STATIC AND CYCLIC STRENGTH OF A CRACKED BODY WHICH STRENGTHENED BY INJECTION TECHNOLOGIES

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Abstract: Using the modern concepts of fracture mechanics, the injection processes of crack-shaped defects in structural elements in different service conditions are modeled. The corresponding calculations, on the bases of which the degree of hardening of the damaget structures by the injection technologies and parameters, which influence their efficiency, have been perfomed. The ways of optimization of injection technology of the damaged structural elements are established.

1. INTRODUCTION

In this paper a possibility of renewing the serviceability of structural elements, damaged by cracks, by using injection of the zones with defects, has been substantiated.

In practice there are technologies of healing of the cracked materials by introducing certain liquid materials in a crack, which can bind with the basic material during hardening (crystallization, polymerization, etc.). As a result the material is strengthened and can bear service loading. A degree of the elements bearing strength and their residual life depends on many factors, in particular, on adhesion strength of materials interface, correlation between elastic properties of materials, geometry of defects, strength of the injection material (instant and long – term), its hardening parameters (increase or decrease of a volume) and on many other.

Let us evaluate the strength, residual life and influence of basic parameters on the injection efficiency using, as an example the Griffith problem – a model of a body with a crack.

2. STATIC STRENGTH OF A BODY WITH A FILLED CRACK

According to the concepts of fracture mechanics, strength and integrity of structures and their loss are related with the presence of cracks and their growth in the material. A Griffith problem about strength of a plate with a crack is a classical example of such an approach (Fig. 1). It has been found that the strength of a body with a crack, calculated within the frames of energy (Griffith, 1920) and force (Irwin, 1957) approaches, is determined by the dependence:

$$\sigma_c = \sqrt{2\gamma E / \pi l} = \frac{K_C}{\sqrt{\pi l}} \quad (1)$$

Here γ is specific energy of fracture, E is Young's modulus, K_C is critical stress intensity factor.

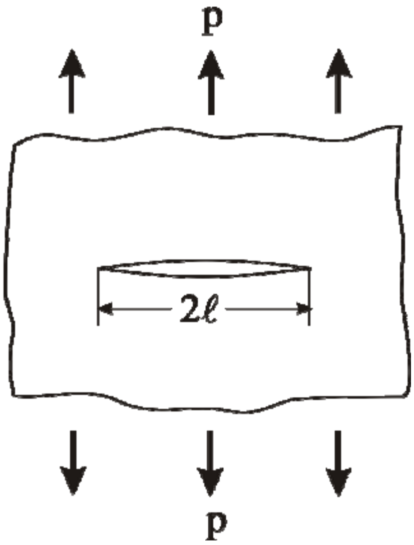

Fig. 1. Model scheme of cracked body.

Let us establish the character of the change of a cracked plate strength, if the crack is filled with the injection material. By using the solution of the problem about a thin inclusion in a plate (Panasyuk et al., 1986) for stresses in an inclusion the following expression is found:

$$\sigma = \frac{p(1+2\beta)\varepsilon}{1+2\beta\varepsilon}, \quad (2)$$

where $\varepsilon = E_1/E \le 1$; $\beta = l/c >> 1$; l,c, are semi – axes of an elliptical cavity, E_1 is Young's modulus of the injection material. The stress intensity factor for a filled crack is written as:

$$K_1 = \frac{p\sqrt{\pi l}\left(1-\varepsilon\right)}{1+2\beta\varepsilon}\ . \tag{3}$$

Correlations (2), (3) allow us the establishment of the strength of a plate with a filled crack.

Let $\sigma_в^*$, be the ultimate strength of the injected material. Since, according to (2) the injection material is in the homogeneous uniaxial tension stress state, then by the theory of strength of maximum stresses, external loading at which fracture of the filled material occurs, is:

$$p_c^* = \frac{\sigma_в^*\left(1+2\beta\varepsilon\right)}{\left(1+2\beta\right)\varepsilon}\ . \tag{4}$$

On the basis of expression (3) and a critical condition of crack propagation $K_I(p_c,l)=K_C$, establish loading at which the growth of the injected crack is possible:

$$p_c = \frac{K_C\left(1+2\beta\varepsilon\right)}{\sqrt{\pi l}\left(1-\varepsilon\right)}\ . \tag{5}$$

It is clear that the process of injection, as a method of strengthening of a cracked structural material is reasonable only under condition that the injection material will not fracture until the stresses loading values will reach the strength of a cracked plate $\sigma_C = K_C \big/ \sqrt{\pi l}$.

It proceeds from (4) that this condition will be satisfied if

$$\sigma_в^* > \frac{K_C\varepsilon\left(1+2\beta\right)}{\sqrt{\pi l}\left(1+2\beta\varepsilon\right)}\ . \tag{6}$$

The choice of the injection material is optimal if the material does not fracture earlier than the basic material, i.e:

$$p_c^* > p_c$$

Thus, taking into account correlations (4), (5) evaluate the required strength of the injected material

$$\sigma_в^* > \frac{K_{ICi}\varepsilon\left(1+2\beta\right)}{\sqrt{\pi l}\left(1-\varepsilon\right)} \tag{7}$$

Under such condition the strength of a plate with a filled crack (a crack is filled) is determined by dependence (5), i.e $\sigma_c^*=p_c$.

To verify dependence (5), experimental investigations of such materials as concrete (basic material) and polyurethane (injection material) have been performed. A testing chart is presented in Fig. 2.

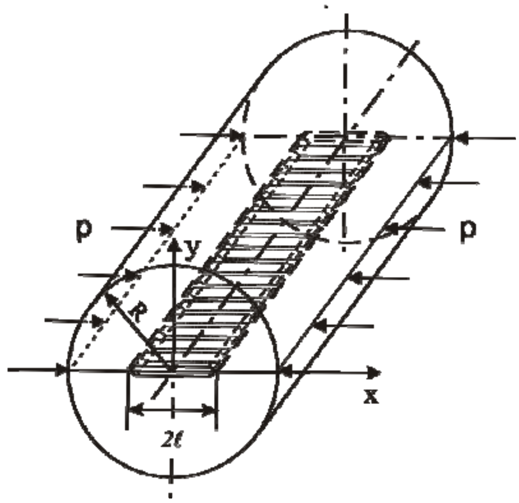

Fig. 2. Scheme of loading of cylinder specimen with a crack

The stress intensity factor for a loaded disc with a crack is found by Yarema and Krestin (1966):

$$K_1 = \frac{P}{R}\sqrt{\frac{l}{\pi}}\left[1+\frac{3}{2}\lambda^2+\frac{3}{4}\lambda^6+O\left(\lambda^8\right)\right], \tag{8}$$

here $O(\lambda^8)$ is a small value of an order of λ^8, $\lambda=l/R$.

It is clear that when $l/R\leq0.25$ the influence of a free surface of a disc on K_I can be neglected and with an error not exceeding 10%, the value of K, for a disc with a crack can be obtained from the solution of the Griffith problem:

$$K_1 = p\sqrt{\pi l}, \quad p = P\big/\pi R\ . \tag{9}$$

It can be proved that under condition $l/R\leq0.25$, expression (3) gives the value of SIF for a filled crack in a disc. In this case equation (5) gives the value of a ultimate load for a cracked disc with the same approximation. The presented data and our considerations allow us to compare theoretical (5) and experimental (obtained according to the chart in Fig. 2) results. In Fig. 3 circles represent experimental data, solid lines – theoretical prediction of the strengthening effect.

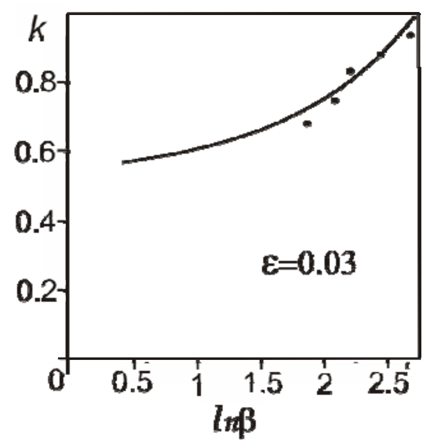

Fig. 3. Relation between parameters $p_c \big/ \sigma_c$ and $\beta = 1\big/c$

As one can see experimental and theoretical data agree well thus testifying to the possibility of application of such calculational models to evaluation of residual instant strength of structural elements with a "healed crack".

3. A WEDGING EFFECT DURING INJECTION

A very undesirable effect (from the viewpoint of structural integrity), that can assist the injection is wedging of a crack by an injection mixture. Supplied to the crack under pressure, the injection mixture exercises a pressure on the crack surfaces. Together with external forces this can cause crack length increment.

Let a defect, loaded by p_0, is filled with injection material, that during hardening has elastic characteristics E, ν, and exercises a pressure on the crack surface with, in general, unknown intensity p. The value of p is determined by two components: a) hydrostatic pressure p_1, of the liquid injection material supply, b) pressure p_2, arising due to the change of material volume during hardening; $p=p_1+p_2$.

The first component p_1 is known. The second is evaluated when solving the corresponding problem of mechanics. Let, when the counter action, (of basic material) is absent the injection material of volume V_0 in a liquid state during hardening occupy some volume $V_1=\alpha\ V_0$. As a result of crack surfaces counteraction and deformations related with that a contact surface of materials is established which corresponds to volume V_2 of the deformed injection material.

Stresses in the injected material, arising due to counteraction of the basic material can be approximately presented as:

$$p_2 = E_* \left(\frac{u_0 + u_1 + u_2}{h} - 1 \right). \tag{10}$$

Here u_0, u_1, u_2 are crack edges displacement caused by forces p_0, p_1, p_2 respectively:

$$h = c\sqrt{1 - x^2/l^2}, \qquad c = \frac{V_1}{\pi l t} = \frac{\alpha l (p_0 + p_1)}{E},$$

t is a plate thickness.

Displacements of the crack surface u_0, u are known, in particular

$$u_0 = \frac{p_0}{E}\sqrt{l^2 - x^2}, \quad u_1 = \frac{p_1}{E}\sqrt{l^2 - x^2},$$

and u_2 is obtained from the solution of the integral equation:

$$\int_{-l}^{l} \frac{u_2'}{t-x} dt - \frac{2\pi E_* (1-\nu) u_2}{E(1+\nu) h} = \frac{2\pi (1-\nu) E_*}{E(1+\nu)} \left(\frac{(p_0 + p_1)\beta}{E} - 1 \right), \tag{11}$$

An exact solution is written as:

$$u_2 = \frac{\varepsilon'(E - (p_0 + p_1)\beta)}{E(1 + \beta\varepsilon')}\sqrt{l^2 - x^2} \tag{12}$$

Thus, on the basis of the above presented, we can write an expression for calculation of the total stress intensity factor due to the action of all factors for a filled crack

$$K_1 = \sqrt{\pi l}\, \frac{(p_0 + p_1 + \varepsilon'(E - (p_0 + p_1)\beta))}{1 + \beta\varepsilon'}. \tag{13}$$

From the condition $K_I < K_C$ determine the pressure p_1 of the injection mixture supply, at which a crack will not increase

$$p_1 < \frac{K_{IC}(1 + \beta\varepsilon')}{\sqrt{\pi l}} - p_0 - \varepsilon' E \tag{14}$$

When the conditions of external loading p_0 and technological pressure of a mixture supply p_1 are set than from inequality $K_I < K_C$ the parameters of rigidity of the hardened injection material at which no crack length increment occurs are found:

$$\varepsilon' < \frac{(p_0 + p_1)\sqrt{\pi l} - K_{IC}}{E\sqrt{\pi l} - \beta K_{IC}}.$$

4. INJECTED CRACKS UNDER CYCLIC LOADING

Let us establish the influence of injection on fatigue crack growth. Consider a plate with a crack, subjected to cyclic loadings under tension, which change from p_{min} to p_{max}. Establish the injection parameters at which fatigue crack stops. We will use a correction that describes a kinetic diagram of fatigue crack growth for this purpose (Yarema and Mikitishin, 1975)

$$\frac{dl}{dN} = v_0 \left(\frac{\Delta K - \Delta K_{th}}{\Delta K_{fc} - \Delta K} \right)^n \tag{15}$$

where v_0, n_1, ΔK_{th}, ΔK_{fc} - are material constants.

The stress intensing factor range in the case of non-filled ΔK and filled cracks ΔK^* is calculated from expressions:

$$\Delta K = (p_{\max} - p_{\min})\sqrt{\pi l}$$

$$\Delta K^* = \frac{(p_{\max} - p_{\min})\sqrt{\pi l}(1 - \varepsilon)}{1 + 2\beta\varepsilon} \tag{16}$$

It proceeds from (15), (16) that fatigue crack due to injection will not propagate in the material, if:

$$\Delta K^* \le \Delta K_{th}$$

or

$$\frac{(p_{\max} - p_{\min})\sqrt{\pi l}(1 - \varepsilon)}{1 + 2\beta\varepsilon} \le \Delta K_{th} \tag{17}$$

So, the rigidity of a filler ε is insufficient to stop fatigue crack propagation.

$$\varepsilon \geq \frac{\left(p_{\max} - p_{\min}\right)\sqrt{\pi l} - \Delta K_{th}}{\left(p_{\max} - p_{\min}\right)\sqrt{\pi l} + 2\beta\Delta K_{th}} \qquad (18)$$

5. SUMMARY AND CONCLUSIONS

By analyzing the problems related with crack injection, we have found that main parameters that have an influence on renewal of the structure carrying ability are the defect geometry (a crack) and a relative rigidity of the injection material.

It is evident that adhesion strength of the surface of materials interface is very important for injection efficiency.

In this paper it was considered that the adhesion strength is sufficient, i.e. not less than the strength of the injection material. However, a more detailed analysis is necessary to study the influence of adhesion in order to understand properly the phenomenon of crack "healing" as a result of the injection technologies application. Both experimental and theoretical investigations are needed. From the practical point view, the wedging effect in a body during injection and compression stresses relaxation in the injected material require a special attention, since under certain conditions these phenomena can be very important for this process.

One more important issue of investigations is account of plastic deformations at the crack fronts. This is especially important for considering cyclic character of external loading.

Some more problems which are worth investigating consider a quantitative analysis of the defects interaction during injection, the influence of a body free surface, transition from a plane model to 3-Δ model, etc.

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STATYCZNA I CYKLICZNA WYTRZYMAŁOŚĆ PĘKNIĘTEGO CIAŁA UMOCNIONEGO ZA POMOCĄ TECHNOLOGII WTRYSKIWANIA

Streszczenie: Zamodelowana, przy użyciu nowoczesnych koncepcji mechaniki pękania, procesy wtryskiwania defektów w kształcie szczeliny do konstrukcyjnych elementów dla różnych warunków pracy. Przedstawiono odpowiednie obliczenia na podstawie których otrzymano stopień utwardzania elementów uszkodzonych w wyniku procesów wtryskiwania. Ustalono sposoby optymalizacji technologii wtryskiwania elementów konstrukcyjnych z uszkodzeniami.