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ANALYSIS OF INFLUENCE OF DISLOCATIONS IN A BEARING CONNECTION WITH BEARING SLACKNESS ON BASIC RATING LIFE OF A BEARING SYSTEM OF A STATICALLY UNDETERMINED SHAFT

Commonly used computations of basic rating life of a bearing system are based on the ISO 281:1990 standard. These computations include dynamic load capacity of a given bearing, its effective load and average rotational speed, whereas they omit distribution of external load acting upon particular rolling parts depending, among other things, on:

– displacement in bearing (displacements in three directions and declination in two planes),

– slackness in bearings.

The aim of the presented theoretical research is to solve a problem of fatigue life of a ball bearing taking into consideration displacement in bearing resulting from elasticity of a three-bearing shaft, elasticity of bearings and their internal slackness.

1. Introduction

Usually, computing fatigue life of a rolling bearing is limited to including dynamic load capacity of the bearing (C), its effective load (P) and medium rotational speed (n) [1]. In more accurate computations of rating life additionally the following elements are included: expected probability of achieving the calculated rating life (coefficient/factor a_1), material of a bearing (a_2) and friction conditions (a_3). Still, the real distribution of outer/external load affecting/influencing particular rolling elements of shaft bearings is not included. Also the fact that due to various reasons bearing rings may not be set in parallel position in relation to each other is omitted. This lack of parallelism causes disturbances in the distribution of load with reference to

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the ideal model by Lundberg and Palmgren [2], from which medium value of load of the most heavily loaded rolling element in the form (1) is derived, according to [1].

$$Q_{\max} = \frac{5 \cdot F_r}{i \cdot Z \cdot \cos\alpha} \tag{1}$$

where:

F_r – radial load of bearing,

i – number of rolling elements in bearing,

Z – number of balls in one row,

α – angle of acting (loading) of a ball.

The basic reason for lack of parallelism of rings are shaft strains.

2. Mathematical approach

A system consisting of a shaft and three bearings is statically undetermined. In handbooks, equations used to solve this problem are supplemented by the equation of shaft elasticity. In line with shaft elasticity, also radial, axial and bending elasticity of bearings is taken into consideration. Radial elasticity is expressed by correlation of bending moment influencing a bearing (and at the same time radial reaction of the bearing) and radial displacement of inner ring in relation to outer ring. Axial elasticity is expressed analogically. Whereas bending elasticity is expressed by correlation of bending moment influencing a bearing (that is a reactive moment of the bearing and angular deflection of outer ring in relation to the inner one). Radial displacement and angular deflection of inner ring corresponds with local deflection and angle of deflection of a shaft line. Subsequently, not only radial and axial reaction but also reactive moment of every bearing is included in equations of the system statics. Feedback in this system should be highlighted here: angles of deflection of bearing rings are determined by the line of shaft deflection, but in turn reactive moments of bearings, depending on the angles of rings deflection, influence this deflection line.

Radial slackness is an additional parameter of ball bearings taken into consideration in the study. The parameter has a triple meaning:

1. Influence on distribution of radial load acting on particular balls (the larger the slackness, the smaller number of balls carry the load).
2. Influence on the value of reactive action moment emerging as a result of angular deflection of the inner ring; the smaller the slackness, the higher „bending rigidity" of the bearing, i.e. the higher reactive moment.
3. Influence on the reaction of supports – rolling bearings.

There exists a nonlinear relation between the angle of deflection of the bearing ring and the reactive moment of the bearing. As the angle increases, contact forces in the bearing increase, in spite of constant character of the forces loading the bearing, which results in rapid shortening of its rating life.

Practically it is not possible to set elastic displacement in a bearing (dislocations and declination of rings) on the basis of its load. It is due to the fact that the number of loaded balls, distribution of loads on balls and angles of action of particular balls in a bearing are not known.

An opposite proceeding is possible, presented in [3] and proposed in publications [4,5]: iterative search of such combination of elastic dislocations in all bearings of a shaft for which forces and reactive moments and forces of bearings correspond with outer loads of the shaft. In the beginning of the procedure, displacements of the inner ring in relation to the outer one in three directions; f_x, f_y,f_z (where axis x is the axis of a bearing, and axes y and z are normal perpendicular in relation to it) and declinations of the inner ring in relation to the outer one in two planes (γ_y – in plane x-y and γ_z – in plane x-z) are assumed, as presented in Fig. 1.

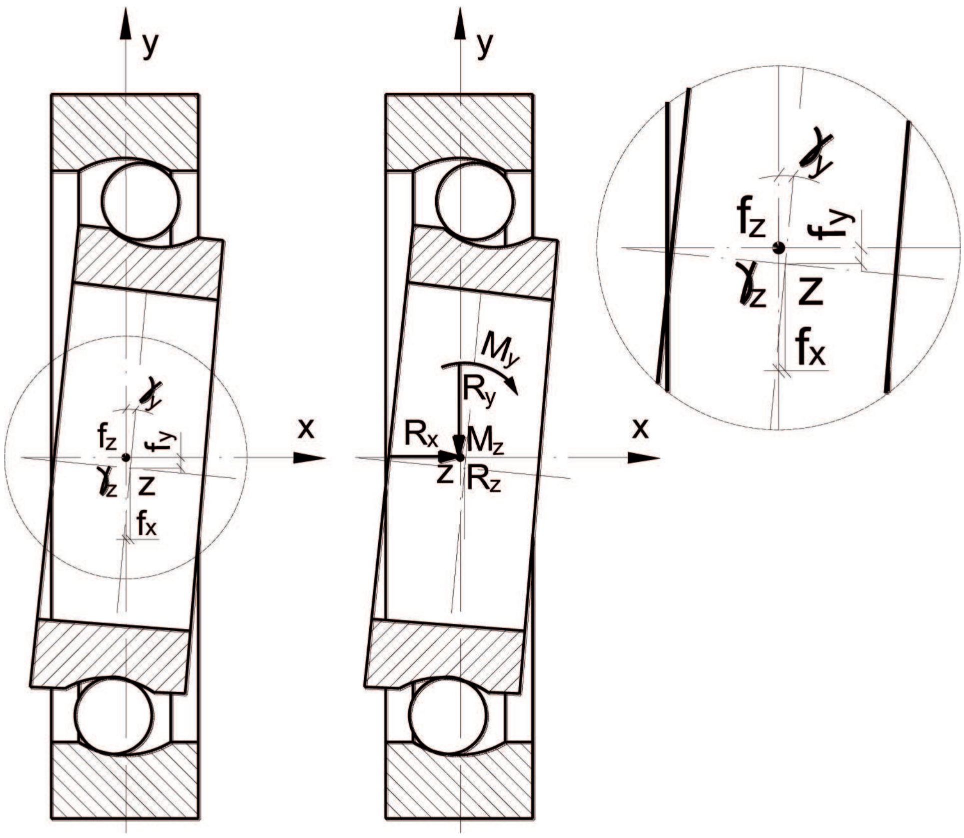

Fig. 1. Dislocations in a bearing [2]

On this basis, the angles of acting of balls and strain at the point of contact of particular balls with rings are calculated, as presented in Fig. 2 and expressed by formulas (2÷7):

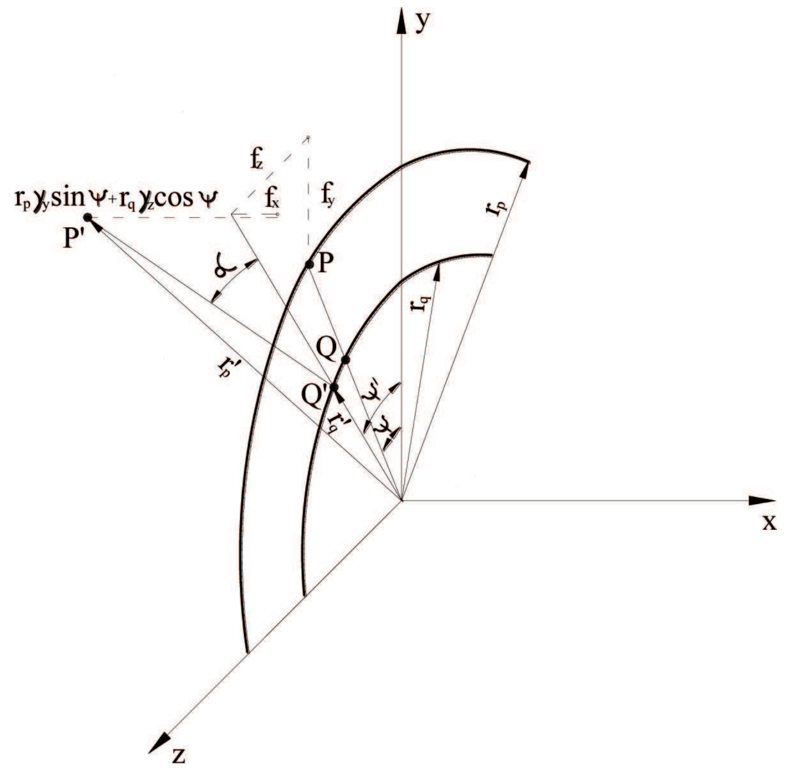

Fig. 2. Dislocation of centre of curvature of the track of the inner ring at the point of contact with a chosen ball, caused by dislocation and inclination of the ring, where: α – the angle of acting (load) of the ball in axial plane of the ball, ψ – nominal angle of the ball setting in frontal plane of the bearing, ψ' – the angle of setting of the ball in frontal plane of the bearing with respect to dislocations, P and Q – centres of lateral curvature of the outer and inner ring track. At the point of contact with a selected ball in a nominal position, P' and Q' – location of the centres as a result of dislocations and rotations of the inner ring in relation to the outer one

$$r_p = \frac{d_m}{2} - \frac{d_k}{2} - \frac{\Delta r}{4} + r_i \tag{2}$$

$$r_q = \frac{d_m}{2} + \frac{d_k}{2} + \frac{\Delta r}{4} - r_o \tag{3}$$

$$r'_p = \left| \begin{array}{l} f_x - r_p \cdot \left(\gamma_y \cdot \sin\psi + \gamma_z \cdot \cos\psi\right) \\ f_y + r_p \cdot \cos\psi \\ f_z + r_p \cdot \sin\psi \end{array} \right. \tag{4}$$

$$r'_q = \left| \begin{array}{l} 0 \\ r_q \cdot \cos\psi' \\ r_q \cdot \sin\psi' \end{array} \right. \tag{5}$$

$$tg\alpha = \frac{f_x - r_p \cdot \left(\gamma_y \cdot \sin\psi + \gamma_z \cdot \cos\psi\right)}{f_y \cdot \cos\psi + f_z \cdot \sin\psi + r_p - r_q} \tag{6}$$

$$\delta = \left|\overline{r'_p} - \overline{r'_q}\right| - \left|\overline{r_p} - \overline{r_q}\right| - \frac{\Delta r}{2} \tag{7}$$

where:

r_p– polar coordinate of the centre of transverse curvature of tracks of inner ring at the contact point with a selected ball in a nominal position (without dislocations),

r_q– polar coordinate of the centre of transverse curvature of tracks of outer ring at the point of contact with a selected ball in a nominal position (without dislocations),

d_m – mean diameter of bearing; d_m = 0,5(d+D), d – internal diameter of bearing, D – external diameter of bearing,

d_k – diameter of the ball in bearing,

Δr – radial slackness in bearing,

r_i – transverse radius of the track of inner ring of bearing,

r_o – transverse radius of the tracks of outer ring of bearing,

δ – sum of strains of the ball in contact with two rings.

Depending on the assumed strains, nominal forces Q_i for all balls are determined:

$$Q_i = \xi \cdot E' \cdot d_k^{0,5} \cdot \delta^{1,5} \tag{8}$$

where:

ξ – shape ratio for calculating contact stress by Hertz, tabled in [3],

E'– Young substitute module; for bearing steel E'=227100 MPa.

The forces are the basis for computing the resultant forces R_x, R_y, R_z and reactive moments of bearing M_y, M_z defined by formulas (9÷13).

Iterations are made long enough for the enumerated forces and moments to meet the equations of balance with outer loads of the shaft:

$$R_x = \sum_{i=1}^{Z} (-Q_i \cdot \sin\alpha) \tag{9}$$

$$R_y = \sum_{i=1}^{Z} (-Q_i \cdot \cos\alpha \cdot \cos\psi) \tag{10}$$

$$R_z = \sum_{i=1}^{Z} (-Q_i \cdot \cos\alpha \cdot \sin\psi) \tag{11}$$

$$M_y = \sum_{i=1}^{Z} (Q_i \cdot r_p \cdot \sin\alpha \cdot \sin\psi) \tag{12}$$

$$M_z = \sum_{i=1}^{Z} (Q_i \cdot r_q \cdot \sin\alpha \cdot \cos\psi) \tag{13}$$

On the basis of normal forces influencing at a defined moment all balls in a bearing, a medium load $Q_{\acute{s}r}$ of any ball is determined (averaged in its circulation around the bearing):

$$Q_r = \left(\frac{1}{Z} \cdot \sum_{i=1}^{Z} Q_i^3 \right) \tag{14}$$

where:

Z – number of balls in a bearing,

and then substitute load of a bearing P:

$$P = Z \cdot \frac{J_r(\varepsilon)}{J_1} \cdot Q_r \tag{15}$$

where:

$J_r(\varepsilon)$ i J_1 – Sjövall integrals, determining relation between outer loads of a bearing and the load of the most loaded ball. In accordance with [6] for durability calculations based on dynamic load capacity of a bearing, the value of integrals is assumed to equal the angle of load $2\psi = \pi$, $J_r(\varepsilon) = 0{,}2288$, $J_1 = 0{,}5625$.

Load capacity L_{10} (in millions of rotations) is determined from the known relation:

$$L_{10} = \left(\frac{C}{P} \right)^3 \tag{16}$$

where:

C – dynamic load capacity of a bearing.

In this way, the rating life has been expressed on the basis of real load of balls in a bearing.

To compare, also calculation of a nominal rating life with a catalogue method has been added, recommended by ISO (basing on the substitutive load determined from the radial force F_r and Arial force F_a) according to [7]

$$P = X \cdot F_r + Y \cdot F_a \tag{17}$$

where:

X – transverse load ratio,

Y – axial load ratio.

In the catalogue method, neither shift nor inclinations of inner ring in relation to the outer ring and reactive moments of bearings are included, and forces in bearings depend only on the load and shape of a shaft.

Thus determined rating life is characteristic for a "rigid" bearing.

3. Calculation results

Below we present results of calculations made for two variations of shape of a three-bearing shaft and five values of slackness with different radial and axial loads. Two assumed variations of shaft shape are presented in Figures 3 and 4. In every variation bearing A locks the shaft axially.

The middle bearing C is settled on a neck with larger diameter than the remaining bearings of the system, and thus it has greater load capacity.

Shaft load is constituted by radial and axial forces applied in two points, as illustrated in Fig. 3 and 4. Values of these forces are assumed as follows: F_{r1} = 2000 N, F_{r2} = 3000 N, F_{a1} = 0, 500, 1000, 1500 N, F_{a2} = 0, 500, 1000, 1500 N.

Values of radial slackness appropriate for selected bearings in five classes (from group 2 to group 5), are assumed the following:

- for group 2: Δr = 0,005 mm,
- for group N: Δr = 0,014 mm,
- for group 3: Δr = 0,020 mm,
- for group 4: Δr = 0,028 mm for bearings "A" and "B" and Δr = 0,037mm for bearing "C",
- for group 5: Δr = 0,038 mm for bearings "A" and "B" and Δr = 0,042mm for bearing "C".

Numerical values of slackness correspond with medium values of slackness for a given range of diameters [8].

The influence of a shaft shape, loads and slackness on "A" bearing rating life is presented numerically (in tables), and the influence of the same parameters on rating life of "B" and "C" bearings is presented in a graphic form.

Numerical presentation has been chosen for "A" bearing, because graphic image has turned out to be of limited legibility as a result of overlapping lines presenting rating life of the bearing.

The analysis of presented tables and charts proves noticeable differences between results of catalogue calculations and the authors' calculations, whereas in some cases the authors' computations indicate longer rating life

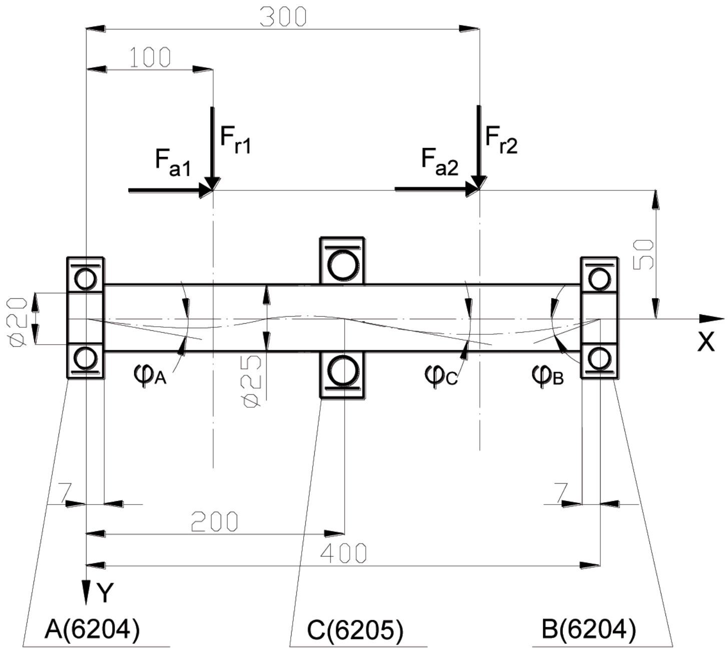

Fig. 3. Bearing system used for calculations – variation I

Table 1.
Value of radial slackness for ball bearings with groove tracks having radial contact with shaft eye [8]

Eye diameter d [mm]		group 2		group N		group 3		group 4		group 5	
above	including	min. [μm]	max. [μm]	min. [μm]	max. [μm]	min. [μm]	max. [μm]	min. [μm]	max. [μm]	min. [μm]	max. [μm]
2,5	6	0	7	2	13	8	23	–	–	–	–
6	10	0	7	2	13	8	23	14	29	20	37
10	18	0	9	3	18	11	25	18	33	25	45
18	24	0	10	5	20	13	28	20	36	28	48
24	30	1	11	5	20	13	28	23	41	30	53
30	40	1	11	6	20	15	33	28	46	40	64
40	50	1	11	6	23	18	36	30	51	45	73
50	65	1	15	8	28	23	43	38	61	55	90
65	80	1	15	10	30	25	51	46	71	65	105

and in some others shorter. It can be noticed that these relations are different in bearing A and in other bearings of the system.

When the bearings load is only radial, for the smallest slackness (group 2) rating life according to the authors' calculation is close to catalogue rating life, while increasing the slackness results in considerable shortening of the rating life. This applies to all bearings in the system, while influence of slackness increase on durability is greater in case of locking bearings (about 60% in case of group 5 slackness).

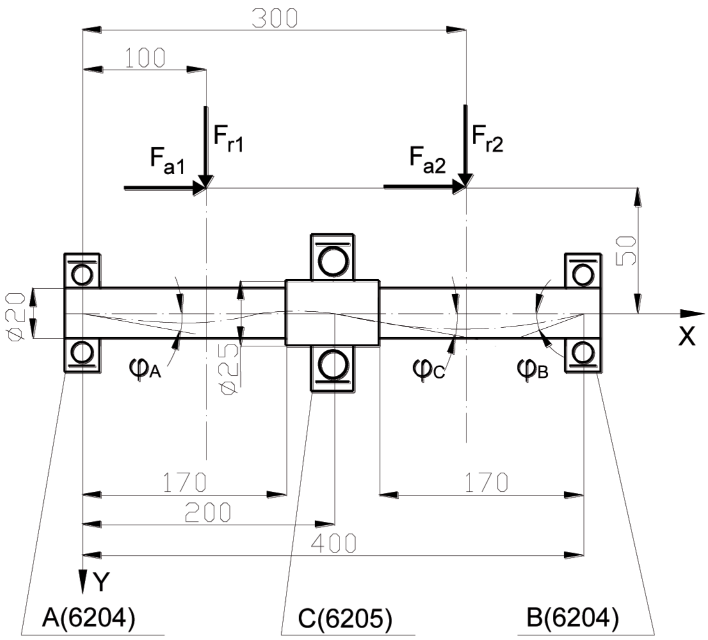

Fig. 4. Bearing system used for calculations – variation II

Table 2.

Rating life of bearing A for variation I of shaft shape

Load	F_{r1}=2000[N], F_{r2}=3000[N]			
	F_{a1}=0[N] F_{a2}=0[N]	F_{a1}=500[N] F_{a2}=500[N]	F_{a1}=1000[N] F_{a2}=1000[N]	F_{a1}=1500[N] F_{a2}=1500[N]
Slackness group	Rating life [in millions of rotations]			
	by catalogue according to authors' calculations			
2	$6.71*10^3$	$2.71*10^2$	$7.31*10^1$	$3.26*10^1$
	$6.31*10^3$	$4.53*10^2$	$9.65*10^1$	$3.75*10^1$
N	$6.71*10^3$	$2.71*10^2$	$7.31*10^1$	$3.26*10^1$
	$5.05*10^3$	$6.22*10^2$	$1.24*10^2$	$4.64*10^1$
3	$6.71*10^3$	$3.32*10^2$	$7.85*10^1$	$3.29*10^1$
	$4.27*10^3$	$7.44*10^2$	$1.45*10^2$	$5.28*10^1$
4	$6.71*10^3$	$4.19*10^2$	$8.76*10^1$	$3.34*10^1$
	$3.42*10^3$	$9.19*10^2$	$1.73*10^2$	$6.19*10^1$
5	$6.71*10^3$	$5.79*10^2$	$1.04*10^2$	$3.55*10^1$
	$2.63*10^3$	$1.13*10^3$	$2.12*10^2$	$7.40*10^1$

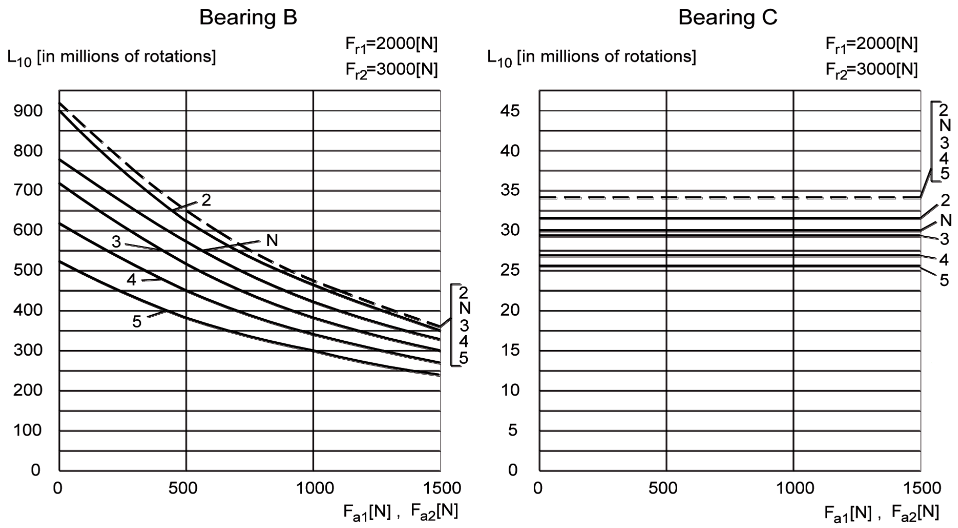

Fig. 5. Rating life L_{10} of ball bearings B and C for variation I for particular slackness groups. Solid line – according to the authors' calculations, dashed line – according to catalogue calculations

Table 3.

Rating life of bearing A for variation II of shaft shape

Load	F_{r1}=2000[N], F_{r2}=3000[N]			
	F_{a1}=0[N] F_{a2}=0[N]	F_{a1}=500[N] F_{a2}=500[N]	F_{a1}=1000[N] F_{a2}=1000[N]	F_{a1}=1500[N] F_{a2}=1500[N]
Slackness group	Rating life [in millions of rotations]			
	by catalogue according to the authors' calculations			
2	$1.33*10^4$	$3.06*10^2$	$7.90*10^1$	$3.46*10^1$
	$1.21*10^4$	$4.72*10^2$	$9.75*10^1$	$3.76*10^1$
N	$1.33*10^4$	$3.06*10^2$	$7.90*10^1$	$3.46*10^1$
	$9.23*10^3$	$6.58*10^2$	$1.26*10^2$	$4.66*10^1$
3	$1.33*10^4$	$3.68*10^2$	$8.38*10^1$	$3.40*10^1$
	$7.59*10^3$	$7.94*10^2$	$1.47*10^2$	$5.31*10^1$
4	$1.33*10^4$	$4.67*10^2$	$9.34*10^1$	$3.49*10^1$
	$5.89*10^3$	$9.88*10^2$	$1.76*10^2$	$6.22*10^1$
5	$1.33*10^4$	$6.52*10^2$	$1.06*10^2$	$3.63*10^1$
	$4.45*10^3$	$1.25*10^3$	$2.16*10^2$	$7.44*10^1$

With complex loads (transverse-axial), in relation to locking bearing it is noticeable that increasing the slackness leads to increasing catalogue rating life as well as the rating life resulting from the author's calculations. It is

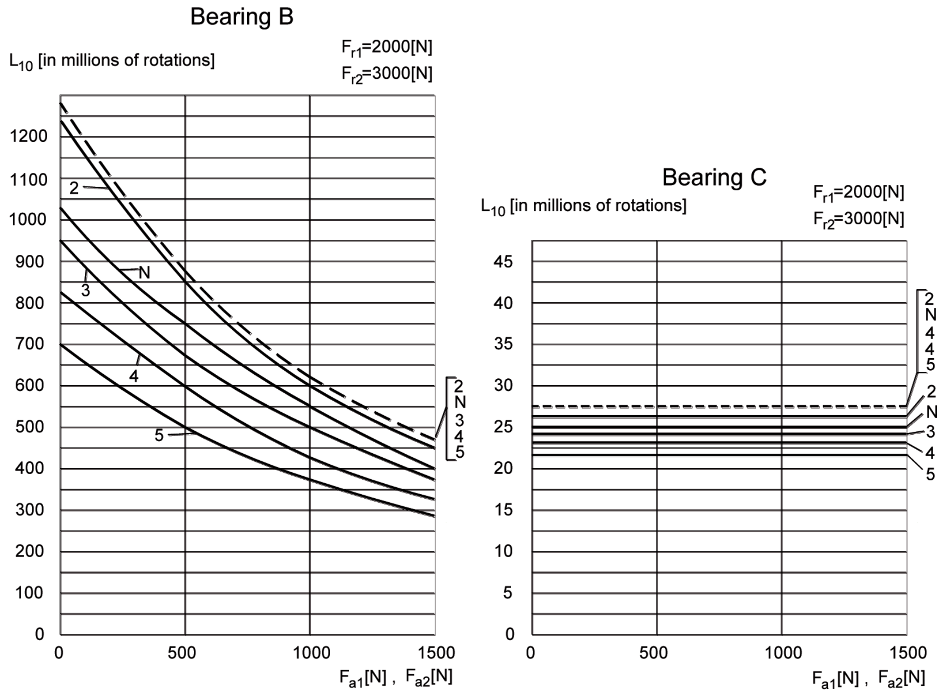

Fig. 6. Rating life L_{10} of ball bearings B and C for variation II for particular slackness groups. Solid line – according to the authors' calculations, dashed line – according to catalogue calculations

worth noticing here that quotient of rating life being a result of the author's calculations and the rating life by catalogue gets longer together with the increase of the bearing slackness, and in extreme cases the quotient is as great as 2.

In the same conditions of loading (it is with transverse-axial shaft loads), rating life of non-locking bearings (B and C) is always shorter according to the authors' calculations than according to catalogue calculations. It gets shorter as the slackness in bearings increases. It can be noticed that relation of these rating lives has little dependence on the value of axial forces and on location of bearing (B or C). It is understandable, because bearings B and C do not carry axial forces and changes of outer axial load cause in them only changes of radial reactions.

These calculation have proven the influence of the shape of shaft on bearing loads and thus on their rating life (compare: tab. II i III and Fig. 5 and 6). For example: with the same load, only the radial one, in the whole range of slackness, catalogue rating life of "A" bearing in variation II is according to the authors calculations 70÷90% longer than catalogue rating life of "A" bearing in variation I (according to catalogue calculations this increase is about 100%). Rating life of "B" bearing in variation II is 35÷40% longer, whereas rating life of C bearing in this variation is 20% shorter. The reason for this phenomenon is the following: reducing the dominating diameter of the shaft from Ø 25 to Ø 20 causes reduction of its rigidity. With the applied scheme of load (radial forces set almost symmetrically between supports), reduction of shaft rigidity result in increase of load of the middle bearing and, at the same time, in load reduction of end bearings. As a result, the middle bearing C rating life shortens, and A and B end bearing rating life gets longer.

The obtained results of calculations confirm restrictions made in literature that catalogue calculations of durability are right only for almost zero slackness in bearing and zero angle of declination from each other of bearing rings. In other cases, these calculations clearly turn out inaccurate. Also acceptable angular deflections quoted in catalogues should be considered as only reference ones, and for more detailed computations they should depend on slackness and load of bearing. Still, in the first place it has been proven that taking into consideration radial, axial and bending elasticity of roller bearings has great meaning for calculations of rating life of a bearing system of a three-bearing shaft.

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Analiza wpływu luzów w łożyskach na trwałość łożyskowania wału statycznie niewyznaczalnego

S t r e s z c z e n i e

Powszechnie stosowane obliczenia trwałości łożyskowania oparte są na normie ISO 281:1994. Obliczenia te uwzględniają nośność dynamiczną danego łożyska, jego efektywne obciążenie i uśrednioną prędkość obrotową. Pominięty jest natomiast rozkład obciążenia zewnętrznego na poszczególne części toczne, który zależy od:

– przemieszczeń w łożysku (przesunięć w trzech kierunkach i przechyleń w dwóch płaszczyznach),

– luzu w łożyskach.

Celem przedstawionych badań teoretycznych jest rozwiązanie zagadnienia trwałości zmęczeniowej łożyska kulkowego z uwzględnieniem przemieszczeń w łożysku, wynikających ze sprężystości wału trzypodporowego, sprężystości łożysk oraz luzu łożysk.