ON ONE MATHEMATICAL MODEL OF THE LASER-INDUCED THERMAL SPLITTING

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Abstract: Distribution of the transient temperature field and the corresponding quasi-static thermal stresses were examined in the system consisted of a bulk substrate and a thin coating of different material deposited on it. Such a system is heated through the outer surface of coating by the pulsed heat flux generated due to absorption of laser pulse irradiation of rectangular or triangular time shape. The dependencies of temperature and stresses on geometrical and thermophysical properties of the substrate and the coating were studied. It was proved that there is the possibility of applying the obtained results to modelling of thermal splitting for brittle materials.

1. INTRODUCTION

Thermal splitting of brittle non-metals (like glass, ceramic materials, granite) is the easiest one, because these materials exhibit big difference between the melting temperature and the temperature of thermal strength. Low thermal conductivity of brittle materials is the cause why considerable thermal stresses are generated in thin subsurface layer in the initial moment. As a result, destruction of the sample takes place shortly after the heating process starts (Kamienkov et al., 1996).

Analytical methods for calculation of the temperature fields and thermal stresses generated by the pulse laser irradiation were developed mainly for homogeneous materials (Dostanko et al., 2002). On the other hand, the material to be splitted, quite often has the form of a protective coating or thin film deposited on the homogeneous substrate. The mathematical model of controlled thermal splitting of homogeneous and piece-wise homogeneous bodies at assumption of uniform distribution of the heat flux intensity was considered earlier (Li et al., 1997; Evtushenko et al., 2005).

2. TEMPERATURE

Controlled thermal splitting is in practice realized with the aid of frictional or laser heating. In such processes the region of heating has elliptical shape with its longer axis parallel to the direction of the detail moving. The heating occurs while the processed surface translates on the distance equal to the length of this ellipse's longer axis. The cooling takes place when the object passes the distance between the heated region and the front boundary of the region to which the cooling agent is applied.

For small values of Fourier's numbers, which correspond to characteristic times of thermal splitting, the large part of the heat flux is directed into the body, perpendicularly to its surface. That makes it possible to consider the generation of temperature fields and thermal stresses as the one-dimensional non-stationary process. Let us consider the system of semi-infinite substrate with the coating of the thickness d (Fig. 1).

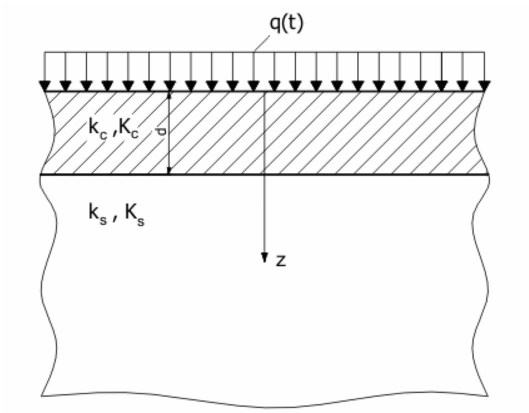

Fig. 1. Heating model of the homogeneous body with coating

Thermophysical properties of the substrate and the coating differ. The coating is heated on the free surface by the heat flux with the intensity q. Perfect thermal contact between the substrate and the coating is assumed.

Temperature distribution in the coating T_c and in the substrate T_s can be found from the solution of the following initial-boundary-value problem of heat conduction:

$$\frac{\partial^2 T_c}{\partial z^2} = \frac{1}{k_c}\frac{\partial T_c}{\partial t},\ 0 < z < d,\ t > 0 \quad (1)$$

$$\frac{\partial^2 T_s}{\partial z^2} = \frac{1}{k_s}\frac{\partial T_s}{\partial t},\ d < z < \infty,\ t > 0 \quad (2)$$

$$T_c(z,0) = 0,\ 0 < z < d \quad (3)$$

$$T_s(z,0) = 0,\ d < z < \infty \quad (4)$$

$$K_c \frac{\partial T_c}{\partial z}\bigg|_{z=0} = -q_0 q^*(t), \quad t > 0 \tag{5}$$

$$T_c(d,t) = T_s(d,t), \quad t > 0 \tag{6}$$

$$K_c \frac{\partial T_c}{\partial z}\bigg|_{z=d} = K_s \frac{\partial T_s}{\partial z}\bigg|_{z=d}, \quad t > 0 \tag{7}$$

$$T_s(z,t) \to 0 \ \text{ at } \ z \to \infty \tag{8}$$

In numerical calculations, in connection with the boundary condition (5), usually the rectangular

$$q^*(t) = \begin{cases} 1, \ 0 < t \le t_s \\ 0, \ t > t_s \end{cases} \tag{9}$$

or triangular

$$q^*(t) = \begin{cases} 2t/t_r, \ 0 < t \le t_r \\ 2(t_s - t)/(t_s - t_r), \ t_r \le t \le t_s \\ 0, \ t > t_s \end{cases} \tag{10}$$

time shape of the laser pulse, is used (Evtushenko et al., 2005). For comparative numerical analysis the parameters of functions (9) and (10) are chosen in such a manner that pulse duration and energy are the same in both cases.

From the solution of the linear initial-boundary-value problem of heat conduction (1)–(8), the temperature distributions in the coating were found in the form

$$T_c(z,t) = T_0 T^*(\zeta,\tau), \ \zeta \ge 0, \ \tau \ge 0 \tag{11}$$

where for the uniform heat flux intensity (9)

$$T^*(\zeta,\tau) = T^{(0)*}(\zeta,\tau) - T^{(0)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s) \tag{12}$$

and for the triangular one (10)

$$T^*(\zeta,\tau) = \frac{2}{\tau_r}\left[T^{(1)*}(\zeta,\tau) - T^{(1)*}(\zeta,\tau-\tau_r)H(\tau-\tau_r)\right] - \frac{2}{\tau_s-\tau_r}\left[T^{(1)*}(\zeta,\tau-\tau_r)H(\tau-\tau_r) - T^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s)\right]$$
$$0 \le \zeta \le 1, \ \tau \ge 0 \tag{13}$$

In the equations (12), (13) $T^{(0)*}$ and $T^{(1)*}$ are the solutions of the transient heat conduction problem (1)–(8) for constant $q^*(\tau) = 1$ and linearly changing with time $q^*(\tau) = \tau$, $\tau > 0$ heat flux intensity, respectively:

$$T^{(k)*}(\zeta,\tau) = \sum_{n=0}^{\infty} \Lambda^n T_n^{(k)*}(\zeta,\tau), \ 0 \le \zeta \le 1, \ \tau > 0, \ k = 0,1 \tag{14}$$

$$T_0^{(0)*}(\zeta,\tau) = u\,\mathrm{ierfc}\left(\frac{\zeta}{u}\right),$$

$$T_n^{(0)*}(\zeta,\tau) = u\left[\mathrm{ierfc}\left(\frac{2n+\zeta}{u}\right) + \mathrm{ierfc}\left(\frac{2n-\zeta}{u}\right)\right], \ n = 1,2,3,.. \tag{15}$$

$$T_0^{(1)*}(\zeta,\tau) = \frac{1}{4}u^3 F\left(\frac{\zeta}{u}\right),$$

$$T_n^{(1)*}(\zeta,\tau) = \frac{1}{4}u^3\left[F\left(\frac{2n+\zeta}{u}\right) + F\left(\frac{2n-\zeta}{u}\right)\right], \ n = 1,2,3,... \tag{16}$$

$$F(x) = \left(1 + \frac{2}{3}x^2\right)\mathrm{ierfc}(x) - \frac{1}{3\sqrt{\pi}}\exp(-x^2) \tag{17}$$

$$\mathrm{ierfc}(x) = \frac{1}{\sqrt{\pi}}\exp(-x^2) - x\,\mathrm{erfc}(x)$$

$$\mathrm{erfc}(x) = 1 - \mathrm{erf}(x) \quad u \equiv 2\sqrt{\tau} \tag{18}$$

$$\Lambda^n = \begin{cases} (-1)^n |\lambda|^n, & -1 < \lambda \le 0 \\ \lambda^n, & 0 \le \lambda < 1 \end{cases} \qquad \lambda = \frac{1-\varepsilon}{1+\varepsilon}$$

$$\varepsilon = \frac{K_s}{K_c}\sqrt{\frac{k_c}{k_s}} \tag{19}$$

In the further part of the paper all quantities connected with the temperature fields $T^{(0)*}(\xi,\tau)$ (14), (15) and $T^{(1)*}(\xi,\tau)$ (14), (16) will be denoted with upper or lower indices in brackets: respectively zero (0) and one (1). Dimensionless parameter $0 < \varepsilon < \infty$ (19), known as the "thermal activity coefficient of the substrate in relation to the coating", is comprised in parameters λ and Λ, which are the constant factors in the solutions (14). It should be noted that the solutions of the corresponding problems of heat conduction for homogeneous half-space are obtained from the first component of the expression (15), (for $n = 0$).

3. STRESSES AND DEFORMATIONS

Experimental examinations of the controlled superficial splitting proved that from the three normal components of the stress tensor – longitudinal, lateral and in the direction of heating – only the lateral component σ_y is useful in thermal splitting (Dostanko et al., 2002). As the result of action of this component, thermal splitting proceeds in the direction of the heat flux movement trajectory. The longitudinal component σ_x is undesirable because when it has enough greater values exceeding tensile strength of materials, the micro cracks oriented at various angles to the direction of splitting are created and divergence between the line of splitting and the direction of heat flow movement occurs. The normal component of stress tensor σ_z has no essential meaning in one-dimensional problem.

On the basis of these data quasi-static normal stresses σ_y induced by the non-stationary temperature field (11)–(19) can be determined from the equations, which describe thermal bending of thick plate of the thickness d with free ends:

$$\sigma_y(z,t) = \sigma_0\,\sigma_y^*(\zeta,\tau), \quad 0 \le z \le d, \ t \ge 0 \tag{20}$$

$$\sigma_y^*(\zeta,\tau) = \varepsilon_y^*(\zeta,\tau) - T^*(\zeta,\tau), \quad 0 \le \zeta \le 1, \ \tau \ge 0 \tag{21}$$

$$\varepsilon_y^*(\zeta,\tau)=\int_0^1 T^*(\zeta,\tau)d\zeta+12(\zeta-0{,}5)\int_0^1(\zeta-0{,}5)T^*(\zeta,\tau)d\zeta\ ,$$
$$0\le\zeta\le 1,\ \tau\ge 0 \qquad (22)$$

When the heating of the plate's surface is realised with the uniform heat flux (9) then the dimensionless lateral stress σ_y^* can be found from the equation:

$$\sigma_y^*(\zeta,\tau)=\sigma_y^{(0)*}(\zeta,\tau)-\sigma_y^{(0)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s),$$
$$0\le\zeta\le 1,\ \tau\ge 0 \qquad (23)$$

for the triangular time shape of the heat pulse (10) one has:

$$\sigma_y^*(\zeta,\tau)=\frac{2}{\tau_r}\left[\sigma_y^{(1)*}(\zeta,\tau)-\sigma_y^{(1)*}(\zeta,\tau-\tau_r)H(\tau-\tau_r)\right]-$$
$$-\frac{2}{\tau_s-\tau_r}\left[\sigma_y^{(1)*}(\zeta,\tau-\tau_r)H(\tau-\tau_r)-\sigma_y^{(1)*}(\zeta,\tau-\tau_s)H(\tau-\tau_s)\right],$$
$$0\le\zeta\le 1,\ \tau\ge 0 \qquad (24)$$

where

$$\sigma_y^{(k)*}(\zeta,\tau)=\sum_{n=0}^{\infty}\Lambda^n\sigma_n^{(k)*}(\zeta,\tau),\ \ 0\le\zeta\le 1,\ \tau\ge 0\ \ k=0,1 \qquad (25)$$

$$\sigma_n^{(k)*}(\zeta,\tau)=Q_n^{(k)}(\tau)-\zeta\, R_n^{(k)}(\tau)-T^{(k)*}(\zeta,\tau),\ \ k=0,1 \qquad (26)$$

$$Q_n^{(k)}(\tau)=4\,I_n^{(k)}(\tau)-6\,J_n^{(k)}(\tau),$$

$$R_n^{(k)}(\tau)=6\,I_n^{(k)}(\tau)-12\,J_n^{(k)}(\tau),\ k=0,1;\ n=0,1,2,... \qquad (27)$$

$$I_0^{(k)}(\tau)=C^{(k)}(u)L^{(k)}\left(\frac{1}{u}\right)\quad J_0^{(k)}(\tau)=uC^{(k)}(u)M^{(k)}\left(\frac{1}{u}\right)$$
$$k=0,1 \qquad (28)$$

$$I_n^{(k)}(\tau)=C^{(k)}\left[L^{(k)}\left(\frac{2n+1}{u}\right)-L^{(k)}\left(\frac{2n-1}{u}\right)\right]$$
$$k=0,1;\ n=1,2,3\ldots \qquad (29)$$

$$J_n^{(k)}(\tau)=$$
$$C^{(k)}\left\{u\left[M^{(k)}\left(\frac{2n+1}{u}\right)-2M^{(k)}\left(\frac{2n}{u}\right)+M^{(k)}\left(\frac{2n-1}{u}\right)\right]-\right.$$
$$\left.-2n\left[L^{(k)}\left(\frac{2n+1}{u}\right)-2L^{(k)}\left(\frac{2n}{u}\right)+L^{(k)}\left(\frac{2n-1}{u}\right)\right]\right\}$$
$$k=0,1;\ n=1,2,3\ldots \qquad (30)$$

$$C^{(0)}(u)=u^2\quad C^{(1)}(u)=0.25u^4 \qquad (31)$$

4. RESULTS AND DISCUSSION

The input dimensionless parameters of the calculations are: spatial coordinate ξ, Fourier's numbers τ, τ_r (dimensionless laser pulse rise time) and τ_s (dimensionless laser pulse duration). Isolines for the normal stresses $\sigma^*=\sigma_y/\sigma_0$ were drawn in the coordinates (ζ,τ) for different temporal profile of the heat pulse. All calculations were conducted for the pulses with dimensionless duration $\tau_s=0.15$, which is characteristic for irradiation done by CO_2 laser, which emits light at wavelength 10.6 μm (Dostanko et al., 2002).

The authors presented the numerical examinations of thermal stresses distribution for the system consisting of ceramic coating ($K_c=2.0$ W/mK, $k_c=0.8\cdot10^{-6}$ m^2/s), ZrO_2 deposited on the 40H steel substrate ($K_s=41.9$ W/mK, $k_s=10.2\cdot10^{-6}$ m^2/s) (Fig. 2).

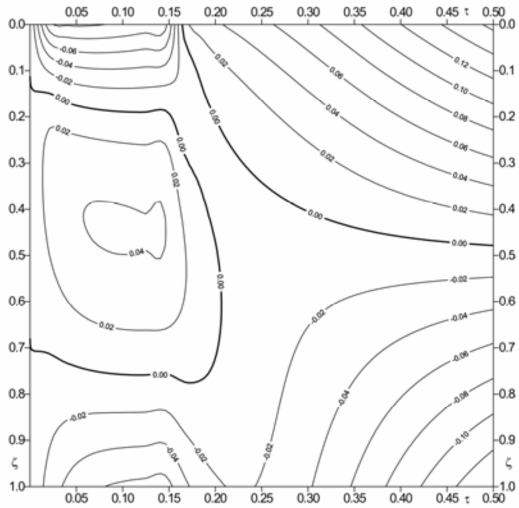

Fig. 2. Isolines of dimensionless lateral stress σ_y^* for ZrO_2 ceramic coating and 40H steel substrate at rectangular laser pulse duration $\tau_s=0.15$

The coefficient of thermal activity for this system found from the equation (19), $\varepsilon=5.866$, and the parameter λ has the value $\lambda=-0.708$. Thermal diffusivity of zirconium dioxide is small when compared with the value for steel. That difference is the cause of high temperatures on the processed surface and considerably higher than for the homogeneous half-space (one order of magnitude) lateral tensile stresses generated in the superficial layer when the heating is finished. So, the thermal processing of the coating from zirconium dioxide leads to the generation of superficial cracks, which divide the surface into smaller fragments. Of course the distribution of cracks at different depths depends on the heat flux intensity, the diameter of the laser beam, pulse duration and other parameters of the laser system. But when using dimensionless variables and parameters the results can be compared and the conclusion is that for the heating duration $\tau_s=0.15$, penetration depth of cracks for coating–substrate system (ZrO_2 –40H steel) is, more than two times greater than for the homogeneous material. The opposite, to the discussed above, combination of thermo-physical properties of the coating and the substrate is represented by the copper–granite system, often used in ornaments decorating interiors of the buildings like theatres and churches. For the copper coating $K_c=402$ W/mK, $k_c=125\cdot10^{-6}$ m^2/s, while for the granite substrate

$K_s = 1.4$ W/mK, $k_s = 0.505 \cdot 10^{-6}$ m^2/s, what means that the substrate is practically thermal insulator and the coating has good thermal conductivity. The distribution of lateral thermal stresses for copper–granite system is on Fig. 3.

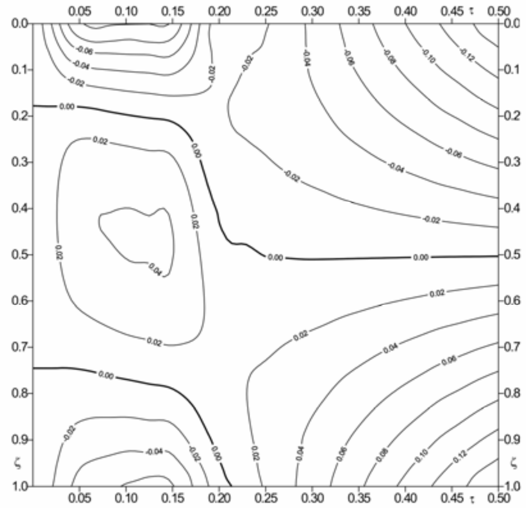

Fig. 3. Isolines of dimensionless lateral stress σ_y^* for copper coating and granite substrate at rectangular laser pulse duration $\tau_s = 0.15$

In this situation, when the thickness of the coating increases, the temperature on the copper surface decreases. Therefore the effective depth of heat penetration into the coating is greater for the better conducting copper than for thermally insulating zirconium dioxide. We note that near to the heated surface $\zeta = 0$ lateral stresses σ_y are compressive not only in the heating phase $0 < \tau < 0.15$ but also during relaxation time, when the heat source is off. Considerable lateral tensile stresses occur during the cooling phase close to the interface of the substrate and the coating, $\zeta = 1$. This region of the tensile stresses on the copper-granite interface can destroy their contact and in effect the copper coating exfoliation can result.

5. CONCLUSIONS

Analysis of the evolution of stresses in the homogeneous plate proves that when it is heated, considerable lateral compressive stresses occur near the outer surface. The value of this stresses decreases when the heating is stopped and after some time the sign changes – what means that the tensile stresses takes place. The time when it happens increases monotonously with increase of a thermal pulse duration (rectangular laser pulses) or with increase of rise time (triangular laser pulses). When the lateral tensile stresses exceed the strength of the material then a crack on the surface can arise. The region of lateral compressive stresses, which occur beneath the surface, limits their development into the material. The presence of the coating (for example, ZrO_2) with thermal conductivity lower than for the substrate results in considerably higher than for the homogeneous material, lateral tensile stresses in the subsurface after the termination of heating. The depth of thermal splitting is also increased in this case. When the material of the coating (for example, copper) has greater conductivity than the substrate (granite) then the stresses have compressive character all the time. The coating of this kind can protect from thermal splitting. The region vulnerable for damage in this case is close to the interface of the substrate and the coating where considerable tensile stresses occur during the cooling phase.

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O PEWNYM MATEMATYCZNYM MODELU PROCESU LASEROWEGO TERMOROZŁUPYWANIA

Streszczenie: Zbadano nieustalone pole temperatury i związane z nim quasi-stacjonarne termiczne naprężenia w układzie składającym się, z podłoża i cienkiej warstwy naniesionej na to podłoże (nagrzewanie impulsowym strumieniem ciepła). Znaleziono zależności temperatury i naprężeń od geometrycznych i cieplno-fizycznych właściwości podłoża i warstwy. Pokazano, że istnieje możliwość wykorzystania otrzymanych rezultatów do modelowania procesu sterowanego termicznego rozdzielania (rozłupywania) kruchych materiałów.

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