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DETERMINING TRANSITION PROBABILITIES IN PROBABILISTIC ALGORITHMS

Abstract: The main problem of the paper is related to the algebraic method for determining transition probabilities in probabilistic algorithms interpreted in finite structures. The correctness of this method is based on a lemma stating that the determinant of a matrix (being of a special form) is different from zero. The paper contains two proofs of this lemma, formulated without a proof in [3].

Key words: probabilistic algorithm, probabilistic program, random generation, final state, looping state, not looping state

1. Introduction

Statistics plays an important part both in our everyday life and in the science. It permits "deriving knowledge" included in huge sets of data and studying different correlations and relationships. Statistical technologies support different kinds of researches and diagnosis. Results and analyses of results obtained in this way can be used in solutions of different kind of problems in the future. This motivates us to concentrate our interests on the so-called "probabilistic algorithms", which take into consideration dynamically changing situations (of analyzed occurrence), and also different possibilities of developing of the actual situation. The algorithms consider not only state in a given moment, but also sequences of successive changes. The analysis of changes occurring one after another (e.g., analysis of changes of the weather in a given area) causes most problems in detecting regularities in them. We replace determinism with probabilistic solution because determinism does not take random factors into consideration and it does not account for

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uncertainty. However, uncertainty is related with every researched occurrence in the real world.

Summing up, probabilistic algorithms are useful in analysis of incomplete information. We use probabilistic technologies in simulation of real processes, for which empirical collected of data is very expensive, time-consuming or even completely impossible.

In the present paper iterative probabilistic algorithms are understood as iterative programs using typical program constructions:

$x := a$,

begin ... end,

if ... then ... else ...,

while ... do ...,

and two probabilistic constructions:

$x := ?$,

either$_{(\rho)}$ ... or ...

interpreted as follows: the first construction corresponds to a random generation of a value of the variable x and the first part of the second construction is chosen with the probability ρ (the second one is chosen with the probability $1-\rho$).

For a finite interpretation of an algorithm $K(x_1,\ldots,x_h)$ in a finite set $A=(a_1,\ldots,a_t)$ we have $n=t^h$ possible valuations of variables from $X^h=(x_1,\ldots,x_h)$. Let us denote them by $v_1,\ldots,v_n$.

The main fact, which uses essentially the aforementioned LEMMA, concerns the following procedure of algebraization (cf. Lemma 3.2 in [3]) consisting in associating with every program $\boldsymbol{K}$ a $n\times n$ matrix K where k_{ij} denotes the probability of passing from an initial valuation v_i to a final valuation v_j.

For each probabilistic program $\boldsymbol{K}(x_1,\ldots,x_h)$ interpreted in a finite universe $A=(a_1,\ldots,a_t)$ we can construct, in an effective way, a $n\times n$ matrix $K=\left[k_{ij}\right]_{i,j=1,\ldots,n}$ (where $n=t^h$), such as if the probabilities of appearing $v_1,\ldots,v_n$ as the input valuations are equal $p_1,\ldots,p_n$, respectively, then the probabilities $q_1,\ldots,q_n$ of appearing $v_1,\ldots,v_n$ as output valuations satisfy:

$$[q_1,\ldots,q_n]=[p_1,\ldots,p_n]\circ K .$$

This fact can be illustrated as follows:

$$v_i \xrightarrow{\ p_i\ } \boldsymbol{K} \xrightarrow{\ q_j\ } v_j\ ,\ i,j=1,\ldots,n .$$

The construction of the matrix K for a given program $\boldsymbol{K}$ is inductive with respect to the number of program constructions used in $\boldsymbol{K}$. The most difficult case is when $\boldsymbol{K}$ is of the form while γ do $\boldsymbol{M}$.

Suppose (inductively) that we have given the matrix for the program $\boldsymbol{M}$. If denote by [γ?] subprogram while $\neg\gamma$ do $x := x$; then the matrix K for the program $\boldsymbol{K}$ is defined as follows (cf. [3]):

$$K = \sum_{i=0}^{\infty} K_i ,$$

where K_i denotes the matrix corresponding to the program:

$$\text{begin } \underbrace{[\gamma?] \text{ if } \gamma \text{ then } \boldsymbol{M}; \ \ldots \ ; [\gamma?] \text{ if } \gamma \text{ then } \boldsymbol{M};}_{\text{i-times}} \ [\neg\gamma?]; \text{ end;}$$

Now, we describe an effective method which enables us to determine the matrix K in a finite number of steps is described in [3]. The starting point is the equivalence of the program $\boldsymbol{K}$ of the form while γ do $\boldsymbol{M}$ and the following program:

$$\text{if } \gamma \text{ then begin } \boldsymbol{M}; \underbrace{\text{while } \gamma \text{ do } \boldsymbol{M};}_{\boldsymbol{K}} \text{ end;.}$$

This equivalence motivates the following equation:

$$K = I_{\neg\gamma} + I_{\gamma} \circ M \circ K ,$$

where

$$I_{\gamma[i,j]} = \begin{cases} 1 & \text{for } i = j \text{ and the valuation } v_i \text{ satisfies the condition } \gamma \\ 0 & \text{otherwise} \end{cases}$$

This equation may be written in the equivalent way as

$$(I\text{-}I_{\gamma} \circ M) \circ K = I_{\neg\gamma} .$$

Since the determinant of the matrix $I\text{-}I_{\gamma} \circ M$ (denoted by $\det(I\text{-}I_{\gamma} \circ M)$) may be equal to 0, to determine the matrix K, we need a more subtle analysis described in details in [3].

Speaking informally, an effective classification is realized and suitable positions of matrix M are determined in the following way:

- First, we determine positions equal to 1 (corresponding to valuations which are not satisfying γ (final states)).

- Next, we put the 0's at positions corresponding to initial valuations, for that all computations are infinite (looping states).
- The remaining part of the matrix M (not looping states) is of the form mentioned in LEMMA, which enables to determine its positions.

1.1. Example

Let $\boldsymbol{K}$ denote the program

while $x \neq 3$ do

if $x = 1$ then $x := ?$;

including one variable interpreted in finite 3-element universe. Let $\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$ be distribution of probability of variable x for instruction $x := ?$.

If denote by $\boldsymbol{M}$ the subprogram if $x = 1$ then $x := ?$ then it is easy to determine, that M is of the form $\begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (cf. [3])).

Since the determinant of the matrix $I\text{-}I_\gamma \circ M = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is equal to zero, thus we determine the matrix K using mentioned procedure. In the case of the program $\boldsymbol{K}$ we have the following states:

- $v_0(x) = 1$ (not looping),
- $v_1(x) = 2$ (looping),
- $v_2(x) = 3$ (not looping),

thus the matrix K is of the form

$$K = \begin{bmatrix} 0 & 0 & ? \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The remaining part of K denoted by "?" is of the form mentioned in LEMMA, which enables to determine this position.

□

Thus, the analysis of probabilistic algorithms essentially depends on the LEMMA. It is formulated in [3] without a proof. This paper contains two proofs of the LEMMA: algebraic and analytical. The sketch of analytical proof has been known since 1993. The question concerning the elementary proof remains open until 1999 (this proof is contained in [1]).

1.2. LEMMA

The determinant of the matrix:

$$M = \begin{vmatrix} 1-m_{11} & -m_{12} & \dots & -m_{1n} \\ -m_{21} & 1-m_{22} & \dots & -m_{2n} \\ \dots & \dots & \dots & \dots \\ -m_{n1} & -m_{n2} & \dots & 1-m_{nn} \end{vmatrix}, \text{ where}$$

$$m_{ij} \geq 0, \qquad i,j = 1,\dots,n; \qquad (1.1)$$

$$\sum_{j=1}^{n} m_{ij} < 1, \qquad i = 1,\dots,n \qquad (1.2)$$

is positive.

□

Element m_{ij} in the LEMMA means the probability of the transition from the state i to the state j.

2. An algebraic proof of the LEMMA

For more readable notation of the proof, we can formulate the LEMMA as follows:

Let M_n be a $n \times n$ matrix *M*. Denote by $IA(M,n)$ (inductive assumption) the properties denoted above by (1.1) and (1.2), i.e., $m_{ij} \geq 0$ for $i,j = 1,\dots,n$ and $\sum_{j=1}^{n} m_{ij} < 1$ for $i = 1,\dots,n$.

If the matrix M_n has the properties $IA(M,n)$ then $\det M_n > 0$.

Let us denote elements of the matrix M_n by t_{ij}, where $i,j = 1,\dots,n$.

We get the matrix:

$$M_n = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{22} & \dots & t_{2n} \\ \dots & \dots & \dots & \dots \\ t_{n1} & t_{n2} & \dots & t_{nn} \end{bmatrix} = \begin{vmatrix} 1-m_{11} & -m_{12} & \dots & -m_{1n} \\ -m_{21} & 1-m_{22} & \dots & -m_{2n} \\ \dots & \dots & \dots & \dots \\ -m_{n1} & -m_{n2} & \dots & 1-m_{nn} \end{vmatrix},$$

where properties $IA(M,n)$ may be written:

- $\sum\limits_{j=1}^{n} t_{ij} > 0$, for $i = 1,\dots,n$; (because $1 - \sum\limits_{j=1}^{n} m_{ij} > 0$, $i = 1,\dots,n$) (2.1)
- $t_{ii} > 0$, for $i = 1,\dots,n$; (from (1.1) and (1.2)); (2.2)
- $t_{ij} \le 0$, for $i \ne j$, $i,j = 1,\dots,n$; (from (1.1)). (2.3)

The proof proceeds by induction on the number n – the common length of rows and of columns. Let us assume that the thesis of the LEMMA is valid for each $n \times n$ matrix satisfying condition $IA(M,n)$.

The fact is obvious for $n = 1$.

Let us consider a $(n+1)\times(n+1)$ matrix M_{n+1} with following properties $IA(M,n+1)$:

- $\sum\limits_{j=1}^{n+1} t_{ij} > 0$, for $i = 1,\dots,n+1$;
- $t_{ii} > 0$, for $i = 1,\dots,n+1$;
- $t_{ij} \le 0$, for $i \ne j$, $i,j = 1,\dots,n+1$.

After the first step of the Gauss Elimination Method we get the following matrix:

$$M'_{n+1} = \begin{bmatrix} t_{11} & t_{12} & t_{13} & \dots & t_{1j} & \dots & t_{1n+1} \\ 0 & t'_{22} & t'_{23} & \dots & t'_{2j} & \dots & t'_{2n+1} \\ 0 & t'_{32} & t'_{33} & \dots & t'_{3j} & \dots & t'_{3n+1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & t'_{i2} & t'_{i3} & \dots & t'_{ij} & \dots & t'_{in+1} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & t'_{n+12} & t'_{n+13} & \dots & t'_{n+1j} & \dots & t'_{n+1n+1} \end{bmatrix}$$

where $t'_{ij} = t_{ij} - t_{1j} \cdot \frac{t_{i1}}{t_{11}}$ for $i = 2,\ldots,n+1$, $j = 1,\ldots,n+1$.

We shall prove that the $n \times n$ matrix:

$$S = \begin{bmatrix} s_{11} & \ldots & s_{1n} \\ \ldots & \ldots & \ldots \\ s_{n1} & \ldots & s_{nn} \end{bmatrix} = \begin{bmatrix} t'_{22} & t'_{23} & \ldots & t'_{2n+1} \\ t'_{32} & t'_{33} & \ldots & t'_{3n+1} \\ \ldots & \ldots & \ldots & \ldots \\ t'_{n+12} & t'_{n+13} & \ldots & t'_{n+1n+1} \end{bmatrix}$$

satisfies the conditions $IA(M,n)$.

Consider the first row of the matrix *S*.

Observe that $-\frac{t_{i1}}{t_{11}} \geq 0$ for $i = 2,\ldots,n+1$ and let $\alpha_i = -\frac{t_{i1}}{t_{11}}$ $i = 2,\ldots,n+1$.

This means that (2.1) holds for the first row of S:

$$s_{11} + \ldots + s_{1n} = t'_{22} + \ldots + t'_{2n+1} = t_{22} + \ldots + t_{2n+1} + \alpha_2(t_{12} + \ldots + t_{1n+1}) > 0.$$

Similarly, we can prove (2.1) for $i = 2,\ldots,n$.

Now, consider the first element of the diagonal in S:

$$s_{11} = t'_{22} = t_{22} - t_{12} \cdot \frac{t_{21}}{t_{11}}$$

Denote by $\varepsilon_i = 1 - \sum_{j=1}^{n} m_{ij}$, where $i = 1,\ldots,n$. By virtue of (1.2) we have $\varepsilon_i > 0$.

Then

- $t_{11} = 1 - m_{11} = m_{12} + m_{13} + \ldots + m_{1n+1} + \varepsilon_1$,
- $t_{12} = -m_{12}$,
- $t_{21} = -m_{21}$,
- $t_{22} = 1 - m_{22} = m_{21} + m_{23} + \ldots + m_{2n+1} + \varepsilon_2$.

Thus

$$s_{11} = t'_{22} = m_{23} + \ldots + m_{2n+1} + \varepsilon_2 + m_{21}\left(1 - \frac{m_{12}}{m_{12} + m_{13} + \ldots + m_{1n+1} + \varepsilon_1}\right) > 0$$

because

$$1 \geq 1 - \frac{m_{12}}{m_{12} + m_{13} + \ldots + m_{1n+1} + \varepsilon_1} \geq 0$$

This means that the first element of the diagonal of S is positive.

Analogously, we can prove that all elements on the diagonal of S are positive, i.e., $s_{ii} > 0$ for $i = 2,\dots,n$. Thus S satisfies (2.2).

In an analogous way we can show (2.3) (the remaining elements of the matrix are less than or equal to zero, because we subtract positive values from negative values):

$s_{ij} = t'_{i+1\,j+1} \le 0$ for $i \ne j$, $i,j = 1,\dots,n$;

Thus, we can apply the inductive assumption to the above matrix S, being part of the matrix M_{n+1}, because the matrix satisfies conditions $IA(M,n)$.

Thus $\det S > 0$. Therefore,

$\det M_{n+1} = t_{11} \cdot \det S > 0$.

By virtue of the principle of induction, the Lemma is valid for each $n \times n$ matrix satisfying condition $IA(M,n)$.

□

3. The analytical proof of the LEMMA

We start with an illustration for $n = 2$.

Let $M_2 = \begin{bmatrix} 1-m_{11} & -m_{12} \\ -m_{21} & 1-m_{22} \end{bmatrix}$ be a matrix.

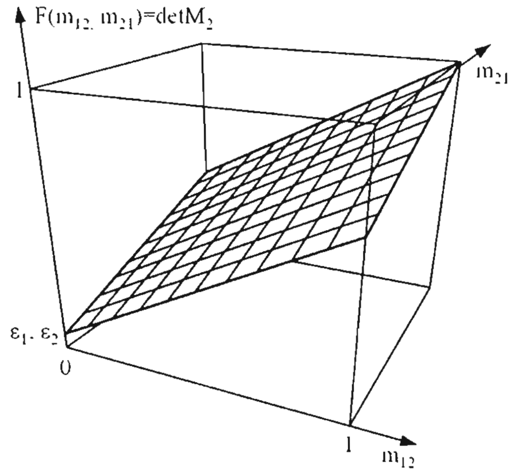
Denote by $\begin{cases} \varepsilon_1 = 1-(m_{11}+m_{12}) \\ \varepsilon_2 = 1-(m_{21}+m_{22}) \end{cases}$. By virtue of (1.2) we have $\varepsilon_1, \varepsilon_2 > 0$.

Consequently, $\det M_2$ can be written as:

$$\det M_2 = \begin{vmatrix} m_{12}+\varepsilon_1 & -m_{12} \\ -m_{21} & m_{21}+\varepsilon_2 \end{vmatrix} = m_{12}\varepsilon_2 + m_{21}\varepsilon_1 + \varepsilon_1\varepsilon_2 .$$

Since $m_{12}, m_{21} \ge 0$, then $\det M_2 > 0$.

We illustrate this fact on the figure below. $\det M_2$ is treated as a function of m_{12}, m_{21}, $F(m_{12}, m_{21}) = \det M_2$ $(0 \le m_{12}, m_{21} < 1)$.



Ill. 1. Graph of function $F(m_{12}, m_{21}) = \det M_2$ for $m_{12}, m_{21} \geq 0$ and $\varepsilon_1, \varepsilon_2 > 0$

It is easy to observe, that F has the minimum at $(0,0)$, moreover, the minimum is positive. Thus all values of F are positive. This means that $\det M_2 > 0$ provided that the elements of M_2 satisfy (1.1) and (1.2)

Let M_n be a $n \times n$ matrix.

The proof proceeds by induction on n. Assume that:

$$\begin{vmatrix} 1-m_{11} & -m_{12} & \dots & -m_{1k} \\ -m_{21} & 1-m_{22} & \dots & -m_{2k} \\ \dots & \dots & \dots & \dots \\ -m_{k1} & -m_{k2} & \dots & 1-m_{kk} \end{vmatrix} > 0$$

for all the determinants of this form (satisfying the assumptions of LEMMA) for $k < n$.

Denote by $\varepsilon_i = 1 - \sum_{j=1}^{n} m_{ij}$, where $i = 1, \dots, n$.

Thus $1 - m_{ii} = m_{i1} + m_{i2} + \dots + m_{ii-1} + m_{ii+1} + \dots + m_{in} + \varepsilon_i$.

By virtue of (1.2) we have $\varepsilon_i > 0$.

Thus the determinant of the matrix M_n can be written as:

$$\det M_n = \begin{vmatrix} m_{12}+\ldots+m_{1n}+\varepsilon_1 & -m_{12} & \ldots & -m_{1n} \\ -m_{21} & m_{21}+m_{23}+\ldots+m_{2n}+\varepsilon_2 & \ldots & -m_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ -m_{n1} & -m_{n2} & \ldots & m_{n1}+\ldots+m_{n,n-1}+\varepsilon_n \end{vmatrix},$$

where $\varepsilon_i > 0$, $i = 1,\ldots,n$.

Let us denote the vector by

$\vec{m} = (\overline{m_{11}},\ldots,m_{1n},m_{21},\overline{m_{22}},\ldots,m_{2n},\ldots,m_{n1},\ldots,\overline{m_{nn}})$, where $\overline{m_{ii}}$ $(i = 1,\ldots,n)$ means, that $\vec{m}$ does not contain the element m_{ii}.

Let us treat the determinant of M_n as the function of $\vec{m}$:

$\det M_n = F(\vec{m}) = F(\overline{m_{11}},\ldots,m_{1n},m_{21},\overline{m_{22}},\ldots,m_{2n},\ldots,m_{n1},\ldots,\overline{m_{nn}})$.

Let us determine the derivative $\dfrac{\partial \det M_n}{\partial m_{12}}$.

Since element m_{12} appears in the first row only, we obtain:

$$\frac{\partial \det M_n}{\partial m_{12}} = \begin{vmatrix} 1 & -1 & \ldots & 0 \\ -m_{21} & m_{21}+m_{23}+\ldots+m_{2n}+\varepsilon_2 & \ldots & -m_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ -m_{n1} & -m_{n2} & \ldots & m_{n1}+\ldots+m_{n,n-1}+\varepsilon_n \end{vmatrix}$$

After adding the first and second columns we obtain:

$$\frac{\partial \det M_n}{\partial m_{12}} = \begin{vmatrix} 1 & 0 & \ldots & 0 \\ -m_{21} & m_{23}+\ldots+m_{2n}+\varepsilon_2 & \ldots & -m_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ -m_{n1} & -(m_{n1}+m_{n2}) & \ldots & m_{n1}+\ldots+m_{n,n-1}+\varepsilon_n \end{vmatrix}$$

After applying the Laplace's development to the first row we obtain:

$$\frac{\partial \det M_n}{\partial m_{12}} = \begin{vmatrix} m_{23}+\ldots+m_{2n}+\varepsilon_2 & -m_{23} & \ldots & -m_{2n} \\ -(m_{31}+m_{32}) & m_{31}+\ldots+m_{3n}+\varepsilon_3 & \ldots & -m_{3n} \\ \ldots & \ldots & \ldots & \ldots \\ -(m_{n1}+m_{n2}) & -m_{n3} & \ldots & (m_{n1}+m_{n2})+\ldots+m_{n,n-1}+\varepsilon_n \end{vmatrix}$$

If we denote:

$$\begin{array}{cccc} m'_{22}=1-(m_{23}+\ldots+m_{2n}+\varepsilon_2) & m'_{23}=m_{23} & \ldots & m'_{2n}=m_{2n} \\ m'_{32}=m_{31}+m_{32} & m'_{33}=1-(m_{31}+\ldots+m_{3n}+\varepsilon_3) & \ldots & m'_{3n}=m_{3n} \\ \ldots & \ldots & \ldots & \ldots \\ m'_{n2}=m_{n1}+m_{n2} & m'_{n3}=m_{n3} & \ldots & m'_{nn}=1-(m_{n1}+\ldots+m_{n,n-1}+\varepsilon_n) \end{array}$$

then we can write:

$$\frac{\partial \det M_n}{\partial m_{12}} = \begin{vmatrix} 1-m'_{22} & -m'_{23} & \ldots & -m'_{2n} \\ -m'_{32} & 1-m'_{33} & \ldots & -m'_{3n} \\ \ldots & \ldots & \ldots & \ldots \\ -m'_{n2} & -m'_{n3} & \ldots & 1-m'_{nn} \end{vmatrix}.$$

It's easy to verify that this determinant satisfies the assumptions of LEMMA, e.g., for $i=2$ we have:

$$m'_{22}+m'_{23}+\ldots+m'_{2n}=1-(m_{23}+\ldots+m_{2n}+\varepsilon_2)+m_{23}+\ldots+m_{2n}=1-\varepsilon_2<1.$$

Analogously, we can show this fact for $i=3,\ldots,n$. In this way we have obtained the $(n-1)\times(n-1)$ determinant of a $(n-1)\times(n-1)$ matrix being in the form investigated in Lemma, therefore by the inductive assumption we obtain $\frac{\partial \det M_n}{\partial m_{12}}>0$.

Analogously, we can prove that:

$\frac{\partial \det M_n}{\partial m_{ij}}>0$, where $i,j=1,\ldots,n$, and $i\neq j$.

Similarly to the case of two variables we shall argue that the function $F(\vec{m})$ has its minimum value at the point $(0,\ldots,0)$, i.e. $F(\vec{m})>F(0,\ldots,0)$.

By the definition of F we have:

$F(0,\ldots,0) = \varepsilon_1 \cdot \ldots \cdot \varepsilon_n$;

Now, let us consider Taylor's formula for the function F around the point $(0,\ldots,0)$ for $n=1$:

$$F(\vec{m}) = F(0,\ldots,0) + \sum_{\substack{i,j=1 \\ i\neq j}}^{n} \frac{\partial F(\theta\vec{m})}{\partial m_{ij}} \cdot m_{ij} = \varepsilon_1 \cdot \ldots \cdot \varepsilon_n + \sum_{\substack{i,j=1 \\ i\neq j}}^{n} \frac{\partial F(\theta\vec{m})}{\partial m_{ij}} \cdot m_{ij}, \quad (3.1)$$

Since $\sum_{j=1}^{n} m_{ij} < 1$, $i = 1,\ldots,n$, and $0 < \theta < 1$, then $\sum_{j=1}^{n} \theta m_{ij} < 1$, for $i = 1,\ldots,n$.

Since $\frac{\partial \det M_n}{\partial m_{ij}} = \frac{\partial F(\vec{m})}{\partial m_{ij}} > 0$, $i,j = 1,\ldots,n$ and $i \neq j$, we have that $\frac{\partial F(\theta\vec{m})}{\partial m_{ij}} > 0$, where $i,j = 1,\ldots,n$ and $i \neq j$. Then, according to (3.1):

$F(\vec{m}) > \varepsilon_1 \cdot \ldots \cdot \varepsilon_n > 0$.

Since at the point $(0,\ldots,0)$ the function has the minimum (in its domain), with positive value at the point $(0,\ldots,0)$, then for all $\vec{m}$ belonging to the domain of the function F $F(\vec{m}) > 0$ and therefore $\det M_n = F(\vec{m}) > 0$.

Thus, we have proved the LEMMA for the $n \times n$ matrix M_n, i.e., $\det M_n > 0$.

□

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WYZNACZANIE PRAWDOPODOBIEŃSTW PRZEJŚĆ W ALGORYTMACH PROBABILISTYCZNYCH

Streszczenie: Poniższa praca zawiera dwa dowody lematu opublikowanego w pracy [3] bez dowodu. Algebraiczny fakt rozważany w lemacie jest punktem wyjściowym dla metody wyznaczania prawdopodobieństw przejść w iteracyjnych algorytmach probabilistycznych interpretowanych w skończonych dziedzinach. Dotyczy on niezerowości wyznacznika macierzy o pewnej specyficznej postaci.

Słowa kluczowe: algorytm probabilistyczny, program probabilistyczny, stan końcowy, stan pętlący, stan nie pętlący