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## MODEL FOR BLADE DIAGNOSIS IN A WORKING ROTOR MACHINE EMPLOYING THE METHOD OF VIRTUAL ELIMINATION OF STOCHASTIC ENVIRONMENT

This paper presents the bases of a new method of monitoring technical condition of turbomachine blades during their operation. The method utilizes diagnostic models such as a quotient of amplitude amplification and a phase shift of a diagnostic signal  $y(t)$  which is a result of blade operation as well as a signal  $x(t)$  of blade environment while a blade tip approaches a sensor, amplitude amplification and phase shift of these signals while the blade tip moves away from the sensor.

The adopted diagnostic models indirectly take into account the existing environment of a blade, represented by the signal  $x(t)$ , without the need to measure it [13, 15]. Thus, the model is sensitive to the changes in technical condition of blades and practically insensitive to a change in environment. The suggested method may prove very important in diagnostics of rotor blades during turbomachines operation (compressors, turbines etc.).

### 1. Introduction

A turbomachine is a complex technical object (it consists of many elements: blades, bearings, shields) that operates in complex environment (the machine is affected by variable: loads, pressure, temperature, vibration etc.). The vital components of the machine, apart from bearings, are blades. Although their construction is very simple, they are difficult to deal with (high risk of evaluation of ability) in terms of their use and operation. Practice proves that a break of one blade only (out of several dozen or even several hundred) almost always causes a serious breakdown of the whole turbomachine (axial-flow compressor, turbine), which is often very expensive. This

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leads to a great interest in methods of current diagnosing (monitoring) blades technical condition during their operation.

Nowadays multiple methods of diagnostics of blades technical condition during turbomachine operation are in use. Diagnostic tests of these methods are based on “non-contact” measurement of value of current movements of the blade tip during short periods of time while the blade tip is in the area of a specialized sensor. Many “non-contact” measuring systems have been developed and introduced, such as these commonly known and used, produced by: Hood, Aqilis, Pratt&Whitney (USA), Rolls Royce (UK), Turbocharges (Switzerland), MTU (Germany) as well as Russian, Chinese and Indian manufacturers. [2, 4÷12, 20, 25].

There is also a number of Polish non-contact measuring systems that are known and used, especially those designed, produced and introduced by the Air Force Institute of Technology (AFIT) in Warsaw.

Among the systems of non-contact measurement of blade movement by AFIT there are:

- blade fracture indicator: SPŁ – 29 [17, 18, 23, 24];
- blade excessive vibration indicator: SNDŁ – 2b [17, 18, 23, 24];
- microwave sensors: MUH, PIT [17, 18, 23, 24].

These systems work successfully in particular operational technical objects (SO-3 engines).

Diagnostic inference used so far in the methods of evaluation of blades technical condition is based solely on modification of signals that are measured during diagnostic tests and that are the result of blade operation without sufficient (according to authors) consideration given to signals (of considerable power) of blade variable environment.

Measurement of blade environment signals during turbomachine operation is difficult and very often even impossible, thus this measurement is not sufficiently considered in blade diagnostics.

Therefore, it can be stated that the methods that have been used so far for evaluation of blades technical condition during turbomachines operation do not fully meet the basic principle of technical diagnostics that demands carrying out a test and technical condition analysis of an object in environment (PN-90/N-04002), thus these methods are neither accurate nor reliable enough.

Therefore, the need arises to develop a new method of diagnostics of blade technical condition during turbomachine operation with taking environment into account, but (if possible) without the need of using the measurement of environment signals that are unavailable and often difficult to measure. This problem can be solved by a blade diagnostics method based on

a special diagnostic model that allows eliminating real existing environment of the blade with the help of special methods.

## 2. Blade operation in stochastic environment

Fig. 1 shows construction and functioning of a blade during its operation in variable environment [17, 18, 21].

The blade consists of two parts: the first one is the operating part, also called the profile part – 1 (blade) and the second one is the fastening part – 2 (of a lock). The operating part comprises also the trailing edge – 3, the edge of attack – 4, the blade tip – 5, the blade ridge – 6, and the trough – 7.

Rotor blades are fastened in a shield with the use of a trapezoidal lock, also called “the dovetail lock”. The slots between blades locks and shield’s notches are filled with a polyester resin. The blades are covered with an epoxy enamel in order to increase their corrosion resistance.

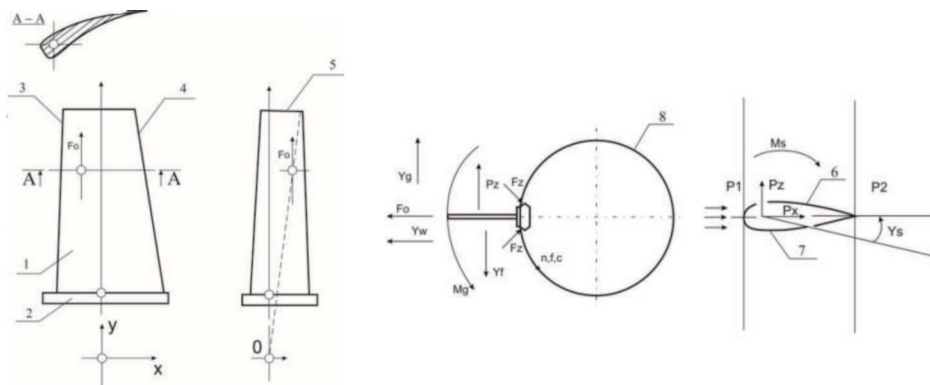


Fig. 1. Turbomachine blade in variable environment.

1 – blade; 2 – blade lock; 3 – trailing edge; 4 – edge of attack; 5 – blade tip; 6 – blade ridge; 7 – blade trough; 8 – rotor drum;  $F_0$  – centrifugal force;  $F_z$  – lock grip force;  $n$  – rotational velocity;  $P_z$  – blade aerodynamic lift;  $P_x$  – resisting force;  $M_s$  – torque moment;  $M_g$  – bending moment;  $P_1$  – gas pressure on rotor rim input;  $P_2$  – gas pressure on rotor rim output;  $Y_g$  – blade deflection;  $Y_s$  – a blade torsion angle;  $Y_w$  – blade longitudinal displacement;  $Y_f$  – various vibration signals (bending, torsional, longitudinal);  $Y_c$  – thermal deformation;  $f$  – vibration signal;  $f_{ob}$  – casing vibration signal;  $c$  – temperature distribution signal

During its use, the blade technical condition changes and, with time, various defects appear (such as fractures, deformations, pits, breaks of blade parts). [21]

Fig. 1 and Fig. 2 show that a blade (of a compressor, a turbine) is a technical object with a complex principle of operation, which results in a multidimensional state of blade deformation.

These deformations originate from environmental factors, and are caused by various interactions such as:

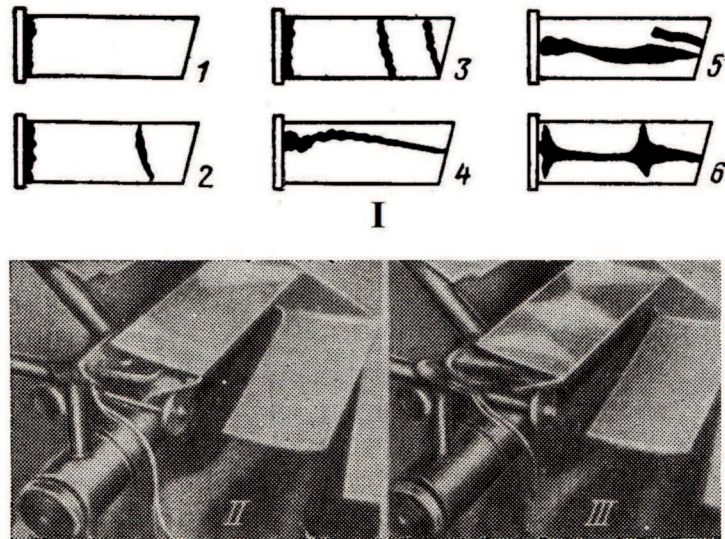


Fig. 2. Forms of vibration and lines indicating traces of vibration nodes  
 Chart I: 1,2,3 – first, second and third form of bending vibration; 4 – first kind torsional vibration; 5 – second kind torsional vibration; 6 – combined bending-torsional vibration.  
 Chart II: photograph of node traces in second form of bending vibration.  
 Chart III: photograph of node traces in third form of bending vibration

- centrifugal force loads,  $F_0$ , that depend on rotational velocity and produce longitudinal and bending strains (Fig.1) –  $Y_w, Y_g$ ;
- gas-dynamic loads,  $P_z$  and  $P_x$ , from the stream of air (gas) depending also on flight velocity and altitude (Fig.1)  $Y_s$  (if a turbomachine is a compressor or an engine turbine);
- loads  $Y_g, Y_s$  due to a curvilinear flight path (Fig. 1);
- dynamic loads originating from mechanical vibration (especially in the resonant range) due to pressure pulsation,  $P_1$  and  $P_2$ , rotational oscillations etc. (Fig. 2)  $Y_f$ ;
- blade and casing vibrations  $f, f_{ob}$  (Fig. 1) and thus  $Y_g, Y_s$ ;
- heat loads  $c$  due to uneven temperature distribution (Fig. 2 – complex strain, eg. I – 6)  $Y_c$ .

From a synthetic point of view, the state of blade operation in an environment can be described by the signal of blade tip displacement  $y(t)$  which is the function of the following signals:  $Y_w, Y_g, Y_s, Y_f, Y_c$ , (Fig. 1 and Fig. 2):

$$y(t) = f(Y_w, Y_g, Y_s, Y_f, Y_c) \quad (1)$$

and the environment signal,  $x(t)$  which is the function of the following signals:  $n, F_0, P_z, P_x, P_1, P_2, f, f_{ob}, c$  (Fig. 1 and Fig. 2):

$$x(t) = f(n, F_0, P_z, P_x, P_1, P_2, f, f_{ob}, c) \quad (2)$$

Blade technical condition  $S_T(\theta)$ , in accordance with diagnostics principles, results from relations between the operation signal  $y(t)$  and the environmental signal  $x(t)$  at the moment of current diagnosing  $\theta_1$  and initial diagnosing  $\theta_0$ . Therefore, the following relationship may be noted:

$$S_T(\theta) = f(y(t)_{\theta_0}, x(t)_{\theta_0}, y(t)_{\theta_1}, x(t)_{\theta_1}, \theta, t) \quad (3)$$

Practice proved many times that there are real difficulties in the process of signals measurement, both for  $y(t)$ , and  $x(t)$  (even to a greater extent), and thus also in the evaluation of blade technical condition during machine operation [13, 15, 17, 18].

### 3. Theoretical bases of the blade diagnostics method during turbomachine operation

A problem of blade diagnostics during turbomachine operation is very complex in its nature because, in order to complete the process of blade diagnostics, only two signals can be used: the first one is the measurable but interfered signal  $y(t)$ , and the second one is the environment signal  $x(t)$  which is practically immeasurable (except of signal  $n$ ). Fig. 3 [23, 24] shows an exemplary course of the signal  $y(t)$ . Fig. 4 [3] shows a course of the multiple signals which altogether constitute the resultant signal  $x(t)$ .

The figure shows the change of amplitude vibration of blade tips (from 3 to 20) caused by the change of rotation velocity of the engine.

Initially, it is assumed that signals  $x(t)$  and  $y(t)$  are temporal, stochastic and interfered. In this situation, it seems reasonable to switch from the time domain "t" of the signals  $x(t)$  and  $y(t)$  to the time domain "τ" of correlation function  $R_{xx}(\tau)$ ,  $R_{yy}(\tau)$  and  $R_{xy}(\tau)$  [13, 15, 17, 18, 19, 21].

The results of such an approach are:

- noise suppression of signals and the possibility to amplify them,
- the possibility to express the signals  $R_{xx}(\tau)$ ,  $R_{yy}(\tau)$  and  $R_{xy}(\tau)$  as simple analytic functions, which allows further conversion of these functions into new (of specific characteristics) functions in frequency domain ( $\omega$ ), which are density functions of singular power of signals  $S_{xx}(\omega)$  and  $S_{yy}(\omega)$  and reciprocal  $S_{xy}(\omega)$ .

The expressions of functions  $x(t)$  and  $y(t)$  as  $S_{xx}(\omega)$ ,  $S_{yy}(\omega)$  and  $S_{xy}(\omega)$  allows, in a very simple manner, to take into account relations between diagnostic signals  $y(t)$  and environment signals  $x(t)$  (Fig. 3 and Fig. 4).

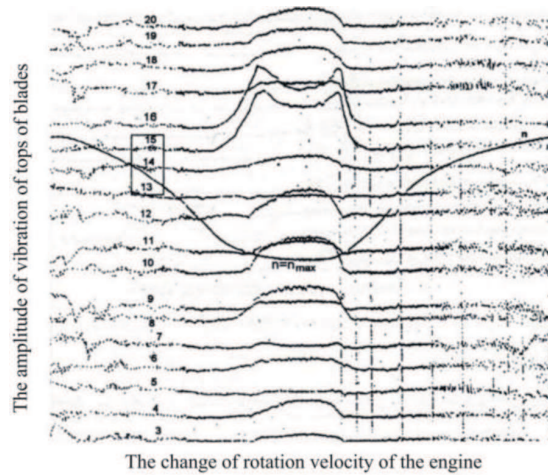


Fig. 3. Course of signal  $y(t)$  for various blades depending on rotational speed

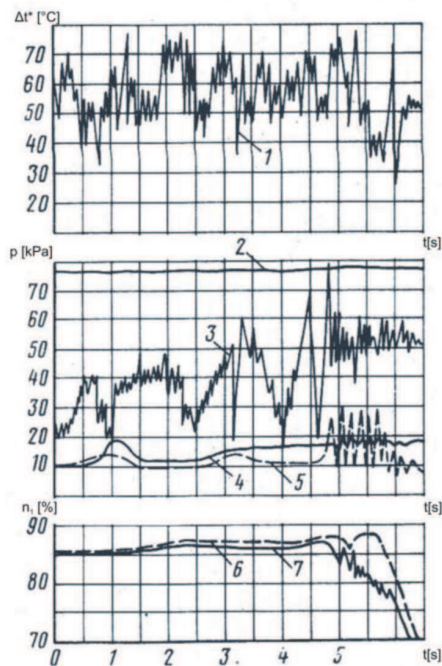


Fig. 4. Change of temperature and pressure of an air stream at the exhaust in specific flight conditions 1 – in the lower part of right engine intake, 2 – at the side of right engine intake, 3 – in the lower part of left engine intake, 4 – in the upper part of left engine intake, 5 – at the side of left engine intake, 6 – change of rotational velocity of right engine rotor, 7 – change of rotational velocity of left engine rotor

We can note that:

$$A_{T01}^2 = \frac{S_{yy}^{T01}}{S_{xx}^{T01}} \quad \varphi_{T01} = \frac{S_{xy}^{T01}}{S_{xx}^{T01}} \quad (4)$$

$$A_{T12}^2 = \frac{S_{yy}^{T12}}{S_{xx}^{T12}} \quad \varphi_{T12} = \frac{S_{xy}^{T12}}{S_{xx}^{T12}} \quad (5)$$

where:  $A_{T01}^2, \varphi_{T01}$  – amplitude amplification and a phase shift of signals x and y while the blade approaches the sensor,  $A_{T12}^2, \varphi_{T12}$  – amplitude amplification and a phase shift of signals x and y while the blade moves away from the sensor.

Further it can be assumed that the observation period of  $T_{1,2}$  occurs shortly (in a few ms) after observation time of signals,  $T_0$ .

In this case it may be assumed that

$$S_{xx}^{T12} = S_{xx}^{T01} \quad (6)$$

Then, a new abstract and a physically interpretable quantity based on equations: 4, 5 and 6 may be obtained in the form of the quotient  $A_{T01}^2$  and  $A_{T12}^2$  of amplitude amplification as well as the difference of phase shifts  $\varphi_{T01}$  and  $\varphi_{T12}$ :

$$A_{T12,T01}^2 = \frac{A_{T12}^2}{A_{T01}^2} = \frac{\frac{S_{yy}^{T12}}{S_{xx}^{T12}}}{\frac{S_{yy}^{T01}}{S_{xx}^{T01}}} \xrightarrow{S_{xx}^{T12}=S_{xx}^{T01}} \frac{S_{yy}^{T12}}{S_{yy}^{T01}} \quad (7)$$

$$\phi_{T12,T01} = \phi_{T12} - \phi_{T01} = \frac{\frac{S_{xy}^{T12}}{S_{xx}^{T12}}}{\frac{S_{xy}^{T01}}{S_{xx}^{T01}}} = \frac{A_{12}e^{-j\phi_{T12}}}{A_{01}e^{-j\phi_{T01}}} = \quad (8)$$

$$A_{T12T01}e^{-j(\phi_{T12}-\phi_{T01})} \xrightarrow{S_{xx}^{T12}=S_{xx}^{T01}} \text{Arg} \frac{S_{xy}^{T12}}{S_{xy}^{T01}}$$

The equation  $A_{T12,T01}^2$  (7) combines the diagnostic signals  $y(t)$  with the environment signals  $x(t)$ , so this is a diagnostic model. Characteristic features of this model are: the possibility of determination only on the basis of the measurable signal  $y(t)$  in observation periods  $T_{01}$  and  $T_{12}$  that occur shortly one after another, and, what is the most important, the possibility of taking environment signal  $x(t)$  into account without the need to measure it, as well as sufficient noise suppression in signal  $y(t)$  [13, 15, 17, 18].

The equation  $\phi_{T12,T01}$  (8) combines diagnostic signals  $y(t)$  with environment signals  $x(t)$  so it constitutes another diagnostic model. As in the case of model  $A_{T12,T01}^2$ , this one is determined without the need to measure a

real environmental signal  $x(t)$ . In order to determine signals  $S_{xy}^{T12}$ ,  $S_{xy}^{T01}$ , a generalized function in the form of a function  $\delta(t, \hat{t})$  must be used. It is assumed that the quotient of the power density function of the signal  $y$  and the distribution  $\delta(t, \hat{t})$ , which represents environment signals  $x$ , eliminates, to a sufficient extent, real environment from model  $\phi_{T12, T01}$  [1, 17, 18, 20].

#### 4. Method of evaluation of current technical condition of blade based on observation of model $A_{T21, T01}$ parameters

The method of current evaluation of changes in technical condition of a turbomachine blade based on observation of parameters of model  $A_{T12, T01}^2$  requires appropriate diagnostic tests.

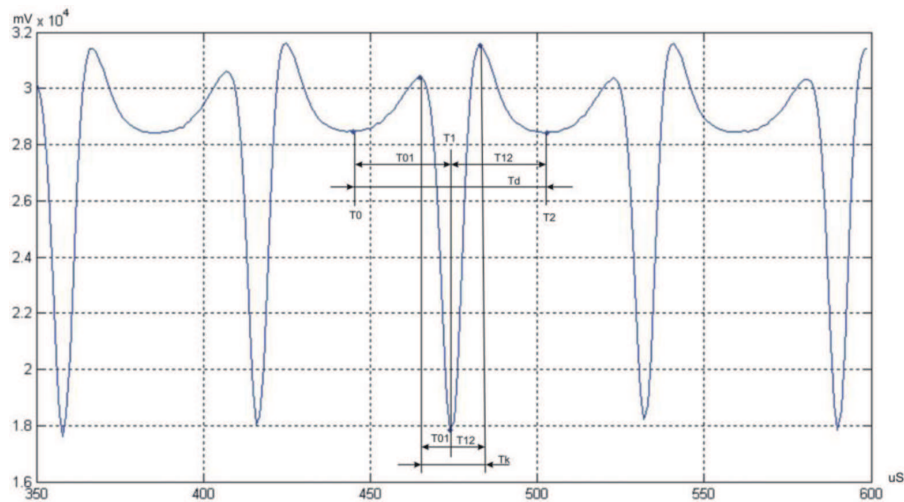


Fig. 5. Inductive sensor signal

$T_d$ ,  $T_k$  – respectively – long and short observation period of a blade tip presence in the sensor area,  $T_0$ ,  $T_1$ ,  $T_2$  – particular moments of observation of a blade tip under the sensor,  $T_{01}$ ,  $T_{12}$  – observation subperiods of a blade tip for  $T_d$  and  $T_k$ , respectively, mV – signal of blade tip displacement, uS – blade displacement time

The distinctive feature of this investigation is that the given observation time  $T$  (value  $T_d$  or  $T_k$ ) of blade translocation below the sensor is divided into two ranges: when blade approaches the sensor.  $T_{01}$ , and when it moves away from it,  $T_{12}$  (moment  $T_1$  is exactly when the blade tip is below the sensor – Fig. 5). The assumption of long  $T_d$  or short  $T_k$  time of blade observation is the result of equation (6)

Then, the estimates of autocorrelation function  $R_{yy}^{*T01}$  and  $R_{yy}^{*T12}$  are determined for  $y(t)$  translocation in the observation periods  $T_{01}$  and  $T_{12}$  and proper analytic expressions fit: [1, 13, 14, 15, 18, 19]



– for  $T_{01}$  period of observation  $T_d$  or  $T_k$

$$R_{yy}^{T01} = \sum_{i=1}^n \alpha_{iT_{01}} e^{-\beta_{iT_{01}} \tau} \cos(\gamma_{iT_{01}} \tau) \quad (9)$$

where:  $\alpha_{iT_{12}}$ ,  $\beta_{iT_{12}}$ ,  $\gamma_{iT_{12}}$  – parameters of analytic form of the correlation function,  $\tau$  – displacement time (space variable of the correlation function),  $n$  – number of correlation function summands.

– for period  $T_{12}$  of observation time  $T_d$  or  $T_k$

$$R_{yy}^{T12} = \sum_{j=1}^m \alpha_{jT_{12}} e^{-\beta_{jT_{12}} \tau} \cos(\gamma_{jT_{12}} \tau) \quad (10)$$

where:  $\alpha_{iT_{12}}$ ,  $\beta_{iT_{12}}$ ,  $\gamma_{iT_{12}}$  – parameters of the analytic form of the correlation function,

$\tau$  – displacement time (different from dynamic time  $t$ ),  $m$  – number of correlation function summands.

The degree of fit between the analytic form of the correlation function and the estimators should be greater than 0.99 and “ $n$ ” should be equal or greater than “ $m$ ”. This may be obtained by selecting an appropriate observation period  $T_d$  or  $T_k$  (Fig. 5) as well as an appropriate measurement window (rectangular, Hamming, Hanning, etc.) [14].

Transition from the physical signals  $x(t)$  and  $y(t)$  to the correlation functions  $R_{xx}(\tau)$  and  $R_{yy}(\tau)$  specified in the displacement period  $\tau$  may be justified not only by the possibility of noise suppression but also by the fact that the function  $C^2$  in time domain  $\tau$  corresponds to a given function  $C$  in the time domain  $t$  and, by the same token, the function  $C^2 \cos(\omega t)$  corresponds to the function  $C \sin(\omega t)$ .

On the basis of analytic forms of singular correlation functions  $R_{yy}^{T01}$  and  $R_{yy}^{T12}$ , we determine the functions of spectral power density  $S_{yy}^{T01}(\omega)$  and  $S_{yy}^{T12}(\omega)$  with the use of Fourier transform:

$$S_{yy}^{T01}(\omega) = F(R_{yy}^{T01}(\tau)) \quad (11)$$

$$S_{yy}^{T12}(\omega) = F(R_{yy}^{T12}(\tau)) \quad (12)$$

where:  $F(\cdot)$  – continuous Fourier transform.

Ultimately, a new abstract diagnostic model may be determined:

$$A_{T_{12}, T_{01}}^2 = \frac{S_{yy}^{T12}}{S_{yy}^{T01}} = \frac{L_0 + L_1 s + L_2 s^2 + \dots}{M_0 + M_1 s + M_2 s^2 + \dots} \quad (13)$$

its parameters give information about technical condition of a diagnosed blade.

The process of monitoring technical condition of a turbomachine blade looks as follows:

- in operation time  $\theta_0$  (start of monitoring) the parameters of an  $A_{T_{12}, T_{01}}^2$  function:  $L_{01}, L_{02}, L_{03} \dots$  and  $M_{01}, M_{02}, M_{03} \dots$  are determined
- in operation time  $\theta_1$  (another monitoring of a blade) the parameters of an  $A_{T_{12}, T_{01}}^2$  function:  $L_{10}, L_{11}, L_{12}, \dots$  and  $M_{10}, M_{11}, M_{12} \dots$  are determined
- a change in technical condition is determined on the basis of relative changes in parameters.

$$\Delta \bar{L}_i = \frac{L_{i1} - L_{i0}}{L_{i0}}; \quad i = 1, \dots, n \quad (14)$$

$$\Delta \bar{M}_i = \frac{M_{i1} - M_{i0}}{M_{i0}}; \quad i = 1, \dots, m \quad (15)$$

– changes of parameters  $\bar{L}_i$  and  $\bar{M}_i$  are diagnostic information to be used in monitoring blade technical condition during turbomachine operation (work).

It may be assumed that, having a sufficient number of parameters  $\bar{L}_i$  and  $\bar{M}_i$ , as well as a large number of their configurations, one can identify various changes in blade technical condition during turbomachine operation.

### 5. A method of evaluation of blade technical condition based on observation of model $\phi_{T_{12}, T_{01}}$ parameters

Courses of signals  $y(t)$  during observation time  $T_{01}$  and  $T_{02}$  are shown in Fig. 5. It is assumed that the environment signal is e.g. high-power white noise which can be correlated with the signal  $y(t)$  [16].

In the result of performed mathematical operations, correlation function estimates were obtained, and then analytical forms of correlation functions:

$$R_{xy}^{T_{01}} = \sum_{k=1}^n \alpha_{kT_{01}} e^{-\beta_{kT_{01}} \tau} \cos(\gamma_{kT_{01}} \tau) \quad (16)$$

$$R_{xy}^{T_{12}} = \sum_{l=1}^m \alpha_{lT_{12}} e^{-\beta_{lT_{12}} \tau} \cos(\gamma_{lT_{12}} \tau) \quad (17)$$

Then, the following is determined:

$$S_{xy}^{T_{01}}(\omega) = F(R_{xy}^{T_{01}}(\tau)) \quad (18)$$

$$S_{xy}^{T12}(\omega) = F(R_{xy}^{T12}(\tau)) \quad (19)$$

and finally

$$\phi_{T12,T01} = \frac{S_{xy}^{T12}}{S_{xy}^{T01}} = \frac{B_0 + B_1s + B_2s^2 + \dots}{A_0 + A_1s + A_2s^2 + \dots} \quad (20)$$

Blade technical condition is specified by parameters of the model  $\phi_{T12,T01}$ .

The process of monitoring blade technical condition of a turbomachine looks as follows:

- in operation time  $\theta_0$  (start of monitoring) the parameters of an  $\phi_{T12,T01}$  function:  $B_{01}, B_{02}, B_{03} \dots$  and  $A_{01}, A_{02}, A_{03} \dots$  are determined;
- in operation time  $\theta_1$  (another monitoring of a blade) the parameters of an  $\phi_{T12,T01}$  function:  $B_{10}, B_{11}, B_{12}, \dots$  and  $A_{10}, A_{11}, A_{12} \dots$  are determined;
- a change in technical condition is determined on the basis of relative changes in parameters

$$\Delta \bar{B}_i = \frac{B_{i1} - B_{i0}}{B_{i0}}; \quad i = 1, \dots, n \quad (21)$$

$$\Delta \bar{A}_i = \frac{A_{i1} - A_{i0}}{A_{i0}}; \quad i = 1, \dots, m \quad (22)$$

– changes in parameters  $\bar{B}_i$  and  $\bar{A}_i$  are diagnostic information to be used in monitoring blade technical condition during turbomachine operation (work).

It may be assumed that a considerable number of parameters  $\bar{B}_i$  and  $\bar{A}_i$  as well as a large number of their configurations enable identification of many various changes in blade technical condition during turbomachine operation.

## 6. Conclusions

Method of current assessment of changes of blade technical condition based on diagnostic models is an innovative method of blade diagnostics, which does not require environment signal measurements.

Method of blade technical condition monitoring may be based on diagnostic model in the form of quotient of output  $y(t)$  signal amplifications to environment signal  $x(t)$  for observation time  $T_{01}$  and  $T_{12}$ . The concept of this method consists in dividing the time  $T$  (Fig. 5) of blade tip movement in sensor area into two ranges: of blade tip approaching the sensor  $T_{01}$  and receding from it  $T_{12}$ .

Both observation periods  $T_{01}$  and  $T_{12}$  of  $y(t)$  are so close in time to each other that the environment signals for these observation periods may be considered identical  $S_{xx}^{T01} = S_{xx}^{T12}$ .

The method of technical condition monitoring is further elaborated by determining power spectral density  $S_{yy}^{T01}, S_{yy}^{T12}$  of signal  $y(t)$  through analytic

forms of autocorrelation functions  $R_{yy}^{T01}$ ,  $R_{yy}^{T12}$ , and, through reciprocal correlation function, of signal  $y(t)$  and noise representing environment  $x(t)$ . The required fit (greater than 0,99) is obtained through proper choice of observation range  $T_d$  and  $T_k$  (Fig. 5) and measurement time-window function: rectangular, Hamming, Hanning etc.

Another distinctive feature of the models  $A_{T12,T01}^2$  and  $\phi_{T12,T01}$  is that they do not require environment signals measurement, although these are indirectly taken into account within special research (two observation periods, determination of diagnostic model as a quotient of models binding diagnostic and environment signals to technical condition parameters).

The method of monitoring of blade technical condition may be also based on diagnostic model in the form of quotient difference of phase shifts of output signal  $y(t)$  and environment signal  $x(t)$  for observation time  $T_{01}$  and  $T_{02}$ . The observation time  $T$  is divided into two ranges.  $\phi_{T12,T01}$ , and one determines a quotient of reciprocal power density  $S_{xy}^{T01}$  and  $S_{xy}^{T12}$  (to determine  $S_{xy}^{T01}$  and  $S_{xy}^{T12}$ , noise distributions of identical form  $\delta(t, \hat{t})$  are used).

The method of current evaluation of a change in blade technical condition on the basis of the diagnostic models  $A_{T12,T01}^2$  and  $\phi_{T12,T01}$  is an innovative method of blade diagnostics in an environment without measuring environment signals.

The method of monitoring blade technical condition may be based on a diagnostic model such as the amplitude amplification quotient of the output signal  $y(t)$  to the environment signal  $x(t)$  for the observation time  $T_{01}$  and  $T_{12}$ . In this method, time  $t$  (Fig. 5) of blade tip displacement  $y(t)$  in the area of a sensor is divided into two ranges: blade tip approaching the sensor  $T_{01}$  and moving away from the sensor  $T_{12}$ .

A distinctive feature of the method of monitoring technical condition of an object is the fact that it determines the abstract model  $A_{T12,T01}^2$  which is a quotient of the spectral power density  $S_{yy}^{T12}$  of the signal  $y(t)$  observed during the next period of time  $T_{12}$  and the spectral power density  $S_{yy}^{T01}$  of the same signal  $y(t)$  observed in the previous period of time  $T_{01}$ . The periods  $T_{01}$  and  $T_{12}$  of signal  $y(t)$  observation are placed so close to each other that the environment signals  $x(t)$  for observation periods of these signals  $y(t)$  may be considered identical  $S_{xx}^{T01} = S_{xx}^{T12}$ .

Another characteristic feature of the method of monitoring technical condition of an object is the fact that the spectral power density  $S_{yy}^{T01}$ ,  $S_{yy}^{T12}$  of the signal  $y(t)$  is determined from analytic forms of the correlation functions  $R_{yy}^{T01}$ ,  $R_{yy}^{T12}$  and  $S_{xy}^{T01}$ ,  $S_{xy}^{T12}$  from the reciprocal correlation functions  $R_{xy}^{T01}$  and  $R_{xy}^{T12}$  of the signal  $y(t)$  as well as from white noise representing the environment  $x(t)$ . The required fit (greater than 0.99) is achieved by an appropriate

choice of observation range width  $T_d$  and  $T_k$  (Fig. 5) and the function of the measurement window (rectangular, Hamming, Hanning, etc.)

As for the models  $A_{T_{12},T_{01}}^2$  and  $\phi_{T_{12},T_{01}}$ , their characteristic feature is the fact that they do not require measurement of environment signals, although indirectly it is taken into account in diagnostic tests conducted for this purpose (two observation periods, determining diagnostic model as the quotient of diagnostic models comprising the diagnostic and the environment signals together with technical condition parameters).

The method of monitoring blade technical condition may also be based on a diagnostic model in the form of the difference of phases shifts of the output signal  $y(t)$  and the environment signal  $x(t)$  for observation time  $T_{01}$  and  $T_{02}$ . The observation time  $T$  is divided into two subsets  $T_{01}$  and  $T_{12}$  and  $\phi_{T_{12},T_{01}}$  is determined that constitutes the quotient of the reciprocal power density  $S_{xy}^{T_{01}}$  and  $S_{xy}^{T_{12}}$  (in order to determine  $S_{xy}^{T_{01}}$  i  $S_{xy}^{T_{12}}$ , the white noise of identical distribution in the form of  $\delta(t, \hat{t})$  can be used).

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### **Model diagnozowania łopatki pracującej maszyny wirnikowej z wirtualną eliminacją jej stochastycznego otoczenia**

#### **Streszczenie**

W artykule przedstawiono podstawy nowej metody monitorowania stanu technicznego łopatek maszyn wirnikowych podczas ich użytkowania. Metoda wykorzystuje modele diagnostyczne w postaci ilorazu wzmocnienia amplitudowego i przesunięcia fazowego sygnału diagnostycznego  $y(t)$  wynikającego z działania łopatki i sygnału  $x(t)$  jej otoczenia podczas zbliżania się wierzchołka łopatki do czujnika i wzmocnienia amplitudowego i przesunięcia fazowego tych sygnałów podczas oddalania się wierzchołka łopatki od czujnika.

Przyjęte modele diagnostyczne pośrednio uwzględnia aktualne otoczenie łopatki  $x(t)$  bez konieczności jego pomiaru [13,15]. Zatem jest on czuły na zmiany stanu technicznego łopatki, a mało wrażliwy na zmianę otoczenia. Proponowana metoda może odegrać istotną rolę w diagnostyce łopatek wirnikowych podczas użytkowania maszyn wirnikowych (sprężarek, turbin, itp).