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ANDRZEJ BUCHACZ *

ANALYSIS OF BEAM HYPERGRAPHS BY MEANS OF EXACT AND APPROXIMATE METHODS AS MODELS OF TRANSVERSE VIBRATING SUBSYSTEMS IN THE SYNTHESIS OF MECHANICAL AND MECHATRONIC SYSTEMS

In this paper, the author compares the of characteristics of subsystems obtained by the approximate and exact method in order to answer to the question – if the approximate method can be used to nominate the characteristics of mechatronic systems. Frequency – modal analysis has been presented for a mechanical system, i.e. transverse-vibrating clamped-free beam. Consequently, the model of the beam was presented in a five-vertex hypergraph. This model, in the case of approximate frequency-modal analysis, can be imitated in a three-vertex hypergraph. Such formulation could be the introduction to synthesis of transverse-vibrating complex beam systems with constant cross-section.

1. Introduction

The problems of analysis of vibrating beam systems, discrete and discrete-continuous mechanical systems by means of the structural numbers methods modelled by graphs and hypergraphs have been investigated in the Gliwice research Centre (e.g.[1, 2]). The problems of synthesis of electrical systems [3] and of a selected class of continuous, discrete - continuous discrete mechanical systems and active mechanical systems have been dealt with [4-8,10]. The continuous-discrete torsionally and transverse vibrating mechatronic systems were considered in [9-12]. The approximate method of analysis, called the Galerkin's method, has been used to obtain the frequency-modal characteristics. To compare the obtained dynamical characteristics – dynamical flexibilities only for mechanical torsionally vibrating bar and transverse vibrating beam being a parts of complex mechatronic systems, we used an

* *Silesian University of Technology, 44-100 Gliwice, 18 A. Konarskiego st, Poland, E-mail: andrzej.buchacz@polsl.pl*

exact method and the Galerkin's method [9, 11, 12]. In this paper, frequency – modal analysis is presented for a mechanical system, a transverse-vibrating clamped-free beam. The model of the beam is presented in a five-vertex hypergraph, which in the case of approximate frequency-modal analysis we can imitate in a three-vertex hypergraph. Such formulation could be the introduction to synthesis of transverse-vibrating complex beam systems with constant cross-section.

2. Vibration free beam as the subsystem of beam-system

2.1. The dynamical flexibility of the beam – solution by the exact method

We consider a beam – treated as a subsystem of a mechatronic system* – with constant cross section, clamped on the left end and free on the right one, with harmonic excitation force in the form $P(t) = P_0 \sin \omega t$. The equation of motion of the beam takes the form

$$EJ_Z\, y(x,t)_{,xxxx} + \rho A y(x,t)_{,tt} = 0, \tag{1}$$

where: $y(x,t)$ – deflection at the time moment t of the lining beam section within the distance x from the origin of the system,E – Young's modulus, ρ – mass density of material of the beam, J_Z – polar inertia moment of the beam cross section, A – area of the beam cross section.

The boundary conditions on the beam ends are the following

$$y(0,t) = 0, \quad y(0,t)_{,x} = 0, \quad y(l,t)_{,xx} = 0, \quad EJ_Z\, y(l,t)_{,xxx} = -P(t), \tag{2}$$

where: l – length of the beam.

The solution $y(x,t)$ to equation (1) is a harmonic function

$$y(x,t) = X(x) \sin \omega t. \tag{3}$$

Determining suitable derivatives of (3) and substituting them into (2) the set of equations, we obtain, after transformations

$$\begin{cases} A(\cosh kl - \cos kl) + B(\sin kl + \sinh kl) = \dfrac{-P_0}{EJ_Z k^3}, \\ A(\sinh kl - \sin kl) + B(\cosh kl + \cos kl) = 0. \end{cases} . \tag{4}$$

which can be written in matrix form

$$\mathbf{WA} = \mathbf{F}, \tag{5}$$

* The mechatronic system was considered in [11].

where:

$$\mathbf{W} = \begin{vmatrix} (\cosh kl - \cos kl), (\sin kl + \sinh kl) \\ (\sinh kl - \sin kl), (\cosh kl + \cos kl) \end{vmatrix}, \mathbf{A} = \begin{vmatrix} A \\ B \end{vmatrix}, \mathbf{F} = \begin{vmatrix} \dfrac{-P_0}{EJ_Z k^3} \\ 0 \end{vmatrix}.$$

The main determinant of the set of equations (5) is equal to

$$|\mathbf{W}| = 2(1 - \cos kl \cosh kl) \tag{6}$$

To determine the constants A, B, we should calculate the following determinants

$$|\mathbf{W}_A| = \begin{vmatrix} \dfrac{-P_0}{EJ_Z k^3} & (\sin kl + \sinh kl) \\ 0 & \cosh kl - \cos kl \end{vmatrix} = -\frac{P_0}{EJ_Z k^3}(\cosh kl - \cos kl), \tag{7}$$

$$|\mathbf{W}_B| = \begin{vmatrix} (\cosh kl - \cos kl) & \dfrac{-P_0}{EJ_Z k^3} \\ (\sinh kl - \sin kl) & 0 \end{vmatrix} = \frac{P_0}{EJ_Z k^3}(\sinh kl - \sin kl). \tag{8}$$

According to (4-8), the constants A, B are equal to:

$$A = -C = \frac{|\mathbf{W}_A|}{|\mathbf{W}|} = -\frac{P_0(\cosh kl - \cos kl)}{2EJ_Z k^3(1 + \cos kl \cosh kl)}. \tag{9}$$

$$B = -D = \frac{|\mathbf{W}_B|}{|\mathbf{W}|} = \frac{P_0(\cos kl + \cosh kl)}{2EIk^3(1 + \cos kl \cosh kl)}. \tag{10}$$

Substituting expressions (9) and (10) into (3), after transformations, we obtain the beam deflection function

$$y(x,t) = -P_0 \sin \omega t \left[\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx) -}{2EJ_Z k^3(1 - \cos kl \cosh kl)} \frac{(\sinh kl - \sin kl)(\cos kx + \cosh kx)}{} \right]. \tag{11}$$

Dynamic flexibility, calculated according to its definition on the basis of (11), takes the form

$$Y = -\frac{(\cosh kl - \cos kl)(\sin x + \sinh kx) - (\sinh kl - \sin kl)(\cos kx + \cosh kx)}{2EJ_Z k^3(1 - \cos kl \cosh kl)}. \tag{12}$$

The graph of absolute value of dynamical flexibility (12) is drawn in Fig. 1 for $x = l$, which means $\alpha_Y = |Y|$.

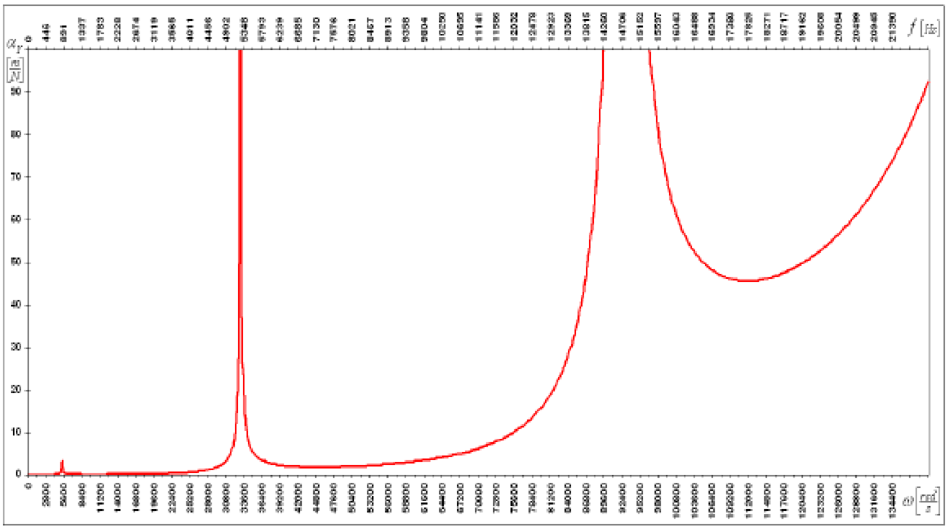

Fig. 1. The plot of dynamical flexibility of transverse vibrating continuous system

2.2. Galerkin's method of calculation of dynamical flexibility of the beam

It should be considered that, if the shaft is under the action of moment with continuous factorization through the beam length with the value $F(x)\sin\omega t$ on the length unit – then the equation of motion of the element of length dx located at the point x is:

$$EJ_Z y_{,xxxx}\mathrm{d}x + \rho A y_{,tt}\mathrm{d}x = F(x)\sin\omega t\mathrm{d}x, \tag{13}$$

To determine the dynamic flexibility, we must find the factors, which depend on the concentrated loading $F(x)\sin\omega t$ acting at the point z. The loading can be considered as a limit of the concentrated loading through the length-as follows:

$$F(x) = \begin{cases} \dfrac{F}{h} & \text{when} \quad z - h \le x \le z, \\ 0 & \text{in other section} \end{cases} \tag{14}$$

and the equation of excited vibrations of the beam can be written as

$$EIy'_{xxxx} + \rho Ay'_{tt} = P_0 \sin\omega t \tag{15}$$

where: $P_0 = \dfrac{F}{h}$.

The deflection of beam – the solution to (15) by means of the Galerkin's method is given in the form

$$y(x,t) = \sum_{n=1}^{\infty} y_n(x,t) = \sum_{n=1}^{\infty} A_n \sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\omega t. \tag{16}$$

Substituting the following derivative of function (16) into (15) we obtain

$$\begin{gathered} EIA_n\left[(2n-1)\frac{\pi}{2l}\right]^4 \sin\left[(2n-1)\frac{\pi}{2l}x\right]\sin\omega t+ \\ \rho AA_n\omega^2 \sin\left[(2k-1)\frac{\pi}{2l}x\right]\sin\omega t = P_0\sin\omega t \end{gathered}. \tag{17}$$

After transformations, the value of amplitude A_n of the vibrations takes the form

$$A_n = \frac{P_0}{\rho A - EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{18}$$

Using equation (18) and putting it into (16), we get the dynamical flexibility

$$Y_{xl}^{(n)} = \frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2 - EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{19}$$

In the global case, the dynamical flexibility at the end of the beam takes the form

$$Y_{xl} = \sum_{n=1}^{\infty} Y_{xl}^{(n)} = \sum_{n=1}^{\infty} \frac{\sin\left[(2n-1)\frac{\pi}{2l}x\right]}{\rho A\omega^2 - EI\left[(2n-1)\frac{\pi}{2l}\right]^4}. \tag{20}$$

The plot of the value of dynamical flexibility defined by expression (20) is shown in Fig. 2 for the sum k =1,2,3.

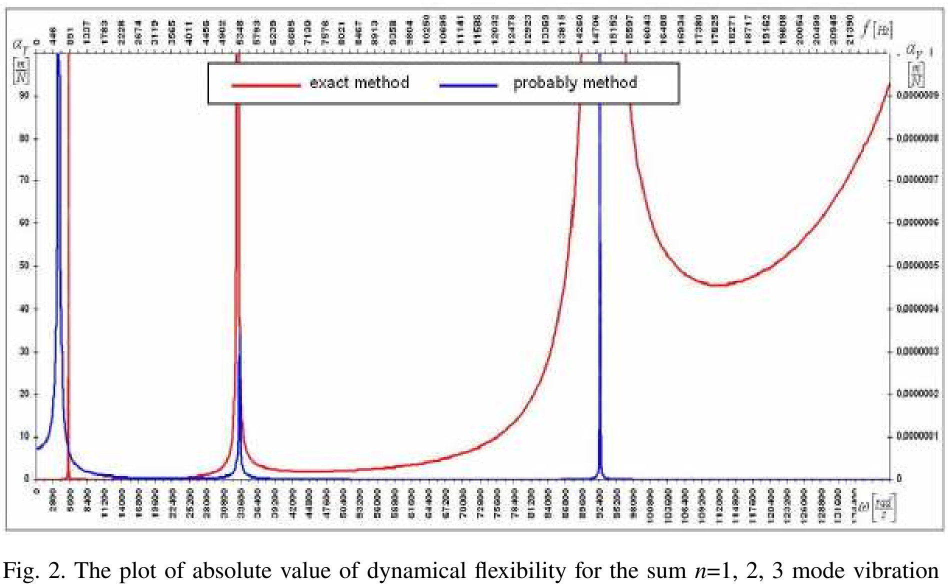

Fig. 2. The plot of absolute value of dynamical flexibility for the sum n=1, 2, 3 mode vibration

On the basis of the obtained formulas, derived by means of the exact method and approximate methods, it is possible to analyze the considered class of vibrating mechanical systems. Moreover, it is possible to analyze mechatronic systems where mechanical parts are vibrating beams using only approximate methods.

When boundary conditions of mechanical parts of mechatronic systems, i.e. beams, are different, it is necessary to use the solutions derived in this paper. These problems will be the subject of future research works.

3. Model of beam system represented by hypergraphs

To specify the meaning of necessary terms and symbols, a review of essential concepts of graph theory will be presented before modeling the torsionally vibrating continuous bar systems. The weighted hypergraphs (in this paper also called the weighted block graphs or weighted graphs of category k) have been applied to modeling the considered mechanical systems. Definitions of graphs as mathematical objects are presented on the basis of the literature. The bibliography of this subject is very extensive and concerns the theory as well as applications of hypergraphs (see [1-6, 14]).

3.1. Basic concepts concerning the class of applied graphs

Using the symbols introduced in papers [1,2,5], we name the following couple

$$X = \left({}_1X, {}_2X \right), \tag{21}$$

a *graph,* where: ${}_1X = \{{}_1x_0, {}_1x_1, {}_1x_2, ..., {}_1x_n\}$ – finite set of vertices, ${}_2X = \{{}_2x_1, {}_2x_2, ..., {}_2x_m\}$ – family of edges being two-element subsets of vertices, in the form of ${}_2x_k = \left({}_1x_i, {}_1x_j\right)$ $(i, j = 0,1, \ldots ,n)$ (of. [14]).

The couple

$$^kX = \left({}_1X, {}_2^kX \right), \tag{22}$$

is called a *hypergraph,* where: ${}_1X$ is the set as in (21), and ${}_2^kX = \left({}_2^kX^{(i)} / i \in \mathrm{N}\right)$, ($k$=2,3, ... ∈N) is a family of subsets of set ${}_1X$; the family ${}_2^kX$ is called a *hypergraph* over ${}_1X$ as well, and ${}_2^kX = \left\{{}_2^kX^{(1)}, {}_2^kX^{(2)}, ..., {}_2^kX^{(m)}\right\}$ is a set of edges [14], called *hyperedges* or *blocks,* if

(i) $\quad {}_2^kX \neq \emptyset (i \in \mathrm{N})$,

(ii) $\quad \bigcup_{i \in \mathrm{I}} {}_2^kX^{(i)} = {}_1X.$

If a subset from the family of subsets of vertices with $n_z \leq n$, is distinguished from the hypergraph ^kX with n vertices, then the *complete graph* of hypergraph ${}^{k}X$ is the graph X_Z. In this graph, each pair of vertices is incident, and graph X_Z has $m=\binom{n}{2}$ edges.

The *skeleton* ${}^{k}X_0$ of hypergraph ${}^{k}X$ is a graph obtained as the result of substitution of each subset of vertices by the tree $\underset{0}{X}$, composed of one-dimensional edges and spread over all vertices of the hypergraph ${}^{k}X$. The tree $\underset{0}{X}$ of graph X with n vertices and m edges is a connected subgraph with the same number of vertices and with $\underset{0}{m} = n - 1$ edges, in which there are no circuits and loops. Then, every skeleton of subsets of vertices is a tree of a substitute-complete graph.

A tree in which every vertex ${}_{1}x_i$ $(i = 1, \ldots, n)$ is incident with the vertex ${}_{1}x_0$ by the edge ${}_{2}x_k = ({}_{1}x_i, {}_{1}x_0)$, $(k = 1, \ldots, \underset{0}{m})$, (see e.g. [1,5]) is called the *Lagrange skeleton*.

Planar geometrical representation of the graphs X and the hypergraphs ${}^{k}X$ is shown in Fig. 3. The sets of edges ${}_{2}X$ are marked by lines, the subsets of family ${}^{k}_{2}X$ (hyperedges or blocks) – two-dimensional continuum with enhanced vertices, have the shape of circles.

In this paper, the hypergraphs – graphs of category k – ${}^{k}X$ $(k = 2,3)$ are used, which will be clearly mentioned each time. We also use the graphs X, called the graphs of the first category – ${}^{1}X$ (see [1, 5, 14]).

The basic notions, written in italics, are shown in Fig. 3.

These elements, taken from the graphs and hypergraphs theory, constitute the collection of formal means, which will be applied in this article and in future works.

3.2. Hypergraphs as models of vibrating beam analyzed by exact and approximate method

We consider the transverse vibrating beam (i) with constant cross-section and constant flexible rigidity $(EJ_z)^{(i)}$(where $E^{(i)}$ – Young's modulus of the beam, $J_z^{(i)}$ – polar moment of inertia of cross-section of the beam) and length $l^{(i)}$. The applied model has the form of a determined and continuous system. In this model, generalized displacements – deflections ${}_{1}s_1^{(i)}$ and ${}_{1}s_2^{(i)}$ correspond to its extreme points. Moreover, generalized displacements ${}_{1}s_3^{(i)}$ and ${}_{1}s_4^{(i)}$ – the slopes of the beam – correspond to its extreme points. These general displacements are measured in the inertial system of reference. The origin of the inertial system of reference has the generalized coordinate ${}_{1}s_0^{(i)} = 0$ assigned to it. Then, the set of generalized displacements of a transverse vibrating beam can be formulated as follows:

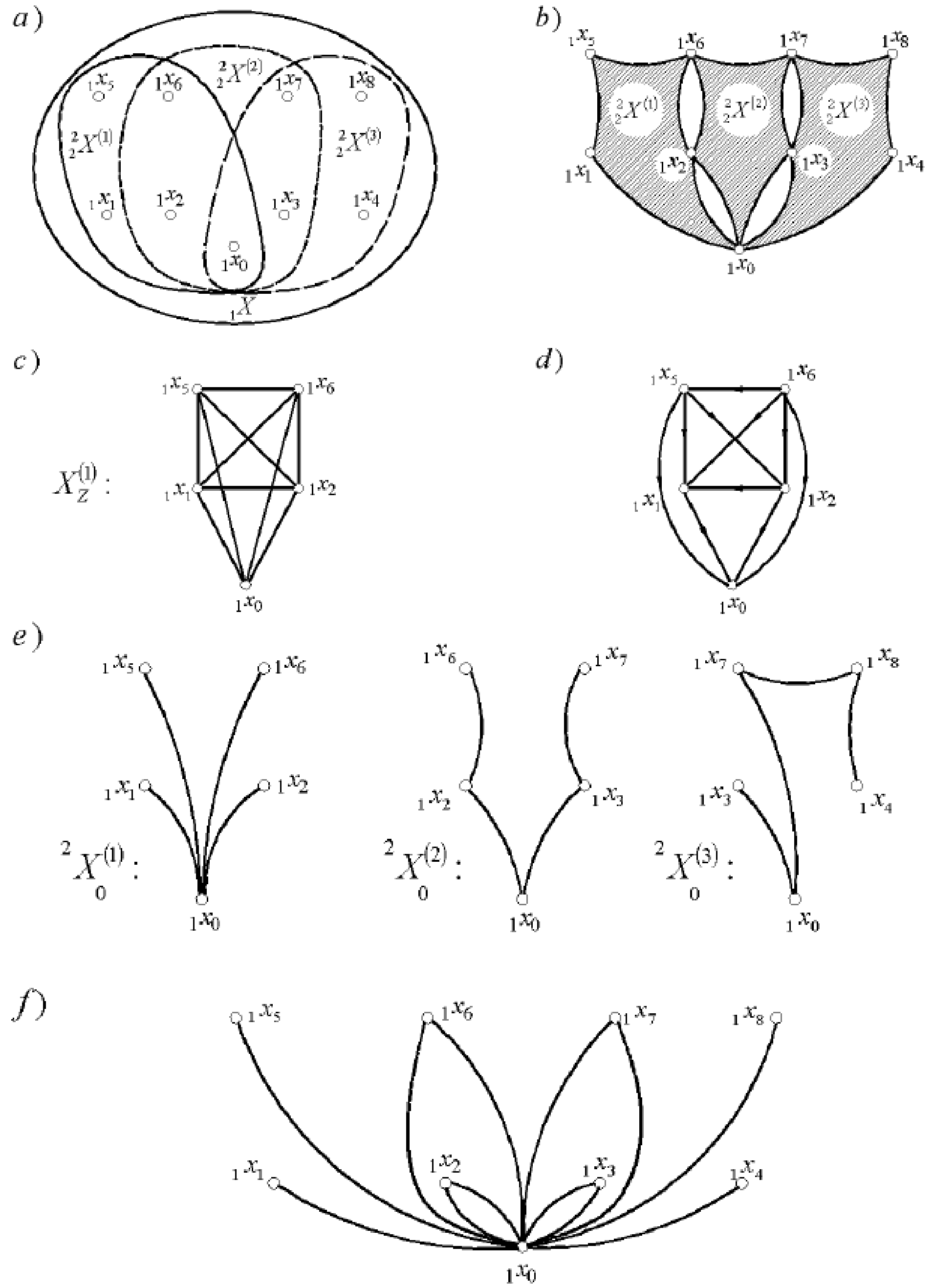

Fig. 3. Basic notions concerning the class of graphs, which are used in this paper: a) set of vertices of hypergraph, b) graphical representation of three-block graph, c) complete graphs of hypergraph blocks, d) complete oriented graphs of three-vertex blocks and of a two-vertex block, e) optionally selected tree-skeletons of hypergraph blocks, f) skeleton of hypergraph, g) Lagrange skeleton of hypergraph

${}_1S^{(\mathrm{i})} = \left\{ {}_1s_0^{(i)}, {}_1s_1^{(i)} = 0, {}_1s_2^{(i)}, {}_1s_3^{(i)} = 0, {}_1s_4^{(i)} \right\}$, while the set of its dynamical flexibilities can be denoted as $Y^{(i)} = \left\{ Y_{ij}^{(i)} \right\} \left(Y_{ij}^{(i)} = Y_{ji}^{(i)}, i, j = 1, ..., 4 \right)$.

By a mutual one-to-one transformation in the form of

$$f : {}_1S^{(i)} \to {}_1X^{(i)}, \tag{23}$$

in such a way that

$$f\left({}_1s_j^{(i)} \right) = {}_1x_j^{(i)}, \tag{24}$$

where: ${}_1s_j^{(i)} \in {}_1S^{(i)}, {}_1x_j^{(i)} \in {}_1X^{(i)},\ {}_1s_j^{(i)} \in {}_1S^{(i)}, {}_1x_j^{(i)} \in {}_1X^{(i)},\ \ j = 0,1,2,3,4$

we obtain the*five-vertex hypergraph as a model of transverse vibrating beam* with constant cross-section

$$\overset{(i)}{\underset{f}{{}^2X}} = \left[\overset{(i)}{{}^2X}, f \right], \tag{25}$$

where: ${}^k_2X^{(i)}$ – one-element family – five-element subset of vertices ${}_1X^{(i)}$.

Graphical representation of transformations (23) made according to (24) in the case of the transverse vibrating beam with constant cross-section is shown in Fig. 4.

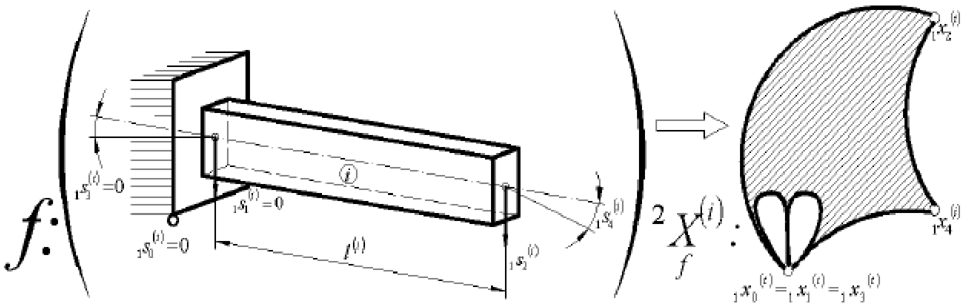

Fig. 4. Hypergraph of model of transverse-vibrating free beam with constant cross-section as graphical representation of transformations (24) and (25)

The couple

$$\overset{(i)}{\underset{1}{{}^2X}} = \left[\overset{(i)}{\underset{f}{{}^2X}}, f_1 \right] \tag{26}$$

is called the*weighted hypergraph,* where: f_1 is the function which assigns the generalized displacements, i.e. deflections: ${}_1s_1^{(i)}$ and ${}_1s_2^{(i)}$ the slopes of the beam – ${}_1s_3^{(i)}$ and ${}_1s_4^{(i)}$ to vertices ${}_1x_j^{(i)}$ of hypergraph $\overset{(i)}{\underset{f}{{}^2X}}$ as

$$f_1\left({}_1x_j^{(i)} \right) = {}_1s_j^{(i)}, \quad j = 0, 1, \ldots, 4. \tag{27}$$

On the basis of, for example the Galerkin's transformation** , the five-vertex hypergraph (Fig. 5a) will be transformed into the three-vertex block graph (Fig. 5b,c).

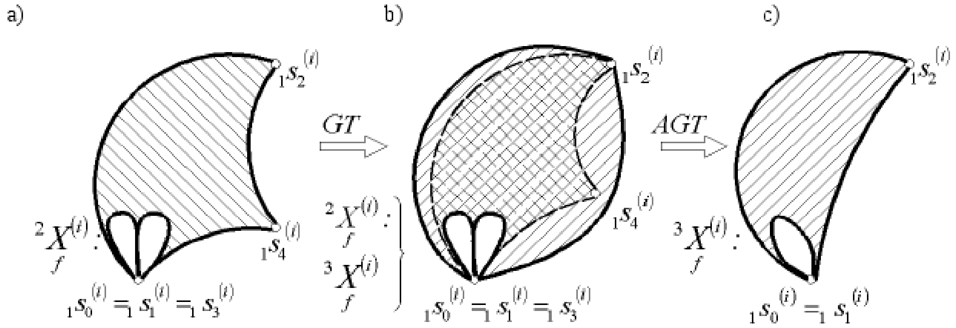

Fig. 5. The illustration of transformation of five-vertex hypergraph into three-vertex one using the Galerkin's method

The complete weighted graph – the substitute graph

$$ {}^{2}X^{(i)}_{\underset{12}{Z}} = \left[{}^{2}X^{(i)}_{\underset{f}{Z}}, f_1, f_2 \right] \tag{28} $$

is obtained:

– after transformations [according to (26-27)] which assign the values of generalized co-ordinates to vertices of complete graph ${}^{2}X^{(i)}_{\underset{f}{Z}}$ of the hypergraph ${}^{3}X^{(i)}_{f}$ (that means the hypergraph after Galerkin's transformation)

and

– after transformation f_2 which assigns dynamical flexibilities to edges of the complete graph , which were defined in following way

$$ f_2\left(\left\{{}_1x_0^{(i)}, {}_1x_0^{(i)}\right\}, \left\{{}_1x_0^{(i)}, {}_1x_2^{(i)}\right\}, \left\{{}_1x_1^{(i)}, {}_1x_2^{(i)}\right\}\right) = \left(\left\{Y_{11}^{(i)}\right\}, \left\{Y_{22}^{(i)}\right\}, \left\{Y_{12}^{(i)}\right\}\right), \tag{29} $$

where: $Y_{11}^{(i)}, Y_{22}^{(i)}, Y_{12}^{(i)}$ are dynamical flexibilities obtained by means of the Galerkin's method.

** There could be a different approximate method of analysis, for example the orthogonalization method [13].

The*weighted Lagrange's skeleton of hypergraph* $^{3}\underset{f}{X}^{(i)}$

$$^{2}\underset{12}{X_0^{(i)}} = \left[{}^{2}\underset{f}{X_Z^{(i)}}, f_1, f_2 \right], \tag{30}$$

is the *weighted subgraph* of the weighted complete graph – the substitute one $^{2}\underset{12}{X_Z^{(i)}}$. Graphical representation of these subgraphs are shown in [1, 2, 5].

In the case of synthesis of n-segment model of the system, composed of subsystems with constant cross-section area, the transverse-vibrating system is modeled by the loaded graph of the third category – after Galerkin's transformation – with n three-vertices-blocks, connected to those vertices to which the corresponding generalized coordinates are assigned.

The use of a weighted hypergraph and its weighted subgraphs (as a model of a transverse vibrating system) in this way may provide the basis for the formalization, which is the necessary condition of discretization of the considered class of continuous mechanical systems.

4. Concluding remarks

On the basis of the obtained formulas and transformations, it is possible to analyze the considered class of vibrating mechanical and mechatronic systems, The presented method could be an introduction to the synthesis of such systems. The problems of synthesis will be the subject of future research works by the author.

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Hipergrafy belek analizowanych metodą dokładną i przybliżoną jako modele podukładów drgających giętnie w syntezie układów mechanicznych i mechatronicznych

S t r e s z c z e n i e

W pracy porównano charakterystyki podukładów otrzymanych metodą przybliżoną i dokładną, aby odpowiedzieć na pytanie: czy metoda przybliżona może być stosowana do wyznaczania charakterystyk układów mechatronicznych. Analizę widmowo-modalną przeprowadzano w przypadku drgającej giętnie belki wysięgnikowej. Następnie model belki odwzorowano w pięciowierzchołkowy hipergraf, który to model, w wyniku zastosowania przybliżonej metody analizy widmowo – modalnej, można przedstawić w postaci hipergrafu trójwierzchołkowego. Takie ujęcie może stanowić wprowadzenie do syntezy drgających giętnie złożonych układów belkowych o odcinkowo stałym przekroju.