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THE PROCEDURE OF MODIFICATION COEFFICIENTS IN PLANETARY GEAR TRANSMISSION

The aim of this article is to present the design procedure for determining modification coefficients of toothed wheels of involutes planetary gear train with internal conjunction of teeth. It is possible to obtain a higher load-carrying capacity which depends also on correction coefficients. For example, we take into consideration a concept of planetary gears in which the teeth can be corrected, which allows better fatigue and contact surface strength.

Two cases are considered when the namely zero center distance (without corrections) of the central and satellite wheels is the same or not, in relation to the zero center distance between the satellite and the sun wheel.

Geometrical dimensions are described with regard to the technological teeth correction scope, and inequality restriction conditions are determined with respect to the ISO standards recommendations and the literature. The procedure can be applied to any other planetary gears with another kinematic connection of wheels.

1. Fundamental design relation and restrictions of planetary gears

The scheme of a stage planetary gear with correction coefficients is shown in Fig. 1. The basic mechanical configuration consists of two co-operating pairs of gears with the same distance of the rotation axis. We assume that the tools which generate the teeth have the module "m", the profile angle "α", the addendum factor y=1 and the tip clearance factor c_o =0.25.

On the basis of a standard rack, cutting tools can generate toothed wheels with different coefficients of corrections. Under these assumptions, the radii of the rolling circle are different then the radii of the pitch diameters and the pressure angle of the co-operating teeth z_1/z_2 and z_2/z_3 are different then

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the profile angle of the tools. In some cases it is possible when the pressure angles α_w" $\neq$ α_w', as indicated in Fig. 1, that the satellite wheel has one radius of the pitch diametric circle, but it has got a pair of rolling circles with the radii r_w' and r_w". the Superscripts (') determine the data for the toothed wheels z_1 /z_2, and the superscripts (") determine the data for the toothed wheels z_2 /z_3.

An example of a gearbox project is presented in Fig. 2, where the central wheel z_1 working without bearings (the floating wheel-9) and the self-aligning bearings (32) of satellites z_2 guarantee uniform redistribution load to each satellite wheel (11).

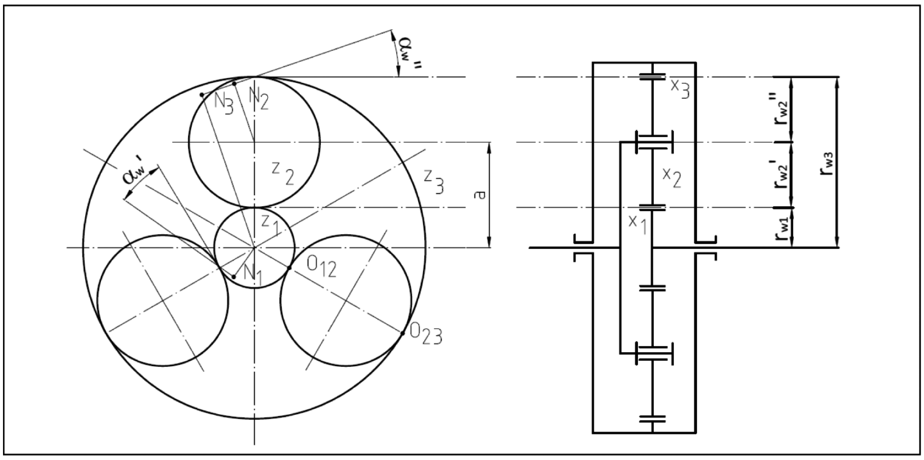

Fig. 1. Geometric diagram of a one stage planetary gear with correction coefficients

If we know the corrections coefficients modification $x_1;x_2;x_3$, then the proportions of toothed engagements in a circumferential cross-section can be found from the formulas presented in literature [1],[3]. The factor "x" changes the relative proportions of the teeth, since the actual shift of the tool r in relation to the blank equal "X=x*m" changes too at the moment the cutting process is completed. The displacement "X" is a distance between the pitch circle of the cutting wheel and the pitch circle or the pitch line of the tool, the sign of "x is positive when displacement "X" is outside of material of the cutting wheel in relation to the central point during the cutting process; otherwise, it is negative.

On account of the proper assembly, it is necessary to set the satellite wheels symmetrically in relation to the central and the sun wheel. A correct distance between the addendum circles of the neighboring satellites requires satisfying some restrictions as to the proper number of teeth $z_1;z_2;z_3$ and the number of satellite wheels as it was presented in [1],[2].

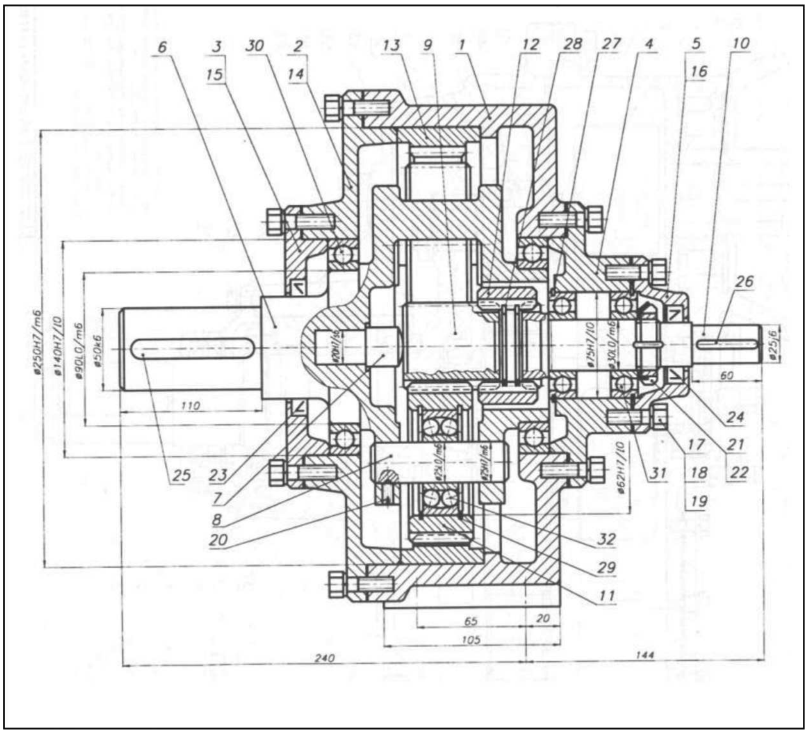

Fig. 2. Structure of a one stage planetary gearbox with 3 satellite wheels

The next important problem is connected with noninterference internal engagements of the satellites and the sun wheel, which requires taking into account the changing geometry caused by corrections coefficients. The condition of noninterference located on the dedendum circle of the satellite wheel [2] is

$$z_2 > z_3 \cdot \{1 - \sqrt{1 - \frac{1}{\sin^2 \alpha''_w}\left[1 - (\frac{r_{a3}}{r_{w3}})^2\right]}\} \tag{1}$$

where: r_{a3} – the addendum radius of the sun wheel, r_{w3} – the radius of the rolling circle of the sun wheel. On the basis of the formula (1) Fig. 3 shows the relations $z_2 > z_2(z_3)$ which secures noninterference located on the addendum circle of the satellite wheel [3], [7] when $x_2 = x_3 = 0$.

Mathematical notation of the condition of noninterference located on the addendum circle of the satellite wheel [2] is more complicated:

$$\phi_3 - \vartheta_3 < (\phi_2 + \vartheta_2) \cdot \frac{z_2}{z_3} \tag{2}$$

$$\phi_3 = ar\cos\frac{r_{a3}^2 - r_{a2}^2 + a_w^2}{2 \cdot a_w \cdot r_{a3}} \quad ; \quad \phi_2 = ar\cos\frac{r_{a3}^2 - r_{a2}^2 - a_w^2}{2 \cdot a_w \cdot r_{a2}} \tag{3}$$

$$\vartheta_3 = inv(\alpha_w'') - inv[ar\cos(\frac{r_{b3}}{r_{a3}})] \;\; ; \;\; \vartheta_2 = inv[ar\cos(\frac{r_{b2}}{r_{a2}})] - inv(\alpha_w'')$$

$$a_w = 0.5(z_3 - z_2)m_w''$$

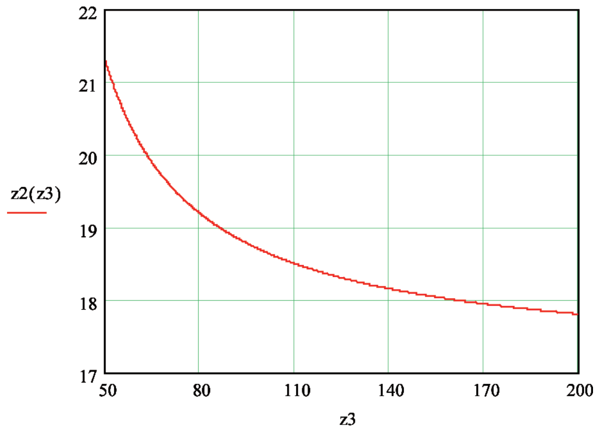

Fig. 3. Minimal number of satellites teeth z_2, y=1, α =20°

The inequality (1) and (2) must be verified when the dimensions of the toothed wheels have been defined.

2. Valuable limit of coefficient of corrections "x"

The original designs of the conjunction of the toothed wheels must satisfy Fölmer's equations by the relations.

$$x_1 + x_2 = C1 = \frac{\{[\tan(\alpha_w') - \alpha_w'] - \tan(\alpha) + \alpha\} \cdot (z_1 + z_2)}{2 \cdot \tan(\alpha)} \quad (4)$$

$$x_2 + x_3 = C2 = \frac{\{[\tan(\alpha_w'') - \alpha_w''] - \tan(\alpha) + \alpha\} \cdot (z_2 - z_3)}{2 \cdot \tan(\alpha)}$$

where: $x_1\,;\,x_2\,;\,x_3$ – correction coefficients modification, $z_1\,;\,z_2\,;\,z_3$ – the number of teeth of the central, satellite and sun wheels, α – the profile angle of tools (e.g. for a sun wheel gear-shaper cutter – Fellows on slotting machine). If $x_1 = x_2 = x_3 = 0$, then the planetary gear is without correction modifications, in addition, when $x_1 + x_2 = 0 \wedge x_3 + x_2 = 0$, we have the corrections named P0; and in both of these cases the pressure angle, according to Fig.1, α_w" = α_w'= α, and the coefficients C1 =C2 =0. When C1≠0 and C2≠0 the corrections has the name P.

For each wheel of external conjunction with z_1 and z_2, the correction coefficient is limited by: no-undercutting condition, minimal thickness of teeth greater than 0.3*m in the radius of the addendum circle r_a (tooth thickness at the top), the radius of the diametric pitch "r" is in the measuring zone $r_a > r > r_f - c$, $c = c_o{*}m$, r_f – the radius of the dedendum circle.

$$x \leq \frac{19}{289} + 10 \cdot \frac{z}{289}\ for\ 10 \leq z \leq 24\ \wedge\ x \leq 1\ for\ z \geq 25 \tag{5}$$

$$x \geq 1 - \frac{z}{17}\ for\ 10 \leq z \leq 32\ \wedge\ x \geq -0.9\ for\ z \geq 33 \tag{6}$$

For the sun wheel z_3 when $z_3 > 30$ the corrections coefficient should be

$$0.5 \geq\ x_3 \geq -0.5\ for\ z_3 \geq 65 \tag{7}$$

For all these reasons, corrections coefficients have been limited and geometric parameters are not identical; that is why it is possible to obtain a higher load-carrying capacity which depends also on correction coefficients.

The application of proper correction coefficients gives us the possibility to find a better fatigue and contact strength of the cooperating teeth.

Let us consider two cases:

Case I, when $z_1+z_2 = z_3-z_2$, the zero center distances of wheels are the same,

Case II, when $z_1+z_2 \neq z_3-z_2$, the zero center distances of wheels are different.

In these cases, the solutions of geometry are different when one applies a rational method of achieving better strength in each separate case. The example of a planetary gear in both cases is presented in Fig. 4. The satellite wheels are located symmetrically in relation to the central and the sun wheel with a correct distance between the addendum circles of the neighboring satellites, at the same time, some restrictions concerning the proper number of teeth z_1; z_2; z_3 are satisfied.

According to Fig. 1, the constant center distance "a" of the wheels forces the pressure angles $\alpha_w^{''}$ *and* $\alpha_w^{'}$ which are given by

$$\cos \alpha_w^{'} = \frac{2 \cdot a}{z_1 + z_2} \cos \alpha\ ,\ \cos \alpha_w^{''} = \frac{2 \cdot a}{z_3 - z_2} \cos \alpha \tag{8}$$

In case I, when $z_1+z_2 = z_3-z_2$, the zero center distances of the wheels are the same, then $\alpha_w^{''} = \alpha_w^{'} = \alpha_w$, and in accordance with (4) C1= – C2= C, the relations between the corrections coefficients are limited by (9)

$$x_1 + x_2 = C\ ;\ x_2 + x_3 = -C \tag{9}$$

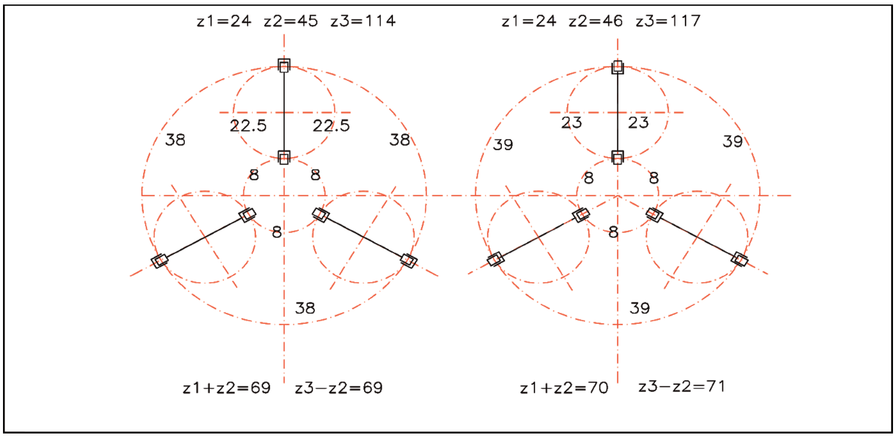

Fig. 4. Case I, when z_1+z_2 =z_3-z_2, the same zero center distance central/ satellite and satellite, /sun wheel. Case II, when $z_1+z_2 \neq z_3-z_2$, zero center distance of wheels are different

If we do not take the correction type P0 into account, then assuming the value of x_2 we obtain other correction factors

$$
\begin{aligned}
&x_1 = C - x_2 \ \wedge \ x_3 = -C - x_2 \ \Rightarrow x_1 - x_3 = 2 \cdot C \\
&r_{w2}^{`} = 0.5 \cdot z_2 \cdot \frac{\cos\alpha}{\cos\alpha_w^{`}} \ ; \ r_{w2}^{``} = 0.5 \cdot z_2 \cdot \frac{\cos\alpha}{\cos\alpha_w^{``}} \\
&r_{w2}^{`} = r_{w2}^{``} = r_{w2}
\end{aligned}
\tag{10}
$$

In case II, when $z_1+z_2 \neq z_3-z_2$, the zero center distances of the wheels are the different, then$\alpha_w'' \neq \alpha_w'$, and in accordance with (4) C1≠C2 the relations between the corrections coefficients are limited by equations

$$
\begin{aligned}
&x_1 + x_2 = C1 \ , \ x_2 - x_3 = C2 \\
&r_{w2}^{`} = 0.5 \cdot z_2 \cdot \frac{\cos\alpha}{\cos\alpha_w^{`}} \ ; \ r_{w2}^{``} = 0.5 \cdot z_2 \cdot \frac{\cos\alpha}{\cos\alpha_w^{``}} \ . \\
&r_{w2}^{`} \neq r_{w2}^{``}
\end{aligned}
\tag{11}
$$

Then the satellite wheel has one radius of the pitch diametric circle but it has a pair of rolling circles with the radii' r_w‘ and r_w“.

The formulas for determining the main dimensions describe the conjunctions of the teeth without backlash (pitch play). In the cutting process of the satellite we have to decrease the correction factor x_2, in order to obtain the operational backlash value L_b expressed by the following formula for amendment Δx_2.

$$
\Delta x_2 = \frac{-2 \cdot L_b}{m \cdot \sin\alpha} \tag{12}
$$

3. Main criteria of fatigue and surface strength

In engineering practice, we use the following ISO [4] formulas to compute the fatigue stresses at the root of teeth σ_F and surface fatigue stresses σ_H in the zone of one pair of contact teeth,

$$\sigma_{F1,2} = \frac{F_t K_F}{b_w m_n} Y_{\mathrm{FS}1,2} Y_\varepsilon Y_\beta \le \sigma_{\mathrm{FP}}, \tag{13}$$

$$\sigma_H = Z_E Z_H Z_\varepsilon Z_\beta \sqrt{\frac{F_t K_H}{b_w d_1} \frac{u+1}{u}} \le \sigma_{\mathrm{HP}}, \tag{14}$$

where: σ_{FP} – is the allowable stress at the root, σ_{HP} – is the allowable surface stress, F_t – is the nominal circumferential load of the cooperating wheels, b_w – the width of the wheels, d_1 – the pitch diameter of a pinion, m_n – the gear module of the pair of cooperating teeth. The calculation variables K_F, K_H, Y_{FS}, Y_ε, Y_β, Z_H, Z_E, Z_ε, Z_β and allowable stresses from the formulas (13), (14) are defined in [4].

The fatigue stresses at the root of teeth σ_F and surface fatigue stresses σ_H depend on relations $Y_{FS}(x,z)$, $Z_H(\rho_{B2})$ where $\rho_{B2}(x,z_1,z_2)$ is the reduced radius of curvature for the mating moment at the contact point of one pair, B_2.

Fig.5a shows the line of contact (actions) ξ, the segment of pressure contact E_1E_2, the radii of basic circles r_{b1}, r_{b2}, the radii of the addendum circles r_{a1}, r_{a2}, the pitch on basic circles p_b and the pressure angle α_w, ε_α – the tooth contact ratio.

Now we can compare the radius ρ_{B2} (x) with $\rho_{B2}\big|_{x_1=x_2=0} = \rho_o$ and build the non-dimensional function for the central and satellite wheels z_1 /z_2 .

$$\kappa(x) = \frac{\rho_{B2}(x)}{\rho_o} \tag{15}$$

In geometrical sense, the strength of the tooth working surfaces depends only on the reduced radius of curvature and the value of function κ(x) must not be less then 1, taking proper corrections coefficients <x>.

The relation $\rho(\xi)$, well known in literature [8] and illustrated in Fig.5b, is presented by the formula

$$\rho(\xi) = \xi \cdot \left(1 - \frac{\xi}{l}\right) \tag{16}$$

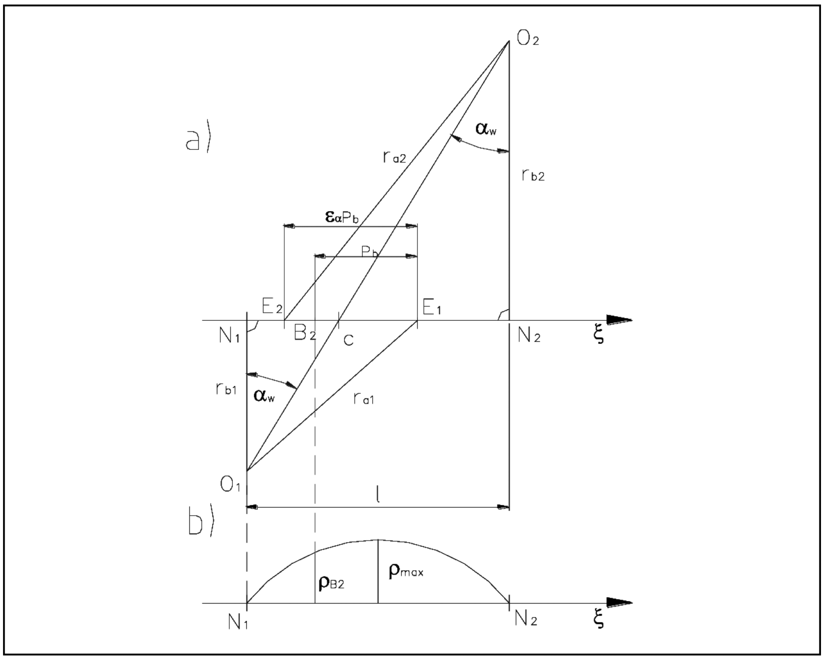

Fig. 5. Engagement of toothed wheels z_1/z_2, a) along the line of action ξ at segment $E_1E_2 \subset N_1N_2$, b) reduced radius of curvature along the line of contact $\rho(\xi)$

Let us transform the formula (15) to apply it to the teeth of matting wheels after replacing its values by the parameters of a toothed gear. Consequently κ(x) can be written thus

$$\rho_{B2}(x) = \eta(x) \cdot (1 - \eta(x)) \cdot \frac{l}{4} \;\; ; \;\; \eta(x) = \frac{\xi_B}{l}$$
$$\rho_o = \eta_o \, (1 - \eta_o) \cdot \frac{l_o}{4} \tag{17}$$

Replacing,

$$\xi_B = \sqrt{r_{a1}^2 - r_{b1}^2} - p_b \;\; ; \;\; l = a \cdot \sin(\alpha_w) = a_o \cdot \cos(\alpha) \cdot tg\,(\alpha_w) \tag{18}$$

then substituting relations (19) into it

$$\frac{p_b}{a_o} = \frac{2 \cdot \pi}{z_1 + z_2} \cos(\alpha) \;\; ; \;\; \frac{r_{b1}}{a_o} = \frac{z_1}{z_1 + z_2} \cos(\alpha) \;\; ; \;\; \frac{r_{a1}}{a_o} = \frac{z_1 + 2 \cdot (1 + x_1)}{z_1 + z_2} \tag{19}$$

then we get η(x) with restriction <x>

$$\begin{aligned}
\eta(x) &= \frac{\sqrt{[z_1 + 2\cdot(1+x_1)]^2 - [(z_1\cos(\alpha))]^2} - 2\cdot\pi\cos(\alpha)}{(z_1+z_2)\cdot\cos(\alpha)\cdot\tan(\alpha_w^{/})} \\
\eta_o &= \frac{\sqrt{[z_1 + 2]^2 - [(z_1\cos(\alpha))]^2} - 2\cdot\pi\cos(\alpha)}{(z_1+z_2)\cdot\cos(\alpha)\cdot\tan(\alpha)}
\end{aligned} \tag{20}$$

$$\cos(\alpha_w) = \frac{a}{a_o}\cos(\alpha)$$

$$x_1 + x_2 = c1 = \frac{\{[\tan(\alpha_w^{/}) - \alpha_w^{/}] - \tan(\alpha) + \alpha\}\cdot(z_1+z_2)}{2\cdot\tan(\alpha)}$$

$$x_{1\,\min} \le x_1 \le x_{1\,\max} \quad \wedge \quad x_{2\,\min} \le x_2 \le x_{2\,\max}$$

Substituting the equations (17), (20) into (15) we can obtain the function $\kappa(x_1, x_2)$ for the central and the satellite wheels z_1 /z_2 .

$$\begin{aligned}
\kappa(x) &= A(x)\frac{1-\eta(x)}{1-\eta_o} \\
A(x) &= \frac{\sqrt{[z_1 + 2\cdot(1+x_1)]^2 - [(z_1\cos(\alpha))]^2} - 2\cdot\pi\cos(\alpha)}{\sqrt{[z_1 + 2]^2 - [(z_1\cos(\alpha))]^2} - 2\cdot\pi\cos(\alpha)}
\end{aligned} \tag{21}$$

In design calculations the stresses of the root of teeth $\sigma_F(x,z)$ determine the fatigue resistance of each toothed wheel; the formula (13) depends on the form factor of the tooth $Y_{F\alpha}(x,z)$.

The Form factor $Y_{F\alpha}(x,z)$ is determined by ISO [4] in the form of relations and diagrams referring to cutting tools parameters. The magnitude of this factor reaches the minimum value of about 1.9 when a standard tool rack has α =20°, h_{ao}/m=1.25, and ρ/m=0.2), where ρ – the radius of tools fillet.

Consequently, we build the non-dimensional function for determining the fatigue resistance of each toothed wheel

$$\lambda(x) = \frac{1.9}{Y_{F\alpha}} \tag{22}$$

At valuables region <x>for each toothed wheel z_1, z_2, z_3 we can evaluate the fatigue strength when λ(x) reaches the maximum value [2], [5].

4. Examples of projects

Case I, Fig.4

We take under consideration a planetary gear with the data: z_1 =24, z_2 =45, z_3 =114, y=1, α =20°, m=4 according to Fig. 1 so that some restrictions concerning the proper number of the teeth z_1; z_2; z_3 are satisfied. An analysis of strength needs a diagram $\lambda(x)$ for each wheel. On the basis of the ISO standard diagram [4] it is easy to build by parabolic interpolation functions such as in Fig. 6, which gives us an opportunity to find some different acceptable solutions. Table 1. contains six propositions: no.1 – a gear without corrections, no.2 – a gear with corrections P0, no.3,4,5 – a gear with correction P and the constant value of C = $x_1 + x_2$, and no.6 – a gear with correction P and the distance of axis $a \neq a_o$.

According to Table 1, the best solution (no.4) yields κ =1.1403 (Table 2) which means that the surface strength increases about 6.8%, λ_1 changing from 0.69 to 0.81, the fatigue strength of the central wheel increases about 17.3%, other wheels have proper fatigue strength λ_2 =0.75, λ_3 =0.81. According to relations (9) and (10) α_w' =21.475^oα_w'' =21.475° and r_{w2}' = r_{w2}''. The Resultant dimensions of teeth engagement are presented in Table 3, and the proof of noninterference of the satellite and the sun wheels is presented in Table 4.

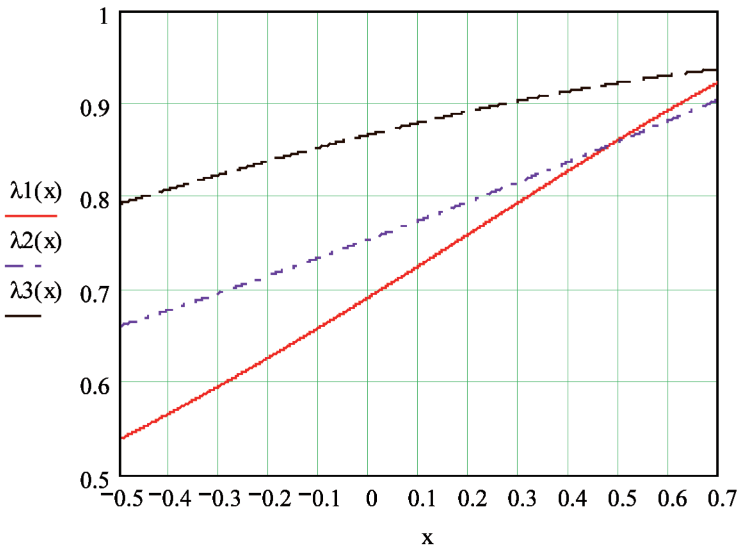

Fig. 6. Fatigue strength $\lambda(x)$ for wheels z_1 =24, z_2 =45, z_3 =114

Table 1.

$x_1 + x_2 = x_2 + x_3 = 0$; $C = 0$; *corrections P0* $x_1 = C - x_2$; $x_3 = -C - x_2$; $C \neq 0$; *corrections P*							
No.	x_3	x_2	x_1	λ_3	λ_2	λ_1	κ
1	0	0	0	0.87	0.75	0.69	1.00
2	0.12	-0.12	0.12	0.88	0.73	0.73	1.037
3	-0.25	0	0.25	0.83	0.75	0.78	1.1139
4	-0.35	0	0.35	0.82	0.75	0.81	1.1403
5	-0.4	0.1	0.2	0.81	0.78	0.76	1.0933
6	-0.3567	0.1	0.1567	0.82	0.78	0.75	1.0759
	a = 139.00 mm ; a_o =138 mm						

Table 2.

$\alpha_{w'} = 0.3748 \qquad \alpha = 0.3491 \qquad x_1 = 0.35 \qquad x_2 = 0$

$$\eta := \frac{\sqrt{\left[z_1 + 2\cdot(1 + x_1)\right]^2 - \left(z_1\cdot\cos(\alpha)\right)^2} - 2\cdot\pi\cdot\cos(\alpha)}{(z_1 + z_2)\cdot\cos(\alpha)\cdot\tan(\alpha_{w'})} \qquad \eta = 0.3288$$

$$\eta_o := \frac{\sqrt{(z_1 + 2)^2 - \left(z_1\cdot\cos(\alpha)\right)^2} - 2\cdot\pi\cdot\cos(\alpha)}{(z_1 + z_2)\cdot\cos(\alpha)\cdot\tan(\alpha)} \qquad \eta_o = 0.298$$

$$A := \frac{\sqrt{\left[z_1 + 2\cdot(1 + x_1)\right]^2 - \left(z_1\cdot\cos(\alpha)\right)^2} - 2\cdot\pi\cdot\cos(\alpha)}{\sqrt{(z_1 + 2)^2 - \left(z_1\cdot\cos(\alpha)\right)^2} - 2\cdot\pi\cdot\cos(\alpha)} \qquad A = 1.1926$$

$$\kappa := A\cdot\frac{1 - \eta}{1 - \eta_o} \qquad \kappa = 1.1403$$

Table 3.

$z_1 = 24 \qquad z_2 = 45 \qquad z_3 = 114 \qquad m = 4$

$r_{w1} = 48.4702 \qquad r_{w2'} = 90.8817 \qquad r_{w2''} = 90.8817 \qquad r_{w3} = 230.2336$

$r_{b1} = 45.1052 \qquad r_{b2} = 84.5723 \qquad r_{b3} = 214.2499$

$r_{a1} = 53.3519 \qquad r_{a2} = 93.9519 \qquad r_{a3} = 225.4487 \qquad \alpha_{w's} = 21.4752$

$r_{f1} = 44.4 \qquad r_{f2} = 85 \qquad r_{f3} = 234.4003 \qquad \alpha_{w''s} = 21.4752$

$r_1 = 48 \qquad r_2 = 90 \qquad r_3 = 228 \qquad a_{o12} = 138 \qquad a_{o23} = 138$

$x_1 = 0.35 \qquad x_2 = 0 \qquad x_3 = -0.3501 \qquad a = 139.3519$

Table 4.

$$Z_2 := z_3 \cdot \left[1 - \sqrt{1 - \frac{1}{\sin(\alpha_{w''})^2} \cdot \left[1 - \left(\frac{r_{a3}}{r_{w3}} \right)^2 \right]} \right] \qquad Z_2 = 19.0926$$

$z_2 > Z_2$

$$\phi_3 := \operatorname{acos}\left(\frac{r_{a3}^2 - r_{a2}^2 + a^2}{2 \cdot a \cdot r_{a3}} \right) \qquad \phi_3 = 0.2126$$

$$\phi_2 := \operatorname{acos}\left(\frac{r_{a3}^2 - r_{a2}^2 - a^2}{2 \cdot a \cdot r_{a2}} \right) \qquad \phi_2 = 0.5308$$

$$\theta_3 := \tan(\alpha_{w''}) - \alpha_{w''} - \tan\left(\operatorname{acos}\left(\frac{r_{b3}}{r_{a3}} \right) \right) + \operatorname{acos}\left(\frac{r_{b3}}{r_{a3}} \right) \qquad \theta_3 = 7.5868 \times 10^{-3}$$

$$\theta_2 := -\tan(\alpha_{w''}) + \alpha_{w''} + \tan\left(\operatorname{acos}\left(\frac{r_{b2}}{r_{a2}} \right) \right) - \operatorname{acos}\left(\frac{r_{b2}}{r_{a2}} \right) \qquad \theta_2 = 0.0146$$

$$L := \phi_3 - \theta_3 \qquad L = 0.205 \qquad Ls := L \cdot \frac{180}{\pi} \qquad Ls = 11.7447$$

$$P := (\phi_2 + \theta_2) \cdot \frac{z_2}{z_3} \qquad P = 0.2153 \qquad Ps := P \cdot \frac{180}{\pi} \qquad Ps = 12.3362$$

$L < P$ no interference

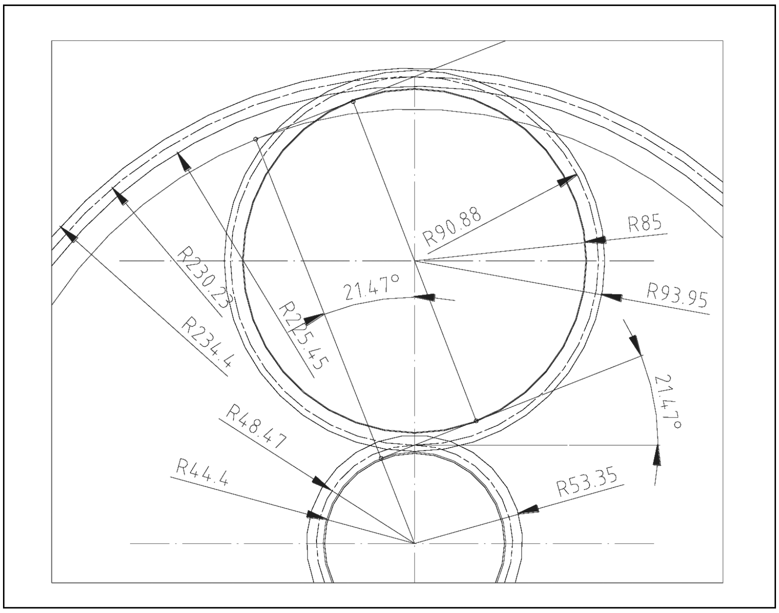

Fig. 7. Geometric parameters of designed gears $z_1 = 24$, $z_2 = 45$, $z_3 = 114$

The fundamental geometric parameters of the designed gears are shown in Fig. 7 and the geometry of the cooperating toothed wheels is demonstrated in Fig. 8.

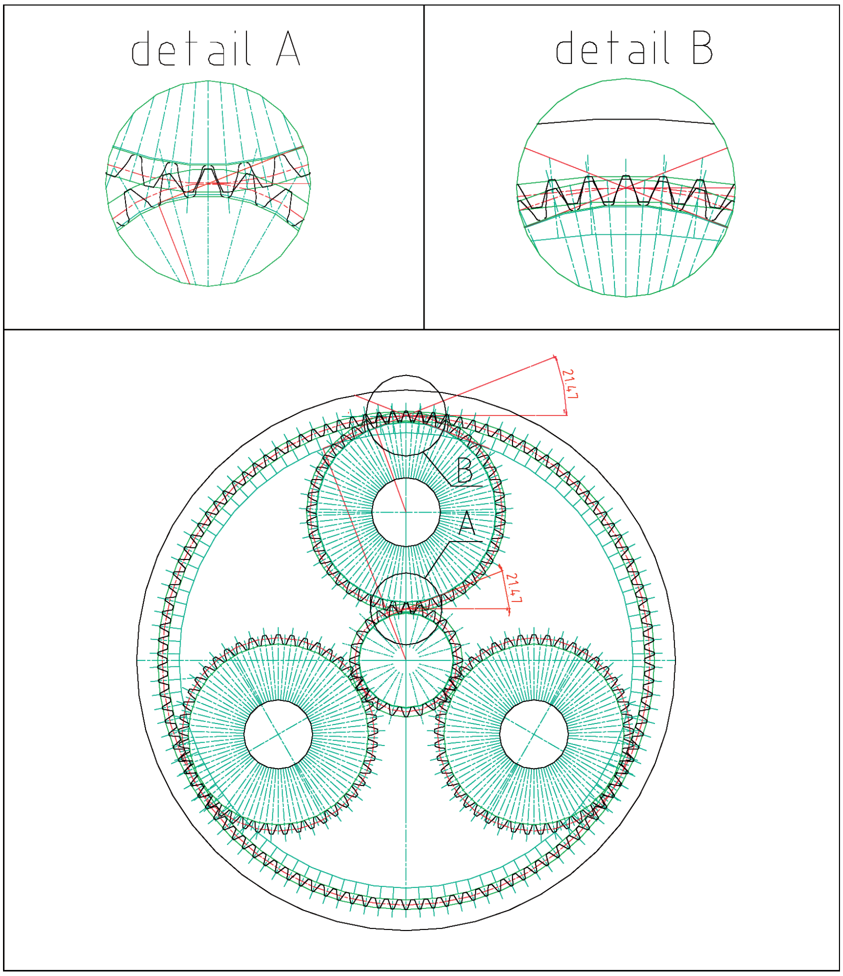

Fig. 8. Geometry of cooperating planetary toothed wheels z_1 =24, z_2 =45, z_3 =114

Case II, Fig.4

We take under consideration a planetary gear with the data: z_1 =24, z_2 =46, z_3 =117, y=1, α =20^o, m=4 according to Fig.1 so that some restrictions concerning the proper number of the teeth z_1; z_2; z_3 are satisfied. An analysis of strength needs a diagram λ(x) for each wheel. On the basis of the ISO standard [4], a diagram such as in Fig. 9, gives us an opportunity to find some acceptable solutions. Table 5 contains propositions with only the correction P possible, as was mentioned before $\alpha_w' \neq \alpha_w''$ and $r_{w2}' \neq r_{w2}''$. In this case, it is more convenient to assume a different distance of the axis "a", taking $x_2 > 0.1$ (Fig. 1) and then checking the results λ_1, λ_2, λ_3, κ the condition of noninterference of the satellite and the sun wheels, and the fulfillment of inequalities (1) and (2).

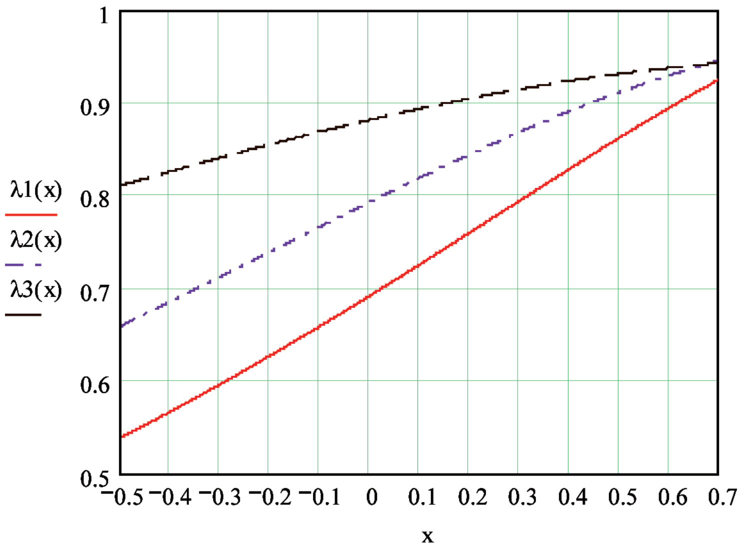

Fig. 9. Fatigue strength λ(x) for wheels z_1 =24, z_2 =46, z_3 =117

Table 5.

$x_1 = C1 - x_2$; $x_3 = C2 - x_2$; *corrections P*							
No.	x_3	x_2	x_1	λ_3	λ_2	λ_1	κ
1	0.0233	0.1	0.2896	0.88	0.82	0.79	1.1284
	a = 141.50 mm ; a_{o12} =140 mm; a_{o23} =142 mm						
2	-0.1	0.1	0.4257	0.87	0.82	0.83	1.1792
	a = 142.00 mm; a_{o12} =140 mm; a_{o23} =142 mm						
3	-0.3266	0.2	0.4647	0.84	0.84	0.84	1.2032
	a = 142.50 mm; a_{o12} =140 mm; a_{o23} =142 mm						

According to Table 5, the best solution (no.3) yields κ =1.2032 that means the surface strength increases about 9.8%, and all the wheels have the proper fatigue strength $\lambda_1 = \lambda_2 = \lambda_3 = 0.84$. On the basis of relation (9) and (10) we obtain $\alpha_w^{'}$ =22.6005°$\alpha_w^{''}$ =20.5452°, and the procedure of computing the coefficient of corrections is presented in Table 6.

Table 6.

$m := 4 \qquad \alpha := \pi\cdot\frac{20}{180} \qquad z_1 := 24 \qquad z_2 := 46 \qquad z_3 := 117$

$a := 142.50 \qquad z_1 + z_2 = 70 \qquad z_3 - z_2 = 71 \qquad \alpha = 0.3491$

$a_{o12} := 0.5(z_1 + z_2)\cdot m \qquad a_{o12} = 140$

$a_{o23} := 0.5(z_3 - z_2)\cdot m \qquad a_{o23} = 142$

$\alpha_{w'} := \mathrm{acos}\left(\frac{a_{o12}}{a}\cdot\cos(\alpha)\right) \qquad \alpha_{w'} = 0.3945 \qquad \alpha_{w''} := \alpha_{w'}$

$\alpha_{ws'} := \alpha_{w'}\cdot\frac{180}{\pi} \qquad \alpha_{ws'} = 22.6005$

$C1 := \left[\left(\tan(\alpha_{w'}) - \alpha_{w'} - \tan(\alpha) + \alpha\right)\cdot\frac{(z_1 + z_2)}{2\cdot\tan(\alpha)}\right] \qquad C1 = 0.6647$

$\alpha_{w''} := \mathrm{acos}\left(\frac{a_{o23}}{a}\cdot\cos(\alpha)\right) \qquad \alpha_{w''} = 0.3586$

$\alpha_{ws''} := \alpha_{w''}\cdot\frac{180}{\pi} \qquad \alpha_{ws''} = 20.5452$

$C2 := \left[\left(\tan(\alpha_{w''}) - \alpha_{w''} - \tan(\alpha) + \alpha\right)\cdot\frac{(z_2 - z_3)}{2\cdot\tan(\alpha)}\right] \qquad C2 = -0.1266$

$x_2 := 0.2 \qquad x_1 := C1 - x_2 \qquad x_1 = 0.4647$

$x_3 := C2 - x_2 \qquad x_3 = -0.3266$

Table 7.

$m_{w12} := m\cdot\frac{\cos(\alpha)}{\cos(\alpha_{w'})} \qquad m_{w12} = 4.0714$

$r_1 := 0.5z_1\cdot m \qquad r_1 = 48 \qquad r_2 := 0.5z_2\cdot m \qquad r_2 = 92 \qquad a_{o12} := r_1 + r_2 \qquad a_{o12} = 140$

$r_{b1} := r_1\cdot\cos(\alpha) \qquad r_{b1} = 45.1052 \qquad r_{b2} := r_2\cdot\cos(\alpha) \qquad r_{b2} = 86.4517$

$r_{w1} := 0.5z_1\cdot m_{w12} \qquad r_{w1} = 48.8571 \qquad r_{w2'} := 0.5z_2\cdot m_{w12} \qquad r_{w2'} = 93.6429$

$r_{f2} := r_2 - (1 - x_2)\cdot m - c \qquad r_{f2} = 87.8 \qquad a := r_{w1} + r_{w2'} \qquad a = 142.5$

$r_{f1} := r_1 - (1 - x_1)\cdot m - c \qquad r_{f1} = 44.8589$

$\Delta h := (x_1 + x_2)\cdot m - (a - a_{o12}) \qquad \Delta h = 0.1589 \qquad h_z := 2.25m - \Delta h \qquad h_z = 8.8411$

$r_{a1} := r_1 + (1 + x_1)\cdot m - \Delta h \qquad r_{a1} = 53.7$

$r_{a2} := r_2 + (1 + x_2)m - \Delta h \qquad r_{a2} = 96.6411$

$c12 := a - r_{f1} - r_{a2} \qquad c12 = 1$

$c21 := a - r_{a1} - r_{f2} \qquad c21 = 1$

The resultant dimensions of teeth engagement z_1/z_2 i.e. the radii of the diametric circle, the basic circle, the rolling circle, the addendum and the dedendum circle, and also the tip clearance c12 i c21 are presented in Table 7. In a similar way, the resultant dimensions of the teeth engagement z_2/z_3 are calculated additionally to verify the thickness of the teeth in the rolling circles s_2, s_3. The results are presented in Table 8. The collected results of the dimensions of teeth engagement are presented in Table 9; Fig. 10 illustrates the main geometric parameters of the designed planetary gears.

Table 8.

$$r_3 := 0.5z_3\cdot m \qquad r_3 = 234 \qquad r_2 := 0.5z_2\cdot m \qquad r_2 = 92$$

$$r_{b3} := r_3\cdot\cos(\alpha) \qquad r_{b3} = 219.8881 \quad r_{b2} := r_2\cdot\cos(\alpha) \qquad r_{b2} = 86.4517$$

$$m_{w23} := \frac{2a}{z_3 - z_2} \qquad m_{w23} = 4.0141$$

$$C2 := \left(\tan(\alpha_{w''}) - \tan(\alpha) - \alpha_{w''} + \alpha\right)\cdot\frac{(z_2 - z_3)}{2\cdot\tan(\alpha)} \qquad C2 = -0.1266$$

$$x_2 = 0.2 \qquad x_3 := C2 - x_2 \qquad x_3 = -0.3266 \qquad \alpha = 0.3491$$

$$m_{w23} := m\cdot\frac{\cos(\alpha)}{\cos(\alpha_{w''})} \qquad m_{w23} = 4.0141 \qquad \alpha_{w''} = 0.3586 \quad \alpha_{w'} = 0.3945$$

$$r_{w3} := 0.5z_3\cdot m_{w23} \qquad r_{w3} = 234.8239 \quad r_{w2''} := 0.5z_2\cdot m_{w23} \qquad r_{w2''} = 92.3239$$

$$a_{o23} := 0.5(z_3 - z_2)\cdot m \qquad a_{o23} = 142 \quad a := r_{w3} - r_{w2''} \qquad a = 142.5$$

$$a_{o12} := 0.5(z_1 + z_2)\cdot m \qquad a_{o12} = 140 \qquad c := 0.25m \qquad c = 1$$

$$r_{f3} := r_3 + (1 - x_3)\cdot m + c \qquad r_{f3} = 240.3066 \quad h_{a3} := (1 + x_3)\cdot m \qquad h_{a3} = 2.6934$$

$$r_{f2} := r_2 - (1 - x_2)\cdot m - c \qquad r_{f2} = 87.8 \qquad h_{f3} := (1 - x_3)\cdot m + c \qquad h_{f3} = 6.3066$$

$$h_z := 2.25m - \Delta h \qquad h_z = 8.8411 \qquad \Delta h := 0.0267$$

$$r_{a3} := r_{f3} - h_z \qquad r_{a3} = 231.4655 \quad r_{a2} := r_{f2} + h_z \qquad r_{a2} = 96.6411$$

$$c23 := r_{f3} - a - h_z - r_{f2} \qquad c23 = 1.1655$$

$$c32 := r_{f3} - r_{a2} - a \qquad c32 = 1.1655$$

$$s_2 := 2\cdot r_{w2''}\left(\frac{\frac{\pi}{2} + 2x_2\cdot\tan(\alpha)}{z_2} + \tan(\alpha) - \alpha - \tan(\alpha_{w''}) + \alpha_{w''}\right)$$

$$s_2 = 6.65$$

$$s_3 := 2\cdot r_{w3}\cdot\left(\frac{\frac{\pi}{2} + 2x_3\cdot\tan(\alpha)}{z_3} + \tan(\alpha_{w''}) - \alpha_{w''} - \tan(\alpha) + \alpha\right)$$

$$s_3 = 5.9607 \qquad s_3 + s_2 = 12.6106 \qquad \pi\cdot m_{w23} = 12.6106$$

Table 9.

$z_1 = 24$	$z_2 = 46$	$z_3 = 117$	$m = 4$		
$r_{w1} = 48.8571$	$r_{w2'} = 93.6429$	$r_{w2''} = 92.3239$	$r_{w3} = 234.8239$		
$r_{b1} = 45.1052$	$r_{b2} = 86.4517$	$r_{b3} = 219.8881$			
$r_{a1} = 53.7$	$r_{a2} = 96.6411$	$r_{a3} = 231.4655$	$\alpha_{w'} = 0.3945$	$\alpha_{ws'} = 22.6005$	
$r_{f1} = 44.8589$	$r_{f2} = 87.8$	$r_{f3} = 240.3066$	$\alpha_{w''} = 0.3586$	$\alpha_{ws''} = 20.5452$	
$r_1 = 48$	$r_2 = 92$	$r_3 = 234$	$a_{o12} = 140$	$a_{o23} = 142$	
$x_1 = 0.4647$	$x_2 = 0.2$	$x_3 = -0.3266$	$a = 142.5$		

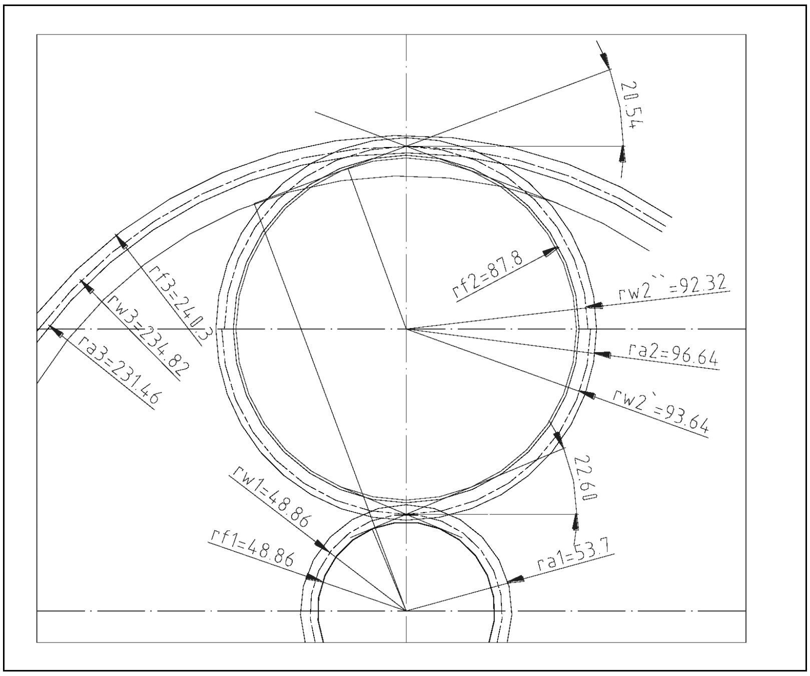

Fig. 10. Geometric parameters of designed gears $z_1 = 24$, $z_2 = 46$, $z_3 = 117$

5. Conclusion

In this study, we have presented a new design procedure for determining modification coefficients of the toothed wheels of a planetary gear train with internal conjunction of teeth. It is possible to obtain a higher load-carrying capacity, which depends also on the correction coefficients.

We have proposed the procedure for planetary gears with internal engagements, in which the non-dimensional strength function $\lambda(x,z)$ and $\kappa(x,z)$

is used. In this way, the new allowable geometrical solution of a planetary gear can be evaluated.

The described relations gives us the opportunity to compute all geometrical parameters of the cooperating wheels, which can be manufactured on an industrial scale.

The presented procedure and the projects may be useful for developing a more effective, optimal computer procedure.

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REFERENCES

[1] Podstawy Konstrukcji Maszyn, *pod redakcją Marka Dietricha,* PWN, Warszawa, 1995.

[2] Dziama L., Michniewicz M., Niedzwiedzki A.: Przekładnie zębate. PWN, Warszawa, 1989.

[3] Ochęduszko K.: Koła zębate, t. 1, WNT, Warszawa, 1990.

[4] PN-ISO 6336-1/2/3.

[5] Kasperek T., Ryś J.: Dobór korekcji w przekładni planetarnej o identycznej geometrii satelitów, Czasopismo Techniczne, z.7 1-M/2010.

[6] Ryś J., Kasperek T.: Geometria przekładni planetarnej z zazębieniem zewnętrznym o uproszczonej technologii, Czasopismo Techniczne, z. 1 1-M/2006.

[7] Ryś J., Sendyka B.: Ustalenie obszaru rozwiązywalności II poprawki kół o zarysach ewolwentowych, Czasopismo Techniczne, z. 6, 1978.

[8] Ryś J., Skrzyszowski Z.: Podstawy konstrukcji maszyn, t. 2, Wyd. PK, Kraków 2003.

[9] Ryś J., Kasperek T.: Project of planetary gear with the same geometry of a satellite wheel, The Archive of Mechanical Engineering, vol. LV, 2008, 3, p. 265-273.

Metoda doboru współczynników korekcji w zębatych przekładniach planetarnych

Streszczenie

Celem pracy jest prezentacja procedury obliczeniowej określającej geometrię i wpływ współczynników korekcji na wytrzymałość postaciową i kontaktową powierzchni zębów w planetarnych przekładniach zębatych z zazębieniem wewnętrznym kół satelitów. Rozważane są przypadki, gdy zerowe odległości osi koła centralnego i satelitów są równe, lub nie zerowej odległości osi satelitów i koła słonecznego. Wymiary geometryczne będą opisane na bazie dopuszczalnej korekcji oraz warunków w odniesieniu do wymagań normy ISO oraz literatury. Procedura obliczeń zaprezentowana na przykładach może być zastosowana do dowolnej przekładni planetarnej o innym kinematycznym połączeniu kół.