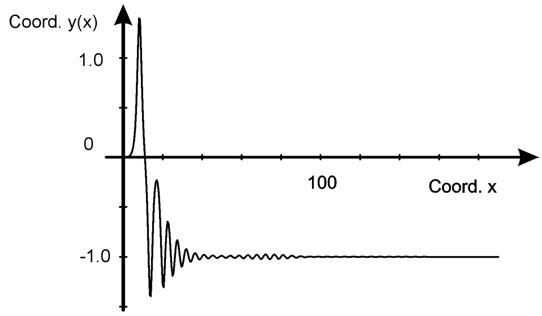

Fig.2. The crack trajectory behavior. Material parameters ara ε = 0.1, γ = 2.19, ω = 0.01, δ= 1; Initial conditions y(0) = 0; y'=0,001

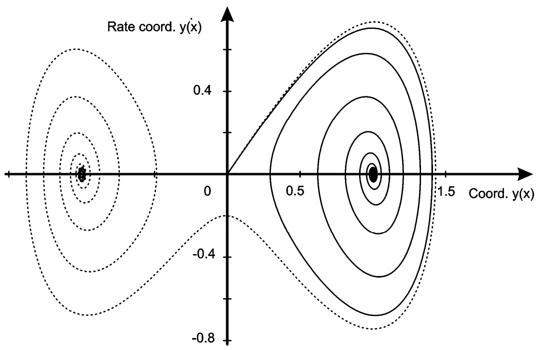

Fig. 3. Phase portrait. Initial conditions y(0) = 0; y'(0) = 0.000001 dotted line, y(0) = 0.001; y(0) = 0,00005 solid line

The considered models of non-homogeneity correspond to the conclusion that a change of non-homogeneity along axis Y ensures a waveguide character of crack propagation along axis Y. In particular, a cut-out may play the role of such waveguide. However, even a rigid determination of the initial and final points is insufficient to have the crack trajectory y(x) to be a deterministically forecasting function.

4. INFLUENCE OF ELECTROMAGNETIC FIELD TO THE TRAJECTORY

In case when in a material the set of actuators is embedded, the electromagnetic field can be presented in a material. The general theory of fracture of piezoelectric materials can be found in Parton and Kudriavcev (1998). A possibility to apply the crack energy density criterion to crack propagation in piezoelectric materials was detail justified earlier (Zuo and Sih, 2000). By using the well-known L. I. Sedov's decomposition of the full crack problem into subproblems, we may present the full crack problem in piezoelectric medium as superposition of mechanical and electrical problems. Let's take the field as pure periodical (sinusoidal). In this case the body fracture energy can be presented in form of

$$Q(x) = C_1 \exp\left(-\left(\delta x\omega - \gamma\sin\left(\omega x\right)\right)/\omega\right) + C_1\cos\Omega x$$

The crack bechaviour in the last case presented in Fig. 4, Fig. 5. and essentialy differ from stable.

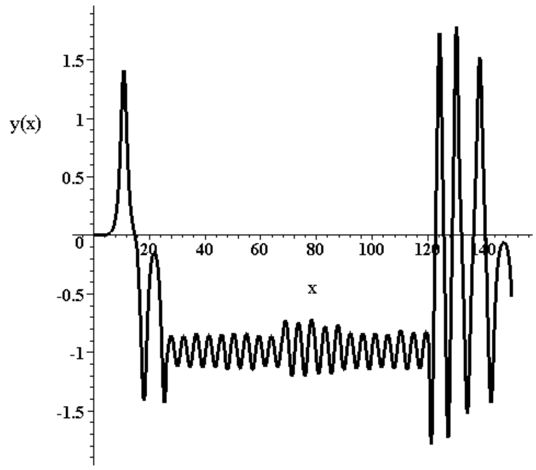

Fig.4. The crack trajectory behavior. Ω=10ω

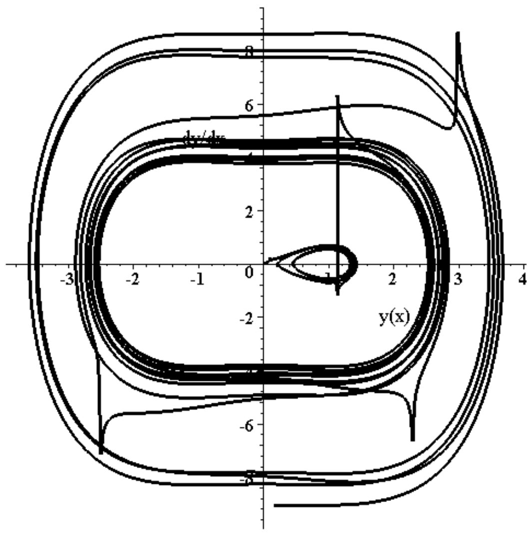

Fig. 5. Phase portrait for crack under electromagnetic field affect.

5. DISCUSSION AND CONCLUSION

The proposed approach allows analysing the dependence of the crack behaviour from the properties of the medium, through which the crack propagates, finding the areas of unstable propagation, and assessing the stochastization (randomization) length (Miklashevich and Chigarev, 2002; Miklaschevich, 2005). Forecasting of the crack trajectory in case of meeting the stochastization condition in possible only through use of probability methods, for example, on the basis of Markov process theory.

The forecasted crack behaviour is logically explained in the model of composites with adhesion links. The initial instability (chaotic state) of propagation of a main crack

may be physically realized only through formation of lateral microcracks in the boundaries of the structural elements
of the composite. In development of the destruction process, the damage will accumulate in the layers passed by the crack, and a weak relaxation on destruction of adhesion links will be insufficient, the energy will start to liberate along the trajectory of the mainline crack and stabilize it (Miklashevich, 2008).

The considered problem may be an example of layered medium synthesis, in which the boundary is a barrier in the crack route, since in the considered example, the boundary may break the condition $y'^2 << 1$, the crack propagates along the boundary, and then along the layer again. Thus, stochastization of the crack trajectory in a layered medium leads to an impossibility to forecast its propagation and, therefore, to impossibility of taking measures, at the stage of designing and manufacturing of an article, to improve crack-resistance. On the other hand, the crack stochastization that enables its propagation along the fibering ensures the crack-resistance of the article, since the shear fracture viscosity is higher than the fracture destruction (Miklashevich, 2002).

In the presence of the electric field, the crack behaviour is much less stable than in the absence thereof. At crack development, bifurcation transitions are possible (sharp outbreaks at phase diagrams). The general analysis of the crack behaviour indicates that the presence of the electric field essentially increases the chaotic behaviour of the trajectory. It follows from Figs. 4, 5 that even in case of high-frequency electromagnetic field, no "suppression" of mechanical oscillations takes place, and the trajectory has multiple bifurcation points (sharp breaks of phase trajectory).

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This work was partially supported by State Scientific Program "Mechanics", sections 2.2 and 4.03. Thanks to V.Barkaline for the discussion of early versions of this paper and for many valuable suggestions and criticism. Thank for Prof. R. Kiencler for very interesting discussions made during the author stay in Bremen, Germany, by the support of DAAD.