RATIONALE FOR MU-SYNTHESIS CONTROL OF FLEXIBLE ROTOR-MAGNETIC BEARING SYSTEMS

Jerzy T. SAWICKI*

*Department of Mechanical Engineering, Cleveland State University, 2121 Euclid Avenue, SH-245, Cleveland, OH 44115, USA

j.sawicki@csuohio.edu

Abstract: The emergence of sophisticated formal control synthesis tools provokes important questions for any prospective user: why learn to use these new tools, what will they offer me? In synthesis of magnetic bearing controllers, it turns out that the range of stabilizing controllers is often quite narrow so that the difference between a poor controller and an "optimal" one may be small. Hence, the product of formal control synthesis tools often looks and performs much like what a reasonably clever control engineer would produce by hand. This paper demonstrates that the real value of these tools lies in a) generation of a performance benchmark which can be used to firmly establish the best performance relative to a specification and b) change of design parameter space to one which is relatively easy to maintain and represents a durable investment from an engineering process view.
Keywords: Robust control, flexible rotor, mu-synthesis, uncertainties.

1. INTRODUCTION

Magnetic bearing systems for rotating machinery represent an archetypal challenge for multi-input, multi-output (MIMO) control: they inherently involve multiple interacting control mechanisms and many conflicting performance objectives. As such, they would appear to be a perfect application of formal MIMO control design techniques such as μ-synthesis. However, the control culture of the magnetic bearings technical community is largely classical and many clever approaches have been developed to enable classical, essentially single-input, single output (SISO) methods to produce reliable and robust solutions to this control problem. A large part of the reason behind this is, quite simply, that classical SISO methods are more widely understood by controls engineers and have a much larger experience base on which to draw. Consequently, most commercial developers view tools like μ-synthesis with considerable trepidation.

Compounding this view is the simple fact that most published examples of μ-synthesis control for AMB systems produce only incremental improvements over hand-synthesized controllers and even this comparison is suspect since optimization of hand-synthesized controllers is largely an art for systems with this level of complexity. We argue here that the primary reason for applying a method like μ-synthesis to AMB control problems is to obtain a better engineering process rather than to obtain substantial performance enhancements. Because μ-synthesis is genuinely an optimization process and because the performance index that it optimizes has a very clear connection to real engineering practice, μ- can at least provide a benchmark against which other controllers may be measured. This alone justifies some level of investment in the method. But more importantly, μ- represents a change in parameter space - the set of knobs that a control designer can turn - and this new parameter space arguably leads to a better engineering process. In particular, investments in this alternate process are easier to translate to different control problems, easier to document, and easier to transfer to new engineers. In order to develop this argument, this paper first outline what is viewed to be the *natural* primary objectives or specifications of AMB controller synthesis. Then the connection between these objectives and the μ-synthesis problem will be developed, highlighting what is retained exactly, what is retained approximately, and what is lost. Then, the actual μ-synthesis problem will be discussed, emphasizing that it is essentially a minor last step once a well structured analysis process for the control objectives has been assembled. That is, most of the engineering effort is applied to developing machinery for assessing the performance of *any* controller relative to these objectives: if this machinery is constructed in a particular way, then μ-synthesis is automatic and requires no significant further effort by the engineer. To illustrate the concepts, an AMB supported machine tool spindle example will be presented. Inevitably, the discussion is heavily invested in the machinery of μ-analysis and synthesis as well as that of H_∞ analysis and synthesis. However, most of the central details can only be provided in sketch because of space limitations: the interested reader is referred to any of numerous authoritative texts on these subjects, for instance (Zhou et al., 1996; Goodwin, 2001; Green and Limebeer, 1995).

2. AMB MACHINING SPINDLE REFERENCE EXAMPLE

To illustrate the concepts presented here, consider the AMB supported machine tool spindle with cross-section shown in Fig. 1. The spindle rotor is supported by two radial bearings and one thrust bearing. The maximum static radial load capacities are approximately 1400 and 600 N for the front and rear bearings, respectively, and the maximum axial capacity for thrust bearing is 500 N. The spindle reaches a rotational speed of 50,000 rpm at 10 kW. The AC motor acts on the rotor between the thrust and rear radial bearing.

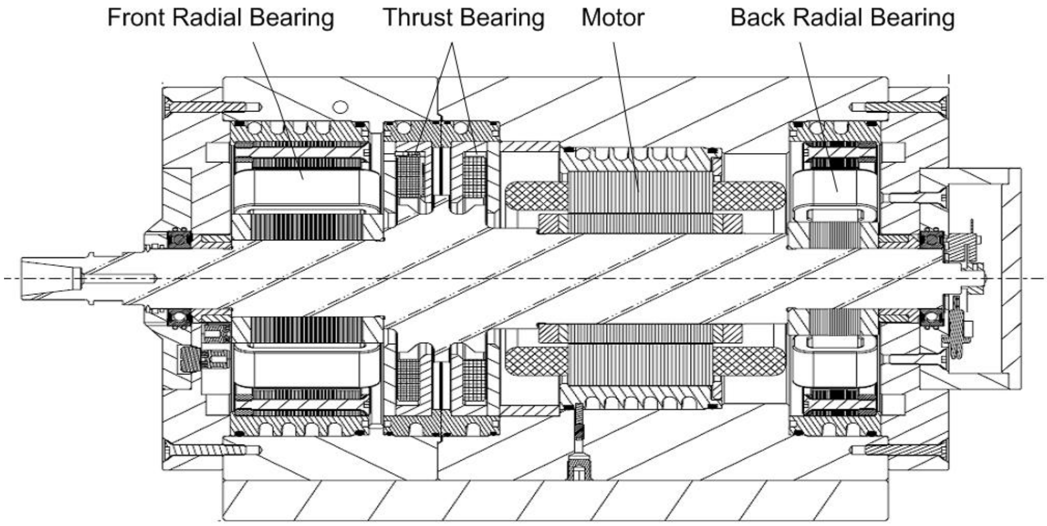

Fig. 1. High-speed machining spindle supported on active magnetic bearings

The total rotor mass is 6.85 kg while the total length is 464 mm. The first two free-free flexible modes are at frequencies 1070 Hz and 1985 Hz. The actuators are driven by transconductance power amplifiers with bandwidth of about 2400 Hz and a gain of 1 volt/amp. Control is implemented digitally with a sampling rate of 10 kSa/sec. The full system model includes rotor (64-element FEM model modally truncated to include two rigid body modes and two flexible modes), actuator properties, amplifier dynamics, computational delay (by Padé approximation), and sensor dynamics.

3. AMB CONTROLLER DESIGN: OBJECTIVES

Most applications of AMB systems for rotating machinery are primarily concerned with steady behavior: analysis focuses on response to steady sinusoial loads such as mass unbalance, shaft bow, aerodynamic loads, and sensor noise. Such an approach is even commonly adopted when considering transient phenomena such as compressor surge. Notable exceptions to this include applications to systems subject to extreme impact loading such as underwater naval vessels. For systems which are linear (really the dominant behavior of AMB systems), this focus on sinusoial response has a deeper theoretical justification which dictates that the “worst case”1 bounded signals that can act on linear (time invariant) systems are sinusoial.

The literature contains many detailed application examples where the performance objectives in AMB controller synthesis are elucidated (Sawicki et al., 2007; Fittro and Knospe, 1999; Sawicki and Maslen, 2006, 2007; Namarikawa and Fujita, 1999). Generally, the obvious objectives include an adequate stability margin and adequate management of external loads. Given the underlying nonlinear character of AMB systems, a common secondary objective is to maintain operation in an essentially linear regime, avoiding numerous sources of nonlinearity including actuator magnetic saturation, amplifier voltage saturation, and actuator nonlinearity due to large journal displacements.

At the most conceptual level, the AMB system may be described by the block diagram indicated in Fig. 2 in which the control inputs u are signals delivered to the power amplifiers, the measurements y are signals received from position sensors, the loads w are forces or electrical noise acting on the system, and performance measures z are those signals that the engineer will monitor in assessing adequate management of the loads w.

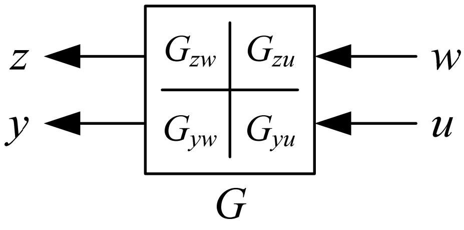

Fig. 2. An AMB system represented as a four block problem

In describing these signals, each will have a natural engineering description and these diverse descriptions will need to be adapted to a uniform and mathematically tractable form for purposes of assessment and design. It is assumed that the signals y and u are self-explanatory and will typically have units of volts. More important is the character of the signals w and z. The exogenous signals w will be a combination of forces (rotor unbalance, gravity load, process loads, impacts) and sources of measurement signal corruption: generally, electrical noise. Some of these have a nice description as a combination of simple basis functions ($\sin\omega t$, $\cos\omega t$, 1.0, e^{at}) with unknown but bounded amplitudes. A simple example is mass unbalance which will typically act at many locations along a rotor and will be described at each location as

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix}_i = me_{u.i}\Omega^2 \begin{Bmatrix} \sin\left(\Omega t + \phi_i\right) \\ \cos\left(\Omega t + \phi_i\right) \end{Bmatrix} \tag{1}$$

in which $me_{u.i}$ is a known level of mass unbalance at each location, but with a relatively confident bound: $me_{u.i} < me_{u.i,\,max}$.

Signals such as electrical noise are a bit more difficult to describe but may be represented in terms of spectral bounds. In this case, the spectrum of the signal is expected to lie below a specific bounding curve: assume that there is a stable transfer function $W_w(s)$ whose magnitude exceeds the expected amplitude of the noise signal at every frequency. This means that there is a choice of signal $\hat{w}$ whose RMS amplitude is less than 1.0 for which $w=W_w\hat{w}$ recovers the expected noise signal. Ideally, the amplitude of W_w is as small as possible at every frequency, while still preserving this relationship. Often, when the spectrum of the signal η might be complicated, W_w significantly overbounds the range in the interest of keeping the complexity of W_w low.

The end result is a description of the exogenous signals which takes the form

$$w\left(s\right) = W_w \hat{w}\left(s\right) \tag{2}$$

in which the elements of $\hat{w}(t)$ are expected to be periodic with amplitude less than or equal to 1.0. The weighting function W_w accounts for spectral bounds which vary with frequency as in the case of mass unbalance (increases with the square of frequency out to some maximum rotation rate).

In the case of the performance signals, the requirement for adequate performance will ideally take the form of

$$z_{i,\min} \le z_i\left(t\right) \le z_{i,\max} \;:\quad -\infty < t < \infty \tag{3}$$

Examples of such performance specifications include rotor contact clearance, actuator magnetic flux density, and power amplifier output voltage and current. Most commonly, the limits are symmetric so that we may require

$$\frac{z_i\left(t\right)}{z_{i,\max}} \le 1 \;:\quad -\infty < t < \infty \tag{4}$$

More generally, this nondimensionalization may be written as

$$z = W_z \hat{z} \;:\quad \hat{z}_i\left(t\right) \le 1.0 \tag{5}$$

in which the scaling of the elements of z is encapsulated by the weighting function W_z. In this manner, W_z is a performance specification in that it stipulates limitations on permissible range of the performance variables z_i.

4. THE SYSTEM MODEL

In its most precise description, the dynamic mapping G indicated in Fig. 2 is nonlinear but it is standard practice to use a linear approximation throughout most of the design process. In the sequel, we will assume specifically that G is linear time invariant (LTI) and may be represented as a matrix of transfer functions. For most AMB systems, such a representation retains sufficient fidelity to permit it to carry the design and analysis nearly to completion. A very thorough design process would conclude by connecting the resulting controller to a fully nonlinear model of the AMB-rotor system and use transient simulations to establish that the linear assumptions have not missed critical performance or stability features. This assumption that G is LTI is central to μ−synthesis and is a first limitation of the design process. Of course, similar assumptions underlie most practical controller synthesis processes: the most notable exceptions would be Lyapunov methods (Tsiotras et al., 2000) or variants such as backstepping (de Queiroz and Dawson, 1996) but these methods have received only very limited attention in the AMB literature and are generally not practical to apply to high ordered models G as arise when rotor flexibility is considered.

In a similar manner, it is common to model the controller as also LTI for the bulk of the design and analysis work. Certainly, commercial AMB controllers often contain nonlinear elements (Lindlau and Knospe, 2002; Cole et al., 2000), but these are assumed either to play a role in extending the linear operating regime (output feedback linearization, for instance) or to operate only when the system is under duress. As such, the controller may be described by a matrix transfer function H and the closed loop system indicated in Fig. 3 maps nondimensional exogenous signals $\hat{w}$ to nondimensional performance measures $\hat{z}$ via

$$\hat{z}(s) = W_z^{-1}\left[G_{zw} + G_{zu}H\left(I - G_{yu}H\right)^{-1}G_{yw}\right]W_w\hat{w} \equiv P\hat{w}(s) \quad (6)$$

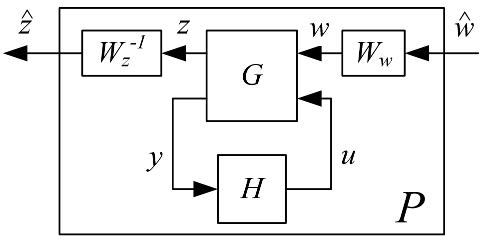

Fig. 3. An AMB system with controller closed loop

Here, the weighted closed loop performance function P can be introduced as a shorthand notation for the more complex expression

$$P = W_z^{-1}\left[G_{zw} + G_{zu}H\left(I - G_{yu}H\right)^{-1}G_{yw}\right]W_w \quad (7)$$

5. PERIODIC FUNCTIONS

The performance problem, then, is to establish that

$$\hat{z}_i(t) \le 1.0 \ : \quad -\infty < t < \infty$$

when $\hat{z}(s) = P\hat{w}(s)$ and $w_i(t)$ is any periodic signal with amplitude less than or equal to 1.0. First, note that if P is stable, then the homogenous responses $\hat{z}_i$ will also be periodic. If they are periodic, then there is a fundamental connection between the amplitudes of $\hat{z}_i$ and their peak temporal values. Hence, assume that the same normalization holds: that $|\hat{z}_i| \le 1$ is a sufficient condition to meet the temporal requirement. Primarily, this assumption means that we neglect the transient response and assume that the engineering analysis is predominantly concerned with steady state (periodic) response. Obviously, very close satisfaction of periodic response bounds may imply that the transient response violates the temporal bound: one approach to managing this shortcoming is to be a bit conservative in establishing the periodic constraint.

With this, the performance requirement becomes

$$|\hat{z}_i(t)| < 1.0, \quad \hat{z}(s) = P\hat{w}(s), \quad |\hat{w}_i(t)| < 1.0 \quad (8)$$

in which both $\hat{z}$ and $\hat{w}$ are assumed to be periodic functions whose amplitudes may be represented in RMS terms. Of course, this condition should be met for the *worst case* choice of $\hat{w}$. In particular, the elements of $\hat{w}$ should be worst case periodic functions and the combination of bounded amplitudes should maximize $\hat{z}$. Fortunately, it may be proved that, for an LTI operator P, the worst case periodic function is a sinusoid of single frequency. Thus, a sufficient condition for satisfying (8) is that

$$|\hat{z}_i(t)| < 1.0, \quad \hat{z}(t) = P(j\omega)\hat{w}\sin\omega t, \quad |\hat{w}_i| < 1.0, \quad \omega \in \Re \quad (9)$$

Relative phase of the exogenous signals in (9) is managed by assuming that the $\hat{w}_i$ are complex numbers. Under this assumption, the functions $\hat{z}_i$ may also be represented as $\hat{z}_i(t) = \hat{z}_i \sin\omega t$ and (9) becomes

$$|\hat{z}_i(t)| < 1.0, \quad \hat{z} = P(j\omega)\hat{w}, \quad |\hat{w}_i| < 1.0, \quad \omega \in \Re \quad (10)$$

6. SINGULAR VALUE ANALYSIS

Equation (10) still represents a worst case condition in that we must assure that none of the elements $\hat{z}_i$ has modulus exceeding 1.0 for any frequency or for any possible combination of $\hat{w}_i$ which are only constrained to have modulus less than 1.0. For the moment, neglect the frequency dependence and focus on the possible combinations of $\hat{w}_i$.

The notion that we need $|\hat{z}_i| < 1$ for any combination of $|\hat{w}_i| < 1$ has a nice engineering interpretation but here we introduce another simplification in order to make the problem more mathematically tractable. Rather than requiring that each element of $\hat{w}$ have modulus less than 1, require that the sum of the squares is less than one: $\sum|\hat{w}_i|^2 = |\hat{w}_i|_2^2 < 1$. Certainly, this condition may only be met if | $\hat{w}_i$ |<1 so it is a sufficient but not necessary constraint on w. Further, rather than requiring that $|\hat{z}_i| < 1$, require that $\sum|\hat{z}_i|^2 = |\hat{z}_i|_2^2 < 1$. Again, this is conservative in that it is a sufficient condition for the stipulation on ˆz but not necessary. Thus, if it is true that

$$|\hat{z}|_2 = |P\hat{w}|_2 < 1, \ \text{for} \ \forall|\hat{w}|_2 < 1 \quad (11)$$

then it is also true that $|\hat{z}_i| < 1$.

The value of the condition indicated by (11) is that it may be tested without performing an exhaustive search on feasible $\hat{w}$. In particular, a necessary and sufficient condition for meeting (11) is that the *maximum singular value* of P is less than 1.0:

$$\bar{\sigma}(P) \le 1 \ \Leftrightarrow \ |\hat{z}|_2 = |P\hat{w}|_2 < 1 \ \ \forall|\hat{w}|_2 < 1$$

Of course, as in (10), P is a function of frequency so that a sufficient condition to meet (10) is that

$$\bar{\sigma}(P) \le 1 \ \ \forall\omega \in \Re$$

or, equivalently

$$\sup_{\omega \in \Re} \overline{\sigma}\left(P\left(j\omega\right)\right) \le 1.0 \qquad (12)$$

The left side of Eq. (12) defines the H_∞ norm of the transfer function $|P|_\infty$ and indicates the worst case possible gain from the signal $\hat{w}$ to the signal $\hat{z}$. It is assumed implicitly that P represents a stable LTI dynamic system.

6. WEIGHTING FUNCTIONS

The weighted plant model for the reference example is shown in Fig. 4. For the machine tool spindle problem, the loads were assumed to act at the bearing locations while each of the position sensors was assumed afflicted with noise. The bearing loads are summarized in Tab. 1 while the sensor noise was 0.6 micrometers broad-band. The performance measures included amplifier voltage (limited to 300 volts), coil current (limited to 7 amps above a 5 amp bias), and journal displacement at the two bearing locations (limited to 50 micrometers at frequencies above 0.002 Hz, and 0.5 micrometers below this).

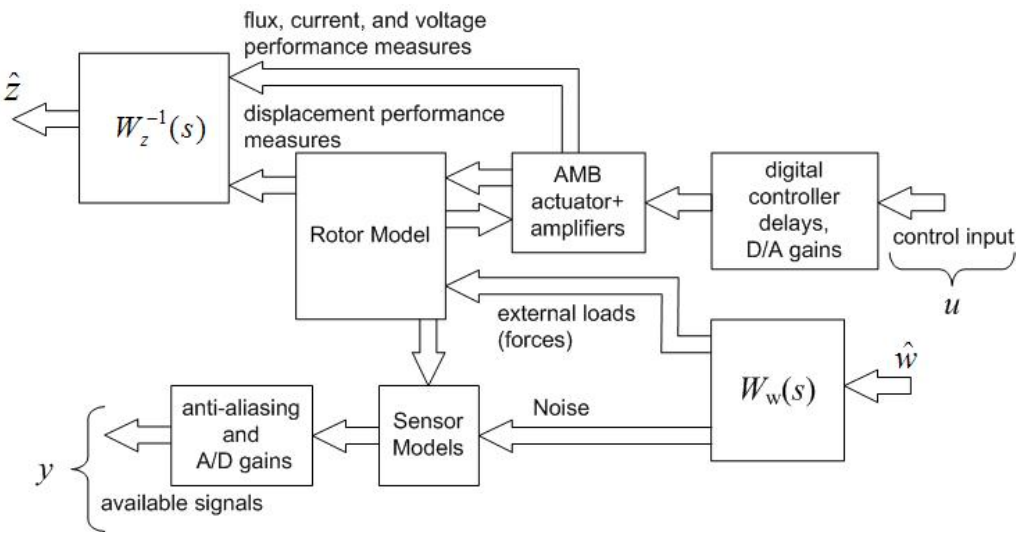

Fig. 4. Weighted model: weighting functions W_w and W_z normalize the load and perfiormance signals

Tab. 1. Bearing load parameters

	tool tip end	drive end
DC load	300 N	130 N
first break	0.0001 Hz	0.001 Hz
midfrequency load	80 N	50 N
second break	40 Hz	40 Hz

7. MODEL UNCERTAINTIES

A significant goal of μ-synthesis is to design controllers which are robust to variations in plant dynamics. A simple example is the effect of gyroscopics: the dynamics of the rotor at standstill are substantially different from those observed when spinning at 16000 RPM. The rotor model contains the rotor spin rate explicitly:

$$M\ddot{x} + \left(D + \Omega G\right)\dot{x} + Kx = f \qquad (13)$$

in which the gyroscopic behavior of the rotor mass is represented by the matrix G and Ω is the spin speed of the rotor. If a controller is designed for the rotor with Ω = 0, then there may be no guarantee that the system will be stable for other values of Ω: obviously undesirable.

In the μ-framework, uncertainties are represented as feedback gains connected to the plant where the nominal value of the feedback gain is zero but it is understood that the gain could lie anywhere inside a real range or complex disk. By convention, the size of this range is chosen to be 1.0. As an example, suppose that our rotor had a seal acting at some location along the shaft. The seal might have some nominal cross-coupled stiffness of 1000 N/m but with uncertainty of ± 300 N/m:

$$\begin{Bmatrix} f_{s,x} \\ f_{s,y} \end{Bmatrix} = \begin{bmatrix} 0 & 1000 \pm 300 \\ -1000 \mp 300 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \qquad (14)$$

This can be represented by

$$\begin{Bmatrix} f_{s,x} \\ f_{s,y} \end{Bmatrix} = \begin{bmatrix} 0 & 1000 \\ -1000 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} + 300 \begin{bmatrix} 0 & \pm 1 \\ \mp 1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} \qquad (15)$$

The first part of the relationship defines the *nominal* behavior and would be included in the core model. The second part defines a feedback with nominal value of zero. The scale 300 N/m would be applied to the input or output matrices tying this feedback into the rotor model and the remnant would be the uncertainty matrix, denoted Δ.

The product of adding weighting functions and uncertainty representations to the base model is depicted in Fig. 5.

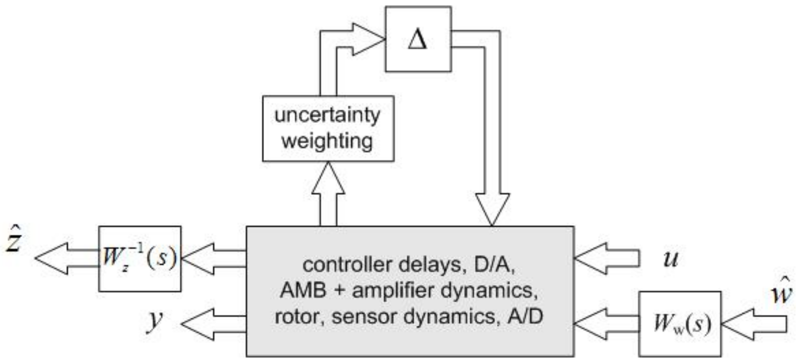

Fig. 5. Model with weighing functions and uncertainty added

For the machine tool spindle, the primary uncertainties were judged to be actuator properties, modal properties for the two bending modes retained, and, of course, rotor speed. The uncertainties in actuator gain and bearing negative stiffness were modeled as 3% and 15% real uncertainties of nominal value, respectively. The modal frequencies of the first and second modes were modeled as 1% complex uncertainty of nominal value for each mode. These latter uncertainties discourage the synthesis machinery from introducing controller dynamics that precisely cancel the dynamics associated with these modes as, for instance, very sharp notch filters. Rotor speed was modeled as 8000 RPM with an uncertainty of 100% in order to obtain a stabilizing controller for the speed range
from 0 to 16000 RPM.

8. THE MU-SYNTHESIS PROBLEM

Having established that assessment of stability robustness and robust performance of an AMB system may be written as a problem in computing the maximum structured singular value, or μ, of the closed loop system, it is natural to consider the possibility that a controller could be automatically synthesized to minimize this μ measure and thereby maximize the robust stability and robust load rejection of the resulting system. This is the objective
of μ-synthesis.

Concisely, μ-synthesis seeks to find that controller H for which the maximum structured singular value of the closed loop system P is minimized. As with H_∞ synthesis, once the specification is established, solving for the controller is a matter of "turning a crank". That is, reasonably effective computational tools exist to solve this problem. An example is the function "dksyn" provided by the Robust Control Toolbox of MatLab (The MathWorks, 2004).

Unlike the H_∞ synthesis problem, solutions to the μ-synthesis problem cannot be found closed-form and require iteration. The most common iteration scheme is called $D-K$ iteration. While the details of this iterative process are beyond the scope of this paper, it is worth pointing out that $D-K$ iteration adds order to the controller beyond the order of a comparable H_∞ controller so that the order of a μ-controller can be substantially larger than that of the plant plus its weighting functions. This iterative character of the solution can also sometimes lead to failure of the solution which, in this case, does not always imply non-existance of a solution.

Several μ-controllers were designed for the system described by Fig. 1 and just two examples are illustrated in Fig. 6, where one of the controllers was optimized to achieve the best machining performance in terms of high surface finish quality. Both controllers were implemented as discrete time, state-space systems with a sampling rate of 10 kHz. The resulting optimized controller was 88th order and was reduced to 44th order by model order reduction using Hankel singular value based algorithms. Differences between the controllers were generated by changing the performance and load weighting functions.

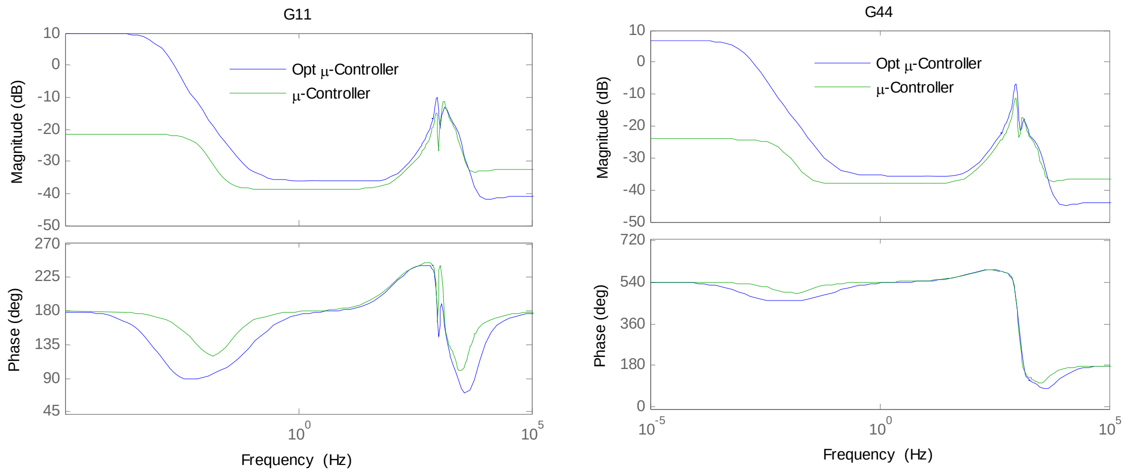

Fig. 6. Comparison of two μ-controllers; one is optimized for machining

To determine the spindle stiffness at the tool tip, with the rotor supported on each the PID, the μ-controller, and the optimized μ-controller, impact testing was carried out with an instrumented hammer. The results presented in the upper plot of Fig. 7 show the advantage of μ-controllers, especially in the vicinity of the first and second modes, where the PID stiffness is significantly lower. Over the wide range of frequencies the PID controller is much less stiff while the optimized μ-controller provides the highest stiffness.

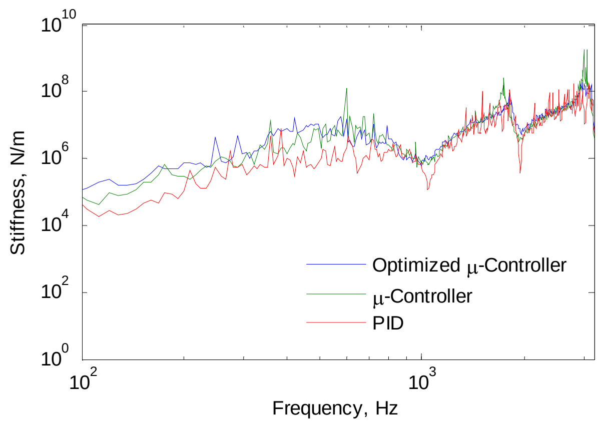

Fig. 7. Stiffness of the spindle at the tool tip extracted from the hammer test for PID and μ-controllers

9. CONCLUSIONS

A well formulated performance assessment tool for evaluating controllers for AMB systems (and practically any other essentially linear system with primarily steady state performance objectives) can be efficiently constructed in the form of a μ-analysis problem. Such a tool is compatible with any linearized AMB/rotor model and any linear controller and, as such, is entirely suitable for assessment of hand-synthesized controllers such as PID with notches and other specialized embellishments. At the same time, having produced such an assessment tool, automated synthesis is just a short step away and involves essentially no investment on the part of the controller design engineer.

The presented simulation and experimental results show the potential of μ-synthesized control of AMB machining spindles for improved cutting performance. In particular, the μ-controllers were able to realize substantially higher broad-band spindle tip stiffness that could be achieved (through manual tuning) by the PID + notch controller. Perhaps a more important advantage is the structure of the synthesis process provided by $\mu-$synthesis. In particular, the synthesis outcome is guided by choice of performance functions and load models and the resulting closed loop performance reflects these functions in a direct manner. Consequently, there is less need for synthesis tricks with the μ- approach. Further, the μ- approach provides

a convenient and rational repository for accumulating system knowledge through model and weighting function refinement. Finally, the μ-approach can provide guarantees of robustness to wide ranges of system parameter such as the operating speed range without requiring gain scheduling or other special techniques: all μ-synthesized controllers developed in the course of this study were stable over the entire operating range while aggressive PID + notch designs did not reliably meet this requirement.

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