VERSITA
ADVANCES IN MANUFACTURING SCIENCE AND TECHNOLOGY Vol. 36, No. 4, 2012
DOI: 10.2478v10264-012-0027-y

A CRITERION DETERMINING THE NUMBER OF THERMAL SENSORS IN A SYSTEM COMPENSATING THERMAL DEFORMATIONS OF A CNC MACHINE FEED SCREW

Mirosław Pajor, Jacek Zapłata

S u m m a r y

The article presents a criterion supporting the selection of the proper number of sensors in a system compensating the effect of thermal deformation of the feed screw on the positioning accuracy of the CNC axis. Moreover, it presents an experimental verification of the criterion and a calculation example along with the tips allowing for the practical use of the criterion.

Key words: thermal error, determining the number of sensors, feed screw, CNC machine tool

Kryterium określające liczbę czujników temperatury w systemie kompensacji odkształceń cieplnych śruby pociągowej obrabiarki CNC

S t r e s z c z e n i e

W artykule przedstawiono opracowane kryterium wspomagające dobór liczby czujników w układzie kompensującym wpływ odkształceń cieplnych śruby pociągowej na dokładność pozycjonowania osi CNC. Prowadzono weryfikację doświadczalną dokładności pozycjonowania dla opracowanego kryterium. Omówiono przykład obliczeń wraz ze wskazówkami pozwalającymi na praktyczne użycie opracowanego kryterium.

Słowa kluczowe: błąd cieplny pozycjonowania, określenie liczby czujników, śruba toczna, obrabiarka CNC

1. Introduction

In addition to the efficiency and geometric accuracy of machining, the costs incurred for the purchase and operation of machine tools are important for its manufacturers and users. In response to the need for cost reduction, researchers seek ways to replace expensive design solutions with new and cheaper ones,

Address: Prof. Mirosław PAJOR, Jacek ZAPŁATA, M.Sc. Eng., West Pomeranian University of Technology in Szczecin, Faculty of Mechanical Engineering, Institute of Manufacturing Engineering, Piastów 19, 70-310 Szczecin, e-mail: Jacek.Zaplata@zut.edu.pl, Miroslaw.Pajor@zut.edu.pl

while maintaining the capability of a machine tool to produce workpieces in a precise way.

According to this strategy, the Centre for Mechatronics of the West Pomeranian University of Technology developed a module for the compensation of heat deformations of the CNC feed axis screw [1]. This solution is subject to patent proceedings. It was awarded the Golden Medal of the Poznań International Fair 2012: Innovation – Technology – Machines.

The developed system eliminates the negative impact of feed screw heat deformations on the accuracy of positioning of feed axis of CNC machine tools. Compensation takes place by means of a module implemented in the control system, which collects information from the temperature sensors located along the feed screw. The presented solution is designed for feed axes with a rotary encoders. It is characterised by a low cost compared to the axes positioned with commercially available magnetic or optical linear encoders.

The use of the developed solution requires a correct solution to the problem of temperature sensors' location in the feed screw. For example, to select the optimal places for the location of sensors in a face lathe a series of experiments allowing to diagnose the main ways of thermal deformations of the machine was performed, and then a selection of appropriate places on the basis of the engineering knowledge have been made [2]. In the paper [3], the authors placed a large number of sensors recording temperature during the measurement of thermal deformations of the machine, and then applied statistical methods for the selection of the optimal ones. To eliminate the time-consuming experiment, the papers [4, 5] proposed a development of an equation of machine heat conduction in a series, and the measurement of temperatures only in the points relevant for the significant series components. The derivation presented in the papers [4, 5] is based on the thermal conductivity equation developed in means of finite element method (FEM), in which the immovability of the heat source and the insignificance of the convection effect was assumed.

In the case of heat deformations of feed screws [6-8], the researchers have not provided a criterion informing how to distribute sensors correctly in the feed screw. In this paper, to solve aforementioned problem, an original criterion supporting the selection of the proper number of sensors was presented.

2. Derivation of the criterion

2.1. Methodology

Analysis of the structure of a conventional feed axis, i.e. using line guides and the ball screw as a gear (Fig. 1a), with locating and non-locating bearing arrangement, allows for fast identification of the main heat sources. These are angular bearing units located in the bearing housings and ball bearings between the feed screw and the nut.

a) b)

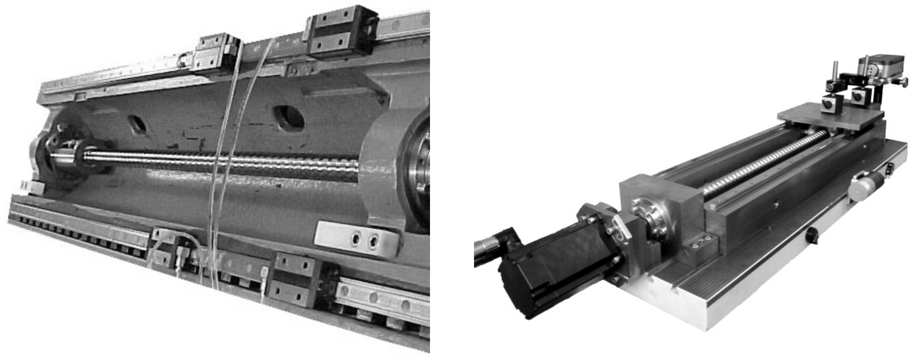

Fig. 1. Feed axis of the AVIA 760 CNC (a), the testing system (b)

Adopting the following assumptions in relation to the issue describing the temperature distribution of the feed screw is as follows:

- the issue has a one-dimensional nature (the ratio of length to the diameter is large enough to make this assumption reasonable),
- the convection coefficient is constant,
- the sectional area of the feed screw is constant,
- the amount of heat generated as a result of friction is constant in time,
- the feed screw is anchored axially at x = 0 – locating bearing unit, the second bearing unit is non-locating,
- the ambient temperature is constant,

it is easy to derive equations describing this distribution. This derivation is presented in Section 2.2.

2.2. Heat balance equations

The balance of heat flux for a situation described, with the above assumptions, is shown in Fig. 2.

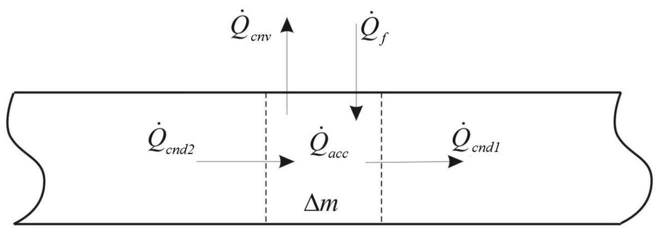

Fig. 2. Heath flow balance

According to the balance of heat flux (Fig. 2), we can write the system of equations:

$$\dot{Q}_{acc} = \dot{Q}_{cnd1} - \dot{Q}_{cnd2} - \dot{Q}_{cnv} + \dot{Q}_f \quad (1)$$

$$\dot{Q}_{acc} = Cp \cdot \Delta m \cdot \frac{\Delta\theta}{\Delta t} \quad (2)$$

$$\dot{Q}_{cnd} = -\lambda \cdot A \frac{\Delta\theta}{\Delta x} \quad (3)$$

$$\dot{Q}_{cnv} = h \cdot P \cdot \Delta x \cdot \theta \quad (4)$$

$$\dot{Q}_f = \dot{q}_f \cdot \Delta x = \begin{cases} \frac{\Delta x}{L_{nm}} \dot{Q}_{fns}, & x \subset area\, of\ nut\, movement \\ \frac{\Delta x}{L_{nm}} \dot{Q}_{fb}, & x \subset area\ of\ bearing\ placement \\ 0, & x \subset remaining\ area \end{cases} \quad (5)$$

where: $\dot{Q}_{acc}$ – heat flux accumulated in mass Δm, W; $\dot{Q}_{cnv}$ – heat flux given off by convection, W; $\dot{Q}_f$ – heat flux provided by friction, W; $\dot{Q}_{cnd1}, \dot{Q}_{cnd2}$ – heat flux conducted in steel, W; $\dot{Q}_{fsn}$ – heat flux generated in the bearings: screw-nut, W; $\dot{Q}_{gb}$ – heat flux generated in a bearing, W; Δm – infinitesimal mass., *Cp* – steel heat capacity, J/kg/°C; θ – difference between the screw temperature and the ambient temperature, °C; λ – conductivity coefficient of steel, W/m^2/°C; *h* – convection heat transfer coefficient at the surface of the feed screw, W/m/°C; *P* – feed screw parameter, m; *A* – sectional area of the feed screw, m^2; ρ – steel density, kg/m^3; L_{nm} – movement range of the nut.

The system of equations (1)-(5) allows to determine the relationship between the temperature, time and x coordinate (along the feed screw axis) in the form of a non-linear partial differential equation (6) [9].

$$\rho\, A\, Cp \frac{\partial\theta}{\partial t} = A\, \lambda \frac{\partial^2\theta}{\partial x^2} \quad - P\, h\, \theta + q_f \quad (6)$$

The equation is easily solved with the application of the numerical methods, using commercially available computational programmes, e.g.: Mathematica, Matlab.

2.3. Estimation of thermal elongation

Heat deformation of the feed screw $\delta(L)$ in the function of the movable table position can be determined by the following formula:

$$\delta(L) = \alpha \int_0^L \theta(x) \mathrm{d}x \tag{7}$$

where: $\delta(L)$ – thermal deformation in the function of distance from the locating bearing, µm, α – thermal elongation coefficient, µm/m, L – current position of the feed axis (position of the moving table), m, x – a coordinate parallel to the feed screw axis, m.

2.4. Criterion

Uncertainty of determining the thermal deformation is dependent on both measurement uncertainties $\Delta\theta(x)$ and ΔL, accuracy of estimation of the thermal expansion coefficient $\Delta\alpha$, and the assumed form of the function approximating the temperature distribution.

$$\theta(x) = \hat{\theta}(x) + E(\hat{\theta}(x)) \tag{8}$$

where: $\theta(x)$ – the actual function of thermal distribution, $\hat{\theta}(x)$ – approximation of thermal distribution functions, $E(\hat{\theta}(x))$ – approximation error.

Since the position can be measured with high accuracy by means of a rotary encoder, the component related to it is irrelevant. The amount α should be determined experimentally [10], with an accuracy ensuring proper operation of the system. For the proper designation of the number of sensors needed for the proper operation of the system for the compensation of thermal deformation, the maximum error resulting from the form of the function approximating the excess of temperature is of great importance.

In the case when an approximation $\hat{\theta}(L)$ is used in the form of linear spline functions, the integral of this approximation is called the Newton-Cotes quadrature [11], and the maximum error resulting from this numerical integration method provides the relationship:

$$E(\int \theta(x)\mathrm{d}x) = -n \cdot \frac{h^3}{12} \max_{\xi \in (0 \div L)} \left(\frac{\partial^2 \theta(x)}{\partial x^2} \bigg|_{x=\zeta} \right) \tag{9}$$

where: h – range length, m; n – number of ranges.

The impact of the uncertainty of numeric integration on the accuracy of heat deformation estimation is given in the relationship:

$$\Delta\delta_{\int \theta(x)dx} = \alpha \cdot E(\int \theta(x)\mathrm{d}x) \tag{10}$$

After the conversion of the equation (6) and neglecting the component connected with the dynamics of temperature change, which for the system in question (Tab. 1) is small enough to make such a negligence, we obtain the following relationship:

$$\frac{\partial^2 \theta(x)}{\partial x^2} = \frac{P\,h}{A\,\lambda}\,\theta(x) - \frac{q_f}{A\,\lambda} \tag{11}$$

Equations (9), (10) and (11) form a criterion which allows to determine the required number of necessary, equidistantly distributed sensors so that the maximum uncertainty of screw heat deformation resulting from the applied approximation of the temperature function was at the level of other components of the uncertainty budget (i. e. statistical errors).

3. Criterion verification

3.1. System parameters

Parameters of the system which were used for the verification of the proposed criterion are presented in the Table below:

Table 1. Geometric parameters of the feed screw and physical values [12, 13]

Feed screw: VNB 2046-2962P	
Nominal diameter and stroke	40 x 16
Total length	1210 mm
Length „from a bearing to a bearing”	1082 mm
Planned maximum speed of movement	v = 50 mm/s
Approximate load on the bearing unit	F = 1600 N
Coefficient of thermal conductivity of the screw material	λ = 60 W/m/°C
Heat transfer coefficient on the screw surface	h = 6 W/m^2/°C

Thermal capacity coefficient of the screw material	*Cp* = 460 KJ/kg/°C
Density of the screw material	ρ = 7850 kg/m^3
Kinematic viscosity of the lubricant in bearings	υ = 100 mm/s^2

3.2. Estimation of the value of heat fluxes

The heat flux generated as a result of the bearing friction can be calculated from the formula:

$$\dot{Q}_f = \frac{2\pi}{60} M_f \cdot n \tag{12}$$

where: M_f – frictional moment, N·m;, n – rotational speed, r/min.

The value of frictional moment is dependent not only on the geometric parameters of the bearing but also on the rotational speed of the bearing, the forces putting strain on the bearing or the lubricant viscosity. The value of this moment for angular bearings should be estimated on the basis of the Palmgren's experimental model [14-16], applying the following formulas:

$$M_t = M_o + M_1 \tag{13}$$

$$M_1 = \tfrac{1}{2}\mu_1 \cdot f_1 \cdot F \cdot d_m \tag{14}$$

$$M_o = 10^{-7} f_o (\upsilon n)^{\frac{2}{3}} d_m \quad \text{dla} \quad \upsilon n > 2000 \tag{15}$$

$$M_o = 160 \cdot 10^{-7} f_o\, d_m \quad \text{dla} \quad \upsilon n < 2000 \tag{16}$$

where: M_1 – frictional moment dependent on the load, N·m; μ_1 – friction coefficient dependent on the bearing load, unitless, d_m – pitch diameter of the bearing, mm; M_o – frictional moment dependent on the viscosity, N·mm; f_o – a coefficient dependent on the type of bearing and the lubrication type, unitless, υ – kinematic viscosity of the lubricant, mm/s^2, n – rotational speed, r/min.

Using the presented dependencies (13)-(16) we obtain an approximate value of the frictional moment and the heat flux generated in a single bearing node:

$$M_{fb} \approx 30\ \text{N} \cdot \text{mm} \tag{17}$$

$$\dot{Q}_{fb} \approx 1.2\ \text{W} \tag{18}$$

Bearing manufacturers often offer ready spreadsheets on their websites, which enable to calculate the frictional moments of bearings [17-19].

To estimate the heat generated in the screw-nut bearing connection, it is helpful to use the manufacturer's data. The friction torque range read from the computer aided design (CAD) drawing of the gear AVIA VNB 2046-2962P is equal to:

$$M_{fns} \approx 0{,}66 \div 1{,}23\ \text{Nm} \tag{19}$$

$$\dot{Q}_{fns} \approx 13\ \text{W} \tag{20}$$

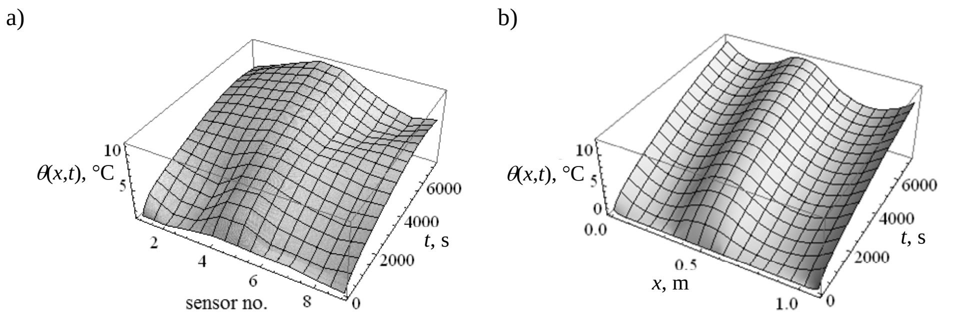

Fig. 3. Temperatures measured by the sensors during the reciprocal movement of the table of the feed axis, movement speed: 50 mm/s, range: 150-300 mm (a) and the temperature field estimated analytically on the basis of the data presented in Table 1 (b) – calculations by means of Mathematica

It has been assumed a priori that half of the heat generated both in the nut and the bearing nodes flows to the feed screw, whereas the second half is transferred to the nut or to the housing respectively. The frictional moment in the feed screw has been adopted at the level of the lower range limit (19).

The comparison of Figures 3a and 3b allows us to state that the proposed method is sufficiently accurate to roughly estimate the temperature fields of the feed screw.

3.3. Calculation of the criterion for the modelled temperature distribution

The system of equations presented in section 2.3, solved for the modelled temperature distribution allows to calculate the maximum uncertainty of feed screw thermal expansion estimation with a given temperature distribution depending on the number of the applied sensors. The obtained results are presented in Fig. 4.

The maximum registered positioning deviations for a working system of heat deformation compensation, decreased by the geometric error component are presented in Fig. 5. The geometric error has been removed by a numerical procedure which was deducting the values measured during the first measurement from the following series. The registered accuracies of positioning, depending on the number of sensors applied for compensation, are 26,9 μm for 2

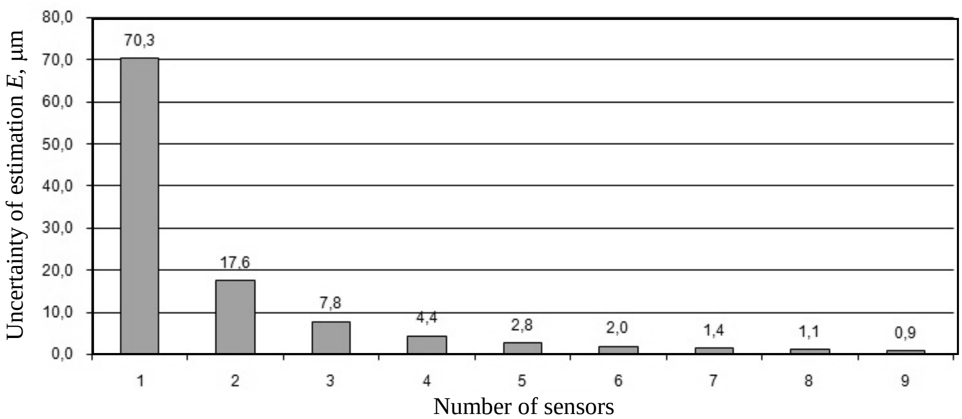

Fig. 4. The maximum uncertainty of feed screw thermal expansion estimation on the basis of the modelled temperature distribution

a) b)

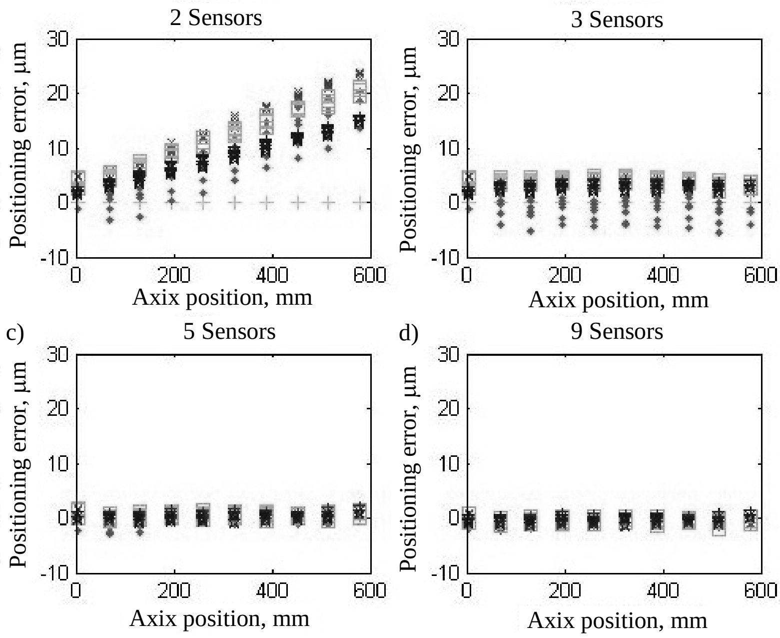

Fig. 5. Deviations of positioning for different temperatures: a) 2 sensor, b) 3 sensors, c) 5 sensors, d) 9 sensors

sensors, 10,4 µm for 3 sensors, 4,6 µm for 5 sensors, 3,3 µm for 9 sensors respectively. The applied number of sensors: 2, 3, 5, and 9, resulted from the physical feasibility of experiment implementation. It is necessary to emphasize that the provided data include also a random component of the positioning error. The obtained results corroborate the correctness of the proposed criterion.

3.4. Calculation of the criterion for the worst, theoretically possible temperature distribution

In the next part of the paper we adopted a simplified heat flux variant from the moving nut to the feed screw. It assumes that the heat transfer occurs in a single fixed point. This approach provides the most adverse temperature distribution, increasing the number of sensors at the same time. This distribution is described with the equation:

$$\theta = \frac{Q_f}{A \cdot \lambda \cdot m \cdot \tanh(m \cdot L)} \quad (21),$$

where in:

$$m = \sqrt{\frac{P \cdot \alpha}{A \cdot \lambda}} \tag{22}$$

The uncertainty of positioning, depending on the number of sensors, for such a temperature distribution is shown in the figure below.

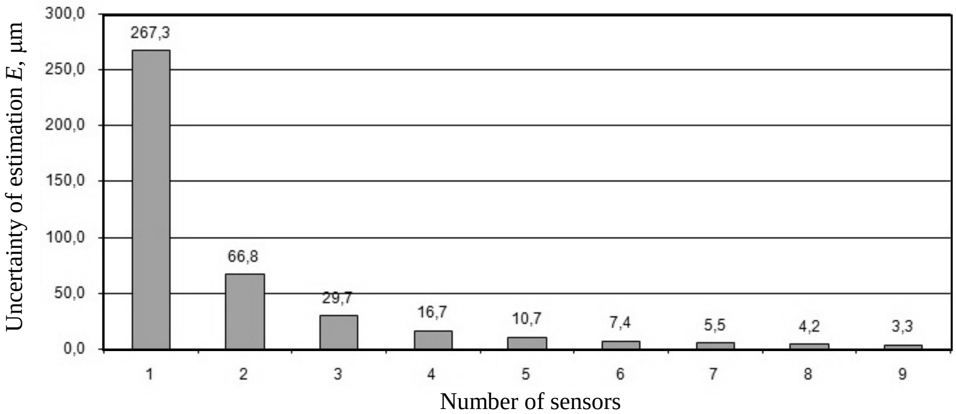

Fig. 6. The maximum uncertainty of thermal expansion estimated of the feed screw, for a temperature distribution possible only theoretically

Figure 6 shows that obtaining the uncertainty estimated under the level of 5 μm (linear encoder accuracy level) [20-22] requires the placement of 8-9 sensors in the feed screw. It is necessary to remember that a given criterion is based on an estimation, therefore, it is always necessary to adopt a certain safety factor, selecting a larger number of sensors than it would result from the estimate itself.

4. Conclusions

The paper presents and verifies a criterion supporting the selection of an appropriate number of sensors to the system of compensation of feed screw thermal deformations depending on the system parameters.

It is postulated that the number of the applied sensors should depend on the feed screw length, frictional moment in the nut and in the bearings, as well as the maximum speed of system operation in accordance with the models allowing for the estimation of the temperature distribution fields and determining numerical errors. It has been shown that the longer the feed screw is, and the larger the system preload is, the more sensors are necessary for the proper operation of the deformation compensation system.

It should be particularly emphasised that the increase in the number of sensors causes a decrease in the numerical errors, not causing any reduction in the random positioning errors. Thus, it is not advisable to increase the number of sensors in an unlimited way.

Acknowledgements

The work was performed within the framework of the grant "Research and Correction of Influence of Machine Tool Thermal Properties on the Accuracy of Forming Machined Workpieces" N N502 280737 sponsored by the Polish National Centre for Research and Development.

References

[1] M. PAJOR, J. ZAPŁATA: Compensation of thermal deformations of the feed screw in a CNC machine tool. *Advances in Manufacturing Science and Technology*, **45**(2011)4, 9-17.
[2] J. YANG, J. YAUN, J. NI: Thermal error mode analysis and robust modeling for error compensation on a CNC turning center. *Inter. Journal of Machine Tools & Manufacture*, **39**(1999), 1367-1381
[3] J-H. LEE, S-H. YANG: Statistical optimization and assessment of a thermal error model for CNC machine tools. *Inter. Journal of Machine Tools & Manufacture*, **42**(2002), 147-155.
[4] J. ZHU, J. NI, A. J. SHIH: Robust machine tool thermal error modelling through thermal mode concept. *Journal of Manufacturing Science and Engineering*, **130**(2008).
[5] J. ZHU: Robust thermal error modelling and compensation for CNC machine tools, Dissertation, The University of Michigan 2008.
[6] C-H. WU, Y-T. KUNG: Thermal analysis for the feed drive system of a CNC machine center. *Inter. Journal of Machine Tools & Manufacture*, **43** (2003), 1521-1528.
[7] T. KODERA, et al.: Real-time estimation of ball-screw thermal elongation based upon temperature distribution of ball-screw. *JSME Inter. Journal, Series C*, **47**(2004), 1175-1181.
[8] S.K. KIM, D. W. CHO: Real-time estimation of temperature distribution in a ball-screw system. *Int. Journal. Math. Tools Manufact*., **37**(1997), 451-464.
[9] HOLMAN J. P.: Heat Transfer, sixth edition. McGraw-Hill Book Company, Singapore 1986.
[10] J. KULESZA, et al.: Pomiary cieplne. WNT, Warszawa 1995.
[11] T. RATAJCZAK: Metody Numeryczne. Wydawnictwo Politechniki Gdańskiej, Gdańsk 2007.
[12] K. RAZNJEVIC: Tablice cieplne z wykresami: dane liczbowe w układzie technicznym i międzynarodowym. WNT, Warszawa 1966.
[13] F. KREITH: Principles of heat transfer. PWS, Boston 1997.
[14] T. HARRIS: Rolling Bearing Analysis. 2nd ed. A Wiley-Interscience, New York 1984.

[15] A. PALMGREN: Ball and Roller Bearing Eng., 3rd ed. Burbank, Philadelphia 1959.
[16] J. BIEDRZYCKI, et al.: Konstrukcja przyrządów i urządzeń precyzyjnych. WNT, Warszawa 1996.
[17] Materiały firmy skf: www.skf.com
[18] Materiały firmy cx: www.cx.pl
[19] Materiały firmy nks: www.nsk.com
[20] Materiały firmy renishaw: www.renishaw.pl
[21] Materiały firmy heidenhain: www.heidenhain.pl
[22] J. W. Li, et al.: Thermal-error modelling for complex physical systems: the-state-of-arts review. *Inter. Journal Advances Manuf. Technol*., **42**(2009), 168-179.

Received in March 2012