EXAMPLES OF A USE OF THE OPTIMAL CONTROL AT ENERGY PERFORMANCE INDEX IN MECHATRONIC APPROACH

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Abstract: Purpose of the paper is to present some examples of application of the optimal control at energy performance index in mechatronic solutions. In the paper were presented methods of vibration surveillance of mechanical systems idealised discretely. These methods were applied in robotics (industrial robots) as well as – for high speed ball end milling processes of flexible details. As example of non-linear system, a possibility of the use of optimal control at energy performance index for wheeled robots was presented.

1. INTRODUCTION

Here is defined general form of energy performance index, which considers changing with time kinetic energy and potential energy of the system, i.e. (Kaliński, 2001):

$$J(t) = \frac{1}{2}(\dot{\mathbf{q}}^* - \dot{\overline{\mathbf{q}}})^T \mathbf{Q}_1 \mathbf{M}^* (\dot{\mathbf{q}}^* - \dot{\overline{\mathbf{q}}}) + \frac{1}{2}(\mathbf{q} - \overline{\mathbf{q}})^T \mathbf{Q}_2 \mathbf{K}^* (\mathbf{q}^* - \overline{\mathbf{q}}) + \frac{1}{2}\mathbf{u}^T \mathbf{R} \mathbf{u} \tag{1}$$

where: Q – matrix of dimensionless weighing coefficients, R – matrix of control command effect, M – matrix of inertia, K – matrix of stiffness, q – vector of generalised coordinates of the real motion trajectory, $\overline{\mathbf{q}}$ – vector of generalised coordinates of the given trajectory, u – vector of control commands.

Vector of generalised displacement $\overline{\mathbf{q}}$ is solution of the following equation:

$$\mathbf{K}^* \overline{\mathbf{q}} = \mathbf{f}_0 \tag{2}$$

where: f0 - vector of non-potential generalised forces of the system, which are system's loads for the given trajectory.
In particular case f0 ≡ f* (see Eq. 4).

Vector of the generalised velocities $\dot{\overline{\mathbf{q}}}$ is time derivative of the $\overline{\mathbf{q}}$ vector.

Matrix Q1 defines influence of the kinetic energy of the vibrations, what is particularly important in process of the free vibrations surveillance, while Q2 – influence of the potential energy, which is important for the vibrations surveillance of the systems with delayed feedback (Kaliński, 2001).

If the task of the control unit in the system is to survey vibrations, then assumption has been made that the control signals do not influence the given trajectory movement. Thus we equal variation of the performance index to zero. After suitable transformations, relationship describing optimal control command has been obtained, i.e.:

$$\mathbf{u} = -\left(\mathbf{R} + \mathbf{R}^T\right)^{-1} \int_{t_0}^{t} \mathbf{B}^T(\tau) \boldsymbol{\Phi}^T(t,\tau) \mathbf{d\tau} \{\mathbf{T}_1^T \left(\mathbf{M}^{*T}\mathbf{Q}_1^T + \mathbf{Q}_1\mathbf{M}^*\right)\left[\dot{\mathbf{q}}^* - \left(\dot{\mathbf{K}}^{*-1}\mathbf{f_0} + \mathbf{K}^{*-1}\dot{\mathbf{f}}_\mathbf{0}\right)\right] + \mathbf{T}_2^T\left(\mathbf{K}^{*T}\mathbf{Q}_2^T + \mathbf{Q}_2\mathbf{K}^*\right)\left(\mathbf{q}^* - \mathbf{K}^{*-1}\mathbf{f_0}\right)\} \tag{3}$$

A very important and thusfar unresolved problem is how to determine values of the components of matrices Q1, Q2, R (Engel and Kowal, 1995). Random selection of these values ought to be avoided, because the surveillance effect depends on them. It is difficult to find effective method for searching of the large dimension space of parameters. One indication leads to a use of computer simulation methods (Górecki et al., 1983).

2. OPTIMAL CONTROL AT ENERGY PERFORMANCE INDEX IN HYBRID SYSTEM

Here is introduced dynamic equation of controlled non-stationary system, described in generalised coordinates:

$$\mathbf{M}^*\ddot{\mathbf{q}}_i + \mathbf{L}^*\dot{\mathbf{q}}_i + \mathbf{K}^*\mathbf{q}_i = \mathbf{f}^* + \mathbf{B}_u^* + \mathbf{u} \tag{4}$$

Signs M*, L*, K*, B*u, q*, f* and u denote respectively matrices of inertia, damping, stiffness and control, and also vectors of generalised displacements, forces and control commands of the system.

2.1. The hybrid system

Further consideration relates to the system decomposition into following ones (Kaliński and Chodnicki, 2007).

1. Modal subsystem, which is described in generalised co-ordinates $\mathbf{q}_m$. Matrices of inertia, damping and stiffness are $\mathbf{M}_{mm}$, $\mathbf{L}_{mm}$, $\mathbf{K}_{mm}$, but vector of generalised forces is $\mathbf{f}_m$. Properties of that subsystem are defined by: $\mathbf{\Omega}_\mathrm{m}$=diag[$\boldsymbol{\omega}_{01}$ $\boldsymbol{\omega}_{02}$ … $\boldsymbol{\omega}_{0\mathrm{mod}}$] – matrix of undamped angular natural frequencies ω_{0k}, k=1, ..., mod, $\boldsymbol{\psi}_\mathrm{m}$=[$\boldsymbol{\psi}_1$ $\boldsymbol{\psi}_2$ … $\boldsymbol{\psi}_\mathrm{mod}$] – matrix of normal modes $\mathbf{\Psi}_k$ corresponding to undamped angular frequencies of the system ω_{0k}, i=1, ..., mod, $\mathbf{Z}_\mathrm{m}$=diag[ζ_1 ζ_2 … ζ_mod] – matrix of dimensionless damping coefficients corresponding to modes k=1, ..., mod, mod – number of modes being considered. Thus, following conditions are fulfilled:

$$\mathbf{q}_m = \mathbf{\Psi}_m \mathbf{a}_m,\ \mathbf{\Psi}_m^T \mathbf{M}_{mm} \mathbf{\Psi}_m = \mathbf{I}_m,\ \mathbf{\Psi}_m^T \mathbf{L}_{mm} \mathbf{\Psi}_m = 2\mathbf{Z}_m \mathbf{\Omega}_m,$$
$$\mathbf{\Psi}_m^T \mathbf{K}_{mm} \mathbf{\Psi}_m = \mathbf{\Omega}_m^2. \tag{5}$$

2. Structural subsystem, described in generalised co-ordinates $\mathbf{q}_s$.
3. Connective subsystem, whose generalised co-ordinates are $\mathbf{q}_c$.

It is assumed that rheonomic–holonomic bilateral constraints are between co-ordinates of modal subsystem qm and connective subsystem qc, that is to say:

$$\mathbf{W}_c \mathbf{q}_c = \mathbf{W}_m \mathbf{q}_m \text{ or } \mathbf{q}_c = \mathbf{W}\mathbf{q}_m \tag{6}$$

and;

$$\mathbf{W} = \left(\mathbf{W}_c^T \mathbf{W}_c\right)^{-1} \mathbf{W}_c^T \mathbf{W}_m = \mathbf{W}(t) \tag{7}$$

Non-stationary controlled system being free-off-constraints becomes stationary.

If we consider constraint reactions' equation, constraints' equations and their time derivatives, we shall obtain description of dynamics of non-stationary controlled system in hybrid co-ordinates ξ, that is to say:

$$\hat{\mathbf{A}}\ddot{\mathbf{q}}_i + \hat{\mathbf{B}}\dot{\mathbf{q}}_i + \hat{\mathbf{C}}\mathbf{q}_i = \hat{\mathbf{p}}(t, t-T) \tag{8}$$

where:

$\boldsymbol{\xi} = \begin{bmatrix} \mathbf{a}_m \\ \mathbf{q}_s \end{bmatrix}$ – hybrid co-ordinates of the whole system,

Now we define energy performance index again, but in the hybrid co-ordinates:

$$J(t) = \frac{1}{2}\left(\dot{\boldsymbol{\xi}} - \dot{\overline{\boldsymbol{\xi}}}\right)^T \mathbf{Q}_{1\xi} \mathbf{M}_\xi \left(\dot{\boldsymbol{\xi}} - \dot{\overline{\boldsymbol{\xi}}}\right) + \frac{1}{2}\left(\boldsymbol{\xi} - \overline{\boldsymbol{\xi}}\right)^T \mathbf{Q}_{2\xi} \mathbf{K}_\xi \left(\boldsymbol{\xi} - \overline{\boldsymbol{\xi}}\right) + \frac{1}{2}\mathbf{u}^T \mathbf{R}\mathbf{u} \tag{9}$$

where: Q1ξ, Q2ξ – matrices of dimensionless weighing coefficients, R – matrix of hybrid control efforts.

Thus, the optimal control command in hybrid co-ordinates has been determined in similar way, as described in Kaliński (2001). That is to say:

$$\mathbf{u} = -\left(\mathbf{R} + \mathbf{R}^T\right)^{-1} \int_{t_0}^{t} \mathbf{B}_\xi^T(\tau) \mathbf{\Phi}_\xi^T(t,\tau) d\tau \left\{ \mathbf{T}_{1\xi}^T \left(\mathbf{M}_\xi^T \mathbf{Q}_{1\xi}^T + \mathbf{Q}_{1\xi} \mathbf{M}_\xi\right) \left[\dot{\boldsymbol{\xi}} - \dot{\overline{\boldsymbol{\xi}}}\right] + \mathbf{T}_{2\xi}^T \left(\mathbf{K}_\xi^T \mathbf{Q}_{2\xi}^T + \mathbf{Q}_{2\xi} \mathbf{K}_\xi\right)\left(\boldsymbol{\xi} - \overline{\boldsymbol{\xi}}\right) \right\} \tag{10}$$

Description of controlled system in hybrid co-ordinates significantly reduces size of the system. The latter is of a great importance, especially in case of large multi-degree-of-freedom systems.

Relationships (Eq. 8 – 10) showed, that for a performance of optimal control in hybrid co-ordinates, here is required matrix Ωm of angular natural frequencies and matrix Ψm of corresponding normal modes of modal subsystem. The latter are time-invariant, due to the modal subsystem being stationary. In order to determine them we can apply:

– Computer software for calculation of eigenfrequencies and corresponding normal modes of systems idealised discretely. In practise, we utilise high-degree-of-freedom calculation models, created by the finite element method.
– Methods of experimental modal analysis.

Both of the approaches above are recommended, with respect to necessity of mutual verification of the results obtained.

2.2. Cutting process dynamics

Dynamic analysis of a slender ball end milling process has been performed, based upon following assumptions (Kaliński, 2001).

– The spindle together with the tool fixed in the holder, and the table with the workpiece, are separated from the machine tool whole structure.
– Here is considered flexibility of the tool and flexibility of the workpiece.
– Coupling elements (CEs) are applied for modelling the cutting process.
– An effect of first pass of the edge along cutting layer causes proportional feedback, but the effect of multiple passes causes delayed feedback additionally.

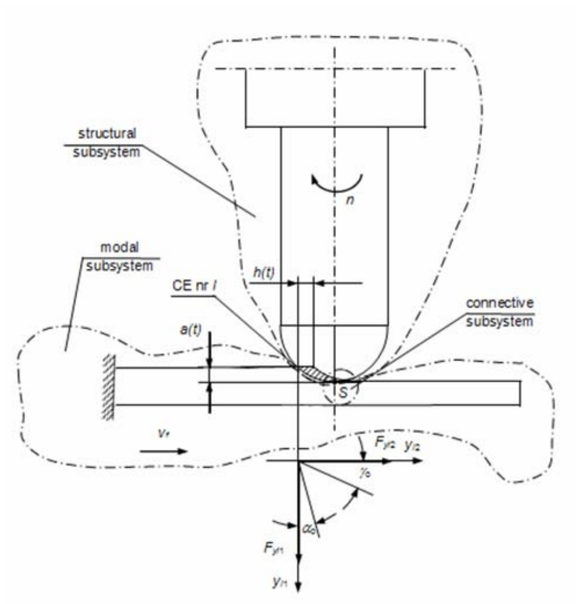

Fig. 1. A scheme of a slender ball end milling of one-side-supported curved flexible billet

As result of a milling process modelling, we get a hybrid system in which are separated (Fig. 1):

– modal subsystem. It is a stationary model of one-side-supported flexible billet, which displaces itself with feed speed vf;
– structural subsystem, that is to say non-stationary discrete model of ball end mill (speed of revolution n) and cutting process;
– abstractive connective subsystem as conventional contact point S between tool and workpiece.

For conventional contact point of tool edge and work-piece (i.e. CE no. l), proportional model of the cutting dynamics is included (Kaliński, 2001; Kaliński et al., 2006). Thus, we can get:

$$\mathbf{F}_l(t) = \mathbf{F}_l^0(t) - \mathbf{D}_{Pl}(t)\cdot\Delta\mathbf{w}_l(t) + \mathbf{D}_{Ol}(t)\cdot\Delta\mathbf{w}_l(t-\tau_l) \tag{11}$$

After transformation of displacements to local co-ordinate system of end mill as well as – flexible billet, equation of dynamics shall get a form (Kaliński et al., 2006):

$$\hat{\mathbf{A}}\ddot{\mathbf{q}}_i + \hat{\mathbf{B}}\dot{\mathbf{q}}_i + \hat{\mathbf{C}}\mathbf{q}_i = \hat{\mathbf{p}}(t, t-T) \tag{12}$$

where:

$$\mathbf{K}^*(t) = \mathbf{K} + \sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{D}_{Pl}\mathbf{T}_l(t) \tag{13}$$

$$\mathbf{f}^* = \sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{F}_l^0(t) + \sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{D}_{Ol}\Delta\mathbf{w}_l(t-\tau_l) \tag{14}$$

But: q – vector of generalised displacements of the system, M, L, K – matrices of decoupled system, F^0_l (t) – vector of desired forces of CE no. l, DPl, , DOl – matrices of proportional and delayed feedback of CE no. l, $\Delta\mathbf{w}_l(t-\tau_l)$ – vector of deflections of CE no. l for time-instant t-τl.

The matrix of transformation Tl(t) is time-dependent, because several edges of the cutter change their positions ourselves.

2.3. Dynamics of milling flexible details as of a hybrid system

Vector of deflections of CE no. l can be described in structural co-ordinates qs and modal co-ordinates am, i.e.:

$$\Delta\mathbf{w}_l = \mathbf{T}_l(t)\cdot\mathbf{q}_s - \mathbf{W}_{ml}(t)\cdot\mathbf{a}_m = \begin{bmatrix}\mathbf{T}_l(t) & -\mathbf{W}_{ml}(t)\end{bmatrix}\cdot\begin{Bmatrix}\mathbf{q}_s\\ \mathbf{a}_m\end{Bmatrix} \tag{15}$$

where: Tl(t) – matrix of transformation of displacements from a structural co-ordinate system to co-ordinate system yl1,yl2,yl3 of CE no l, Wml(t) – matrix of constraints between displacements in modal coordinates and displacements in coordinate system yl1,yl2,yl3 of CE no l.

Finally we shall obtain description of non-stationary model of dynamics of the milling process in hybrid co-ordinates, i.e.:

$$\begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix}\cdot\begin{Bmatrix}\ddot{\mathbf{q}}\\ \ddot{\mathbf{a}}\end{Bmatrix} + \begin{bmatrix}\mathbf{L} & \mathbf{0}\\ \mathbf{0} & \mathbf{2Z\Omega}\end{bmatrix}\cdot\begin{Bmatrix}\dot{\mathbf{q}}\\ \dot{\mathbf{a}}\end{Bmatrix} + \begin{bmatrix}\mathbf{K} + \sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{D}_{Pl}(t)\mathbf{T}_l(t) & -\sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{D}_{Pl}(t)\mathbf{W}_{ml}(t)\\ -\sum_{l=1}^{i_l}\mathbf{W}_{ml}^T(t)\mathbf{D}_{Pl}(t)\mathbf{T}_l(t) & \mathbf{\Omega}^2 + \sum_{l=1}^{i_l}\mathbf{W}_{ml}^T(t)\mathbf{D}_{Pl}(t)\mathbf{W}_{ml}(t)\end{bmatrix}\cdot\begin{Bmatrix}\mathbf{q}\\ \mathbf{a}\end{Bmatrix} = \begin{bmatrix}\sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{F}_l^0(t)\\ -\sum_{l=1}^{i_l}\mathbf{W}_{ml}^T(t)\mathbf{F}_l^0(t)\end{bmatrix} + \begin{bmatrix}\sum_{l=1}^{i_l}\mathbf{T}_l^T(t)\mathbf{D}_{Ol}(t)\Delta\mathbf{w}(t-\tau_l)\\ -\sum_{l=1}^{i_l}\mathbf{W}_{ml}^T(t)\mathbf{D}_{Ol}(t)\Delta\mathbf{w}(t-\tau_l)\end{bmatrix} \tag{16}$$

where: il – number of „active" coupling elements.

3. WHEEL MOBILE ROBOT

Mobile robots can be purposed for variety of applications, like for example, inspection and surveillance. Performance of mobile robots during their tasks depends on good integrity of their components and control algorithms. Global positioning and task planning usually takes a lot of time. However it is very hard to achieve good performance without using an algorithm making fast corrections of motor control signals during performance of desired trajectory. Such low level control procedures should run not only fast but should also be based at dynamic model of the robot for providing optimal control commands.

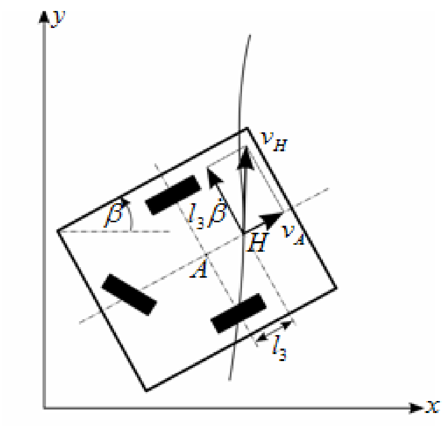

Fig. 2. 2-wheel mobile platform

This part of the paper is devoted to description of a use of the energy performance index for low level control of the 2-wheel mobile platform (Fig. 2). Two-wheel mobile platform is equipped with two independently driven wheels. Third wheel is used as third point of support for the platform and has ability of rotating freely and can achieve any orientation, together with the robot frame. Assumptions are made that robot moves over flat, horizontal surface without slippage and the wheels and other parts of the robot do not experience any deformations during the movement. Mathematical model of two-wheel mobile robot is strongly nonlinear (Giergiel at al., 2002). Two-wheel robot is characterized by non-holomic constraints. Supervising the movement of the two-wheel mobile robot is not trivial task, because of nonlinearities and kinematic constraints between instantaneous velocities of characteristic points.

3.1. Optimal control at energy performance index

Let us assume that control unit is minimising errors in courses of the velocity of the characteristic point A and the angular velocity $\dot{\beta}$ of the robot frame. In such a case, errors in courses of surveyed, generalised velocities could be a source of position errors, which are not being surveyed (Kaliński and Mazur, 2007).

Let us denote mobile platform point H, which follows the path. Velocity components of that point $\dot{x}_H$ and $\dot{y}_H$ have to be modified in such a way that they will correct position errors. Projections on the global coordinate system of the position errors could be divided by time-interval during which we are going to correct this error.

Obtained additional velocity components of the characteristic point H allow us to define additional angular velocities of the driving wheels, with a use of the problem of inverse kinematics.

We assume such obtained additional velocities to the velocities obtained from the question of inverse kinematics for the desired trajectory. A superposition rule is not valid towards non-linear systems. Thus, computation should be performed only for adequately short period of time. Without making a great error, it is allowable to consider that motion parameters are not changing during the period of the
assumed step of time.

Investigated mobile robot moves over horizontal surface; hence potential energy of the robot does not change in time. Here is defined energy performance index, which refers to changing with time kinetic energy of the system (Kaliński and Mazur, 2007):

$$J(t) = \frac{1}{2}(\dot{\mathbf{q}} - \dot{\bar{\mathbf{q}}} - \dot{\hat{\mathbf{q}}})^T \mathbf{Q}\,\mathbf{M}\,(\dot{\mathbf{q}} - \dot{\bar{\mathbf{q}}} - \dot{\hat{\mathbf{q}}}) + \frac{1}{2}\mathbf{u}^T \mathbf{R}\mathbf{u} \tag{17}$$

where: $\mathbf{Q}$ – matrix of dimensionless weighing coefficients, $\mathbf{R}$ – matrix of control command effect, $\dot{\mathbf{q}}$ – vector of generalised velocities of the real motion trajectory, $\dot{\bar{\mathbf{q}}}$ –vector of generalised velocities of the given trajectory, which was obtained from the question of inverse kinematics, $\dot{\hat{\mathbf{q}}}$ – vector of additional, generalised velocities, which are the results of difference between actual and desired position of the mobile platform, divided by assumed time-step.

Further we consider a system with kinematics constraints. For such the system it is possible to define variations of generalised velocities. We shall obtain an optimal control command as follows:

$$\mathbf{u} = -(\mathbf{R} + \mathbf{R}^T)^{-1} \int_t^{t+\Delta t} \mathbf{B}_u{}^T(\tau)\mathbf{\Theta}^T(t,\tau)d\tau \cdot \tag{18}$$
$$\cdot\, \mathbf{T}^T(\mathbf{M}^T\mathbf{Q}^T + \mathbf{Q}\mathbf{M})(\dot{\mathbf{q}} - \dot{\bar{\mathbf{q}}} - \dot{\hat{\mathbf{q}}})$$

where: $\mathbf{T} = [\mathbf{I}\ \ \mathbf{0}]$, $\mathbf{B} = [\mathbf{B}_u^T \mathbf{M}^{-T}\ \ \mathbf{0}]^T$, $\mathbf{x} = [\dot{\mathbf{q}}^T\ \ \mathbf{q}^T]^T$, $\mathbf{\Theta}(t,\tau)$ – is solution to homogeneous differential equation $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$, .

Computer simulations proved that optimal control of the two-wheel mobile robot with the energy performance index (Kaliński and Mazur, 2007) allows us to achieve very accurate trajectories. Performance of the real wheeled mobile robot depends strongly on the unit ability to generate optimal control commands. Because of the non-linearity, control signal should be generated very frequently and thus measurements should also be made very often. Encoders allow us to make nearly instantaneous measurements with good resolution. Such measurements are indirect. Spinning the wheel caused by non-optimal control signals can be a source of measuring errors. It is impossible to eliminate the need of the use of direct methods of measurement, but for short distance and for supervising of the movement presented method should allow us to achieve very good results. Better performance at the short distance should improve overall performance of the mobile robot.

3.2. Computer simulation

For a research of presented method there have been developed the author computer programmes written in the C code. Given trajectory has three main stages. In the 1-st stage mobile platform accelerates from zero to given velocity v_A of the characteristic point *A*. Next, the platform has to
follow a quarter of the circle with constant speed of characteristic point *A*. At last stage the platform has to follow straight line also with constant speed. Presented path was smoothed, because of using clothoid.

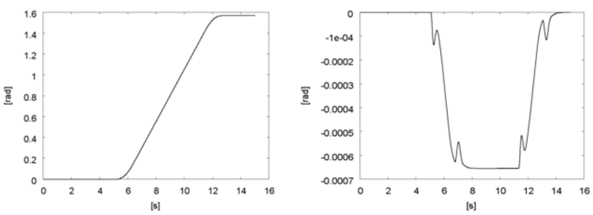

Fig. 3. Course of orientation angle and error of the orientation angle β

Fig. 3 shows that course of the orientation angle *β* of the mobile platform at the optimal control is very good. Errors are significantly small and they are approaching zero by the end of simulation.

3.3. Real 2-wheel platform

In the (Fig. 4) is shown 2-wheel mobile robot constructed and assembled for the test of the presented control algorithm.

Initial tests of the optimally controlled two wheel mobile robot with the performance index were succeeded (Fig 5). Measured trajectory of characteristic point H is very close to the desired trajectory. It was shown that robot performed the trajectory very well. These measurements were based on counting the encoders impulses. The latter means that jerks and spin effects have not been detected during these initial tests. Complete tests will be set up in the nearest future. It is also expected to make experiments with implementations of various control algorithms.

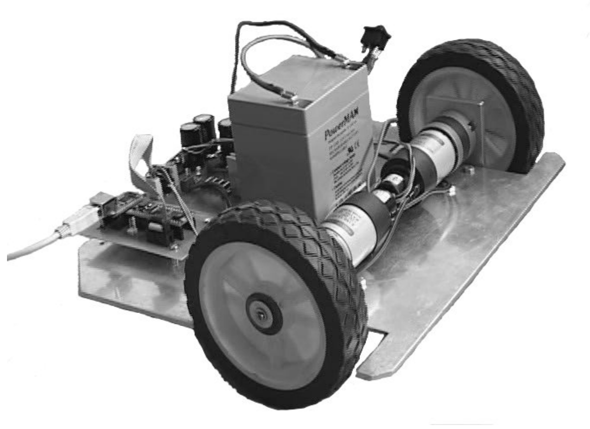

Fig. 4. Constructed mobile robot

Presented method for low level control of 2-wheel mobile platform appeared to be very effective. The latter is supported with following arguments.

– During simulation achieved trajectories were very accurate.
– Presented algorithm seems to be practically stable, because errors were eliminated and desired trajectories stay close to the obtained during simulation time.
– Results of the initial tests on real object appeared to be very good.

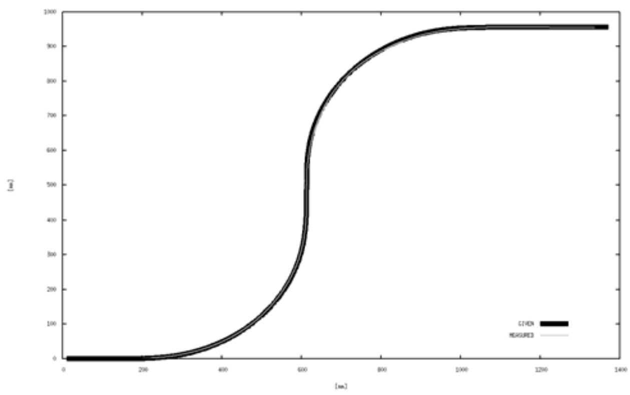

Fig. 5. Measured and desired trajectory of the point *H*

4. SUMMARY

An optimal control at energy performance index for a surveillance of various mechanical discrete systems appeared to be effective rule for new ideas of mechatronic solutions. Meaning of the latter is evidenced by theoretical derivations, computer simulations as well as by experiments on real structures.

The novel method of vibration surveillance during machining of curved flexible details is purposed for development with success. We use modal model of the workpiece whose parameters are identified during modal tests and analytical derivations. Employment of the FEM model compatible with a real billet and creation of the hybrid non-stationary system of the milling process lead to assure an efficiency of vibration surveillance.

The computation results are comparable with real behaviour of 2-wheel mobile platform. It means that such simulation is valuable tool in the process of designing of the motion path and control unit of real mobile platforms. Research showed that proposed surveillance algorithm is reliable and effective. According to that 2-wheel mobile platform is an example of non-linear system, it means that such algorithm could be used also for a surveillance of non-linear systems.

REFERENCES

1. **Engel Z., Kowal J.** (1995), *Sterowanie procesami wibro-akustycznymi,* Kraków, Wydawnictwa AGH.
2. **Giergiel M. J., Hendzel Z., Żylski W.** (2002), *Modelowanie i sterowanie mobilnych robotów kołowych*, PWN
3. **Górecki H., Fokusa S., Korytkowski A., Mitkowski W.** (1983), Warszawa, PWN.
4. **Kaliński K.** (2001), *Nadzorowanie drgań układów mechanicznych modelowanych dyskretnie*, Politechnika Gdańska.
5. **Kaliński K., Chodnicki M.** (2004), Nadzorowanie drgań za pomocą sterowania optymalnego w układzie hybrydowym na przykładzie procesu skrawania, *Teoria maszyn i mechanizmów* (Red. J. Wojnarowski, T. Uhl). t. 2, 197-202, Kraków, Kated. Robotyki i Dynam. Masz., AGH
6. **Kaliński K., Chodnicki M.** (2007) : *Wybrane zagadnienia analizy modalnej konstrukcji mechanicznych*, ed. T. Uhl, AGH, Kraków.
7. **Kaliński K., Galewski M., Sawiak S.** (2006), *Reduction of vibration level by the spindle speed optimisation during ball end milling of flexible details*, Ecole Nat. Supérieure d'Arts et Métiers de Metz
8. **Kaliński K., Mazur M.** (2007), *Prediction of the stability during optimal control of the 3-wheel mobile platform* Wybrane zagadnienia analizy modalnej konstrukcji mechanicznych, AGH.