

APPLICATION OF THE THEORY OF SEMI-MARKOV PROCESSES TO THE DEVELOPMENT OF A RELIABILITY MODEL OF AN AUTOMOTIVE VEHICLE

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Summary

A possibility of applying the theory of semi-Markov (semimarkovian) processes to the description of the reliability of an automotive vehicle has been presented, with taking a passenger car as an example. In the car considered, such constructional systems (functional components) as engine with fuel, lube oil, and coolant feeding systems, clutch, gearbox, drive shaft, driving axle, steering and suspension system, braking system, electrical system, body with chassis, and measuring and monitoring equipment have been distinguished. A reliability model of the car has been developed in the form of a semi-Markov process, which is a single-state model. The set of reliability states has been built as consisting of one state of car serviceability and ten states of unserviceability of the functional components as specified above. A graph of changes in these states has been shown and an initial distribution and a functional matrix representing changes in the said reliability states of the car have been defined. Formulas have been derived that define the boundary distribution of the process of changes in the technical states of such a car. This distribution represents the probability of the car being serviceable and unserviceable due to a failure of any of the functional components mentioned above. The possibility of using statistics to estimate the probabilities of changes in the said car reliability states has been presented. A possibility of applying the theory of semi-Markov processes to the investigation of car reliability in the case of defining the instantaneous distribution of the process of changes in the car reliability states has also been mentioned.

Keywords: reliability, semi-Markov process, automotive vehicle.

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1. Introduction

In the recent years, significant technological progress has been observed in the production of modern automotive vehicles, not only passenger cars, characterised by high energy, durability, and reliability indices recorded in normal conditions, i.e. the conditions for which each vehicle type has been designed. These indices have a significant impact on the driving safety of such vehicles; therefore, the vehicles also show high safety indices. In consequence, if modern automotive vehicles are reasonably operated then road accidents in most cases result from drivers' faults rather than from technical condition of the vehicles. The reasonable operation of the vehicles means, in most general terms, the starting and loading of vehicle engines (and other functional components of the vehicles) during drive in a way ensuring the least possible wear of these components and the carrying out of the preventive maintenance operations prescribed in result of technical diagnostic examinations of the vehicles. If these conditions are observed then the vehicle defects occurring during drive will be random events of low probability, resulting from wear of vehicle components.

However, the reasonable use of technical diagnostics in order to identify the technical condition of automotive vehicles is not an easy job, chiefly because a diagnosis about the technical condition of a vehicle or any of its constructional system (functional component) has the form of a hypothesis. As an example, the diagnosis is formulated like this: "The technical condition of the vehicle is such and such because the observed vector of the diagnostic parameters is such and such." In this case, the sentence "the observed vector of the diagnostic parameters is such and such" describes a certain event but the sentence "the technical condition of the vehicle is such and such" describes an event that is considered as probably true. For the probability of such a sentence to be higher, the likelihood or rightness of the diagnosis should be known [5]. It should be remembered, moreover, that automotive vehicles are often operated in conditions significantly differing from those considered normal, which results from both external circumstances (road surface condition, terrain relief, precipitation, wind power and direction, etc.) and driving skill of the vehicle driver. This leads to vehicle failures becoming increasingly frequent with time. Obviously, the frequency of such failures will depend to a different degree on individual constructional systems (functional components) of the vehicles, such as engine with fuel, lube oil, and coolant feeding systems, clutch, gearbox, drive shaft, driving axle, steering and suspension system, braking system, electrical system, body with chassis, and measuring and monitoring equipment. The vehicle failure frequency may additionally rise due to inadequate preventive maintenance as against the actual technical condition of the vehicle or various previous defects of components of the said functional components. Therefore, the probability of the occurrence of such failures should be known so that the vehicles are operated in a reasonable way. In statistical terms, this probability may be interpreted as the number the relative frequency of random events of this kind fluctuates around. In this connection, it becomes important to determine the vehicle reliability, which may be defined, in terms of valuation, as the likelihood of correct functioning of any automotive vehicle.

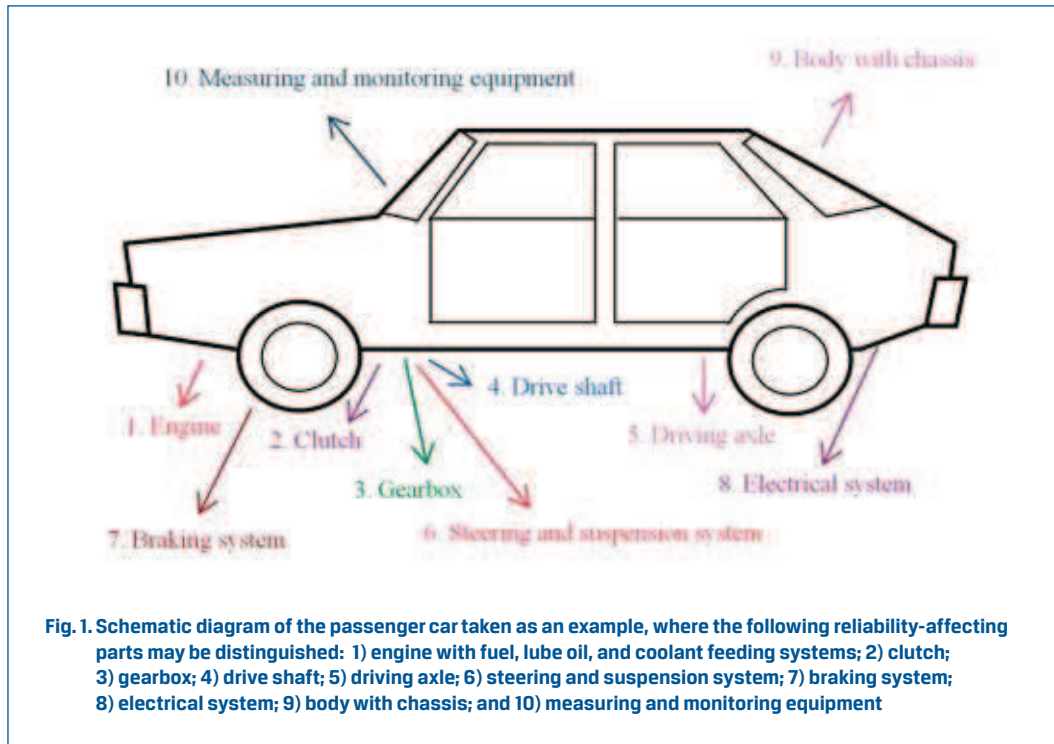
To describe the reliability of automotive vehicles or their functional components as mentioned above, classical reliability theory models can be used. In such a case, various structures of the models are employed, with the serial one being most popular. When comparing, however, these models to reliability models, e.g. the graphs of states – transitions, one can observe that the latter are more useful for describing the reliability of automotive vehicles, as they are capable to represent the changes that result from the specific nature of operation of the vehicles and their functional components. For this reason, the transition graph models are referred to as reliability-functional models of equipment, inclusive of automotive vehicles. In addition to this, such models have been proven particularly usable because they offer a possibility of relatively easy determining of reliability indicators for both the complete vehicles and vehicle components. The indicators may be obtained with the use of either analytical formulas or computer simulation methods. If specific conditions regarding the properties of probability distributions of random quantities (variables) and the properties that characterise vehicles as objects of reliability investigations are met, the theory of semimarkovian processes may be employed [1, 2, 4].

In this study, an attempt has been made to show that the reliability-functional models are more useful for describing the reliability of any automotive vehicle as against the reliability structures of the classical theory of equipment reliability.

2. Formulation of the problem of determining the reliability of an automotive vehicle

In practice, passenger cars being in use undergo various failures [6]. The same applies to the operation of other types of automotive vehicles. Therefore, information about the reliability of such vehicles is necessary for the vehicles to be operated in a reasonable way. The reliability description of a specific vehicle type may be presented in accordance with the same principles as those applicable to other equipment items [1, 3]. From now on, therefore, this discussion will only cover the reliability of a passenger car consisting, as an example, of the following functional components (meant as reliability-affecting parts) [6]: 1) engine with fuel, lube oil, and coolant feeding systems; 2) clutch; 3) gearbox; 4) drive shaft; 5) driving axle; 6) steering and suspension system; 7) braking system; 8) electrical system; 9) body with chassis; and 10) measuring and monitoring equipment (Fig. 1).

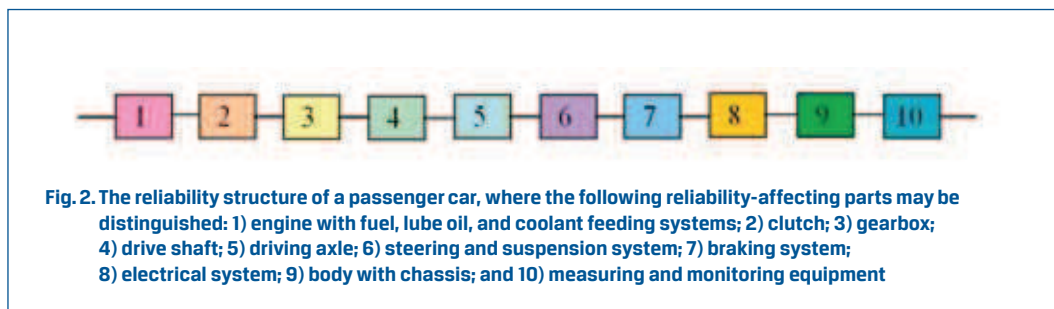
The reliability description of every automotive vehicle is formulated with the serial structure being taken as a basis. This is because the vehicle is serviceable only when all its reliability-affecting parts (engine with fuel, lube oil, and coolant feeding systems, clutch, gearbox, drive shaft, driving axle, steering and suspension system, braking system, electrical system, body with chassis, and measuring and monitoring equipment) are serviceable. Such a structure stems from the functional structure of the vehicle because any disturbance to this structure causes the whole vehicle to be unserviceable. The schematic diagram of the reliability structure of the vehicle consisting of the functional



components as shown in Fig. 1 has been presented in Fig. 2. Hence, the reliability function of such a vehicle is:

$$R(t) = P\{T \geq t\} = \prod_{i=1}^{10} R_i(t) \quad (1)$$

When using formula (1), we must bear in mind the fact that it describes the reliability of a vehicle in which the only reliability-affecting parts are the functional components specified above and that these components (like the said parts) can only be in one of two states excluding each other, i.e. they may be either serviceable or unserviceable, and



they cannot be in any intermediate state. Obviously, this condition applied to automotive vehicles is hardly acceptable in the vehicle operation practice.

To determine $R_i(t)$, i.e. the reliability of the i^{th} functional component of the vehicle, one should know the distribution of time of correct operation of this specific component, i.e. $F_i(t)$, or its risk function, i.e. $\lambda_i(t)$, because [4]:

$$R_i(t) = 1 - F_i(t), \quad t > 0 \quad (2)$$

or

$$R_i(t) = \exp \left[- \int_0^t \lambda_i(\tau) d\tau \right], \quad t > 0 \quad (3)$$

Therefore, the reliability function of the car may be written as follows:

$$R(t) = \prod_{i=1}^{10} [1 - F_i(t)] \quad (4)$$

or

$$R(t) = \exp \left[- \sum_{i=1}^{10} \int_0^t \lambda_i(\tau) d\tau \right] \quad (5)$$

The use of formulas (4) or (5) to determine the reliability of the car is reasonable when the following assumptions may be made:

- The random variables $T_i (i = 1, 2, \dots, 10)$, which represent the time of correct operation of the functional components treated as reliability-affecting parts of the car with distribution functions $F_i(t)$, are independent of each other;
- When the car is in standby operation mode, none of its functional components may become damaged;
- During preventive maintenance, damage repair, and renovation servicing operations, none of the serviceable functional components of the car may become damaged;
- Defective functional components (reliability-affecting parts) of the car are replaced with new units instead of being repaired.

In practice, defective functional components of any car and their structural components are usually restored to serviceable condition by carrying out appropriate servicing operations (of course, if this is cost-effective). This means that the last assumption is wrong in most cases. Conversely, the other three may be considered reasonable. An additional limitation in the use of the serial reliability structure for describing the reliability of an automotive vehicle is the necessity to adopt, as previously mentioned, only a two-value reliability state (i.e. serviceable and unserviceable) of the vehicle and its individual functional components.

The use of the semimarkovian model of changes in the reliability states of the vehicle under consideration makes it possible to take into account preventive maintenance operations and to consider more than two reliability states of the vehicle and its functional components. In a three-state reliability model of the car, similar to the reliability model of a compression-ignition engine presented in publication [1], three states may be distinguished: full (complete) serviceability, partial (incomplete, limited) serviceability, and unserviceability. It is obvious that there may be as many states of intermediate serviceability (i.e. the distinguishable states between the states of full serviceability and unserviceability) as necessary for the obtaining of a practically useful description of the reliability state of an automotive vehicle and its functional components. The essence of this model is the fact that it has been developed as a semimarkovian model where such serviceability states have been distinguished as full (complete) serviceability, incomplete (partial, limited) serviceability, and unserviceability. In the case of a car, an identical three-state model may be considered as well. However, the semimarkovian model of the process of changes in the reliability states of the car may also be analysed as a semimarkovian process $\{W(t): t \geq 0\}$ with a set of states $S = s_i; i = 0, 1, \dots, 10$. The state symbols $s_i \in S (i = 0, 1, \dots, 10)$ should be interpreted as follows: s_0 – full serviceability of the whole vehicle; s_1 – unserviceability of engine with fuel, lube oil, and coolant feeding systems; s_2 – unserviceability of clutch; s_3 – unserviceability of gearbox; s_4 – unserviceability of drive shaft; s_5 – unserviceability of driving axle; s_6 – unserviceability of steering and suspension system; s_7 – unserviceability of braking system; s_8 – unserviceability of electrical system; s_9 – unserviceability of body with chassis; s_{10} – unserviceability of measuring and monitoring equipment. Changes in the said states $s_i (i = 0, 1, \dots, 10)$ take place at the successive instants $t_n (n \in N)$, with the vehicle state at the instant $t_0 = 0$ being s_0 . The state s_0 turns into another one when any of the functional vehicle components fails. Each of the states $s_i (i = 1, \dots, 10)$ lasts till the instant of a breakdown of any functional vehicle component having been repaired or replaced if the repair was considered cost-ineffective.

The state s_0 , referred to as vehicle serviceability state, takes place when all the functional vehicle components are either new or worn to so small a degree that they may be safely burdened with any load within the whole load range for which they were prepared at the designing and manufacturing stage. This means that the vehicle user may then successfully carry out all the tasks for which the vehicle has been designed. Conversely, the states $s_i (i = 1, 2, 3, \dots, 10)$ take place when the functional components of the vehicle are worn to such an extent that the vehicle cannot be used for what it was intended in accordance with the principles of reasonable operation of the vehicle. Such states are detected by appropriate diagnosing systems (DGS) in the period between successive preventive maintenance operations carried out on the vehicle. This is possible if the DGS systems are prepared to identify the technical condition of individual vehicle components. When the DGS systems are not adequately prepared to identify the technical condition of the said vehicle components or they are defective then they cannot detect excessive vehicle wear at a sufficiently early stage, which may cause the vehicle to fail when performing its functions. For the occurrence of such a situation at the vehicle operation stage to be taken into consideration, the vehicle operation process must be described in probabilistic terms with taking into account the probabilities of the occurrence of the said possible states $s_i (i = 0, 1, 2, 3, \dots, 10)$ at specific instants $t_0, t_1, \dots, t_{n-1}, t_n$ during the vehicle operation.

An assumption may be made that the vehicle state at the instant t_{n+1} and the time interval of duration of the state reached at the instant t_n do not depend on the states that occurred at the instants t_0, t_1, \dots, t_{n-1} and on the time intervals during which the states took place. In consequence, the process $\{W(t): t \geq 0\}$ be considered semimarkovian [2, 4]. A graph of changes in the states of this process has been presented in Fig. 3.

The initial distribution of this process is as shown below:

$$P\{W(0) = s_i\} = \begin{cases} 1 & \text{dla } i = 0 \\ 0 & \text{dla } i = 1, 2, \dots, 10 \end{cases} \quad (6)$$

and the functional matrix has the following form:

$$Q(t) = \begin{bmatrix} 0 & Q_{01}(t) & Q_{02}(t) & Q_{03}(t) & \cdot & \cdot & \cdot & Q_{07}(t) & Q_{08}(t) & Q_{09}(t) & Q_{0,10}(t) \\ Q_{10}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{20}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{30}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{40}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{50}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{60}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{70}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{80}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{90}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ Q_{10,0}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

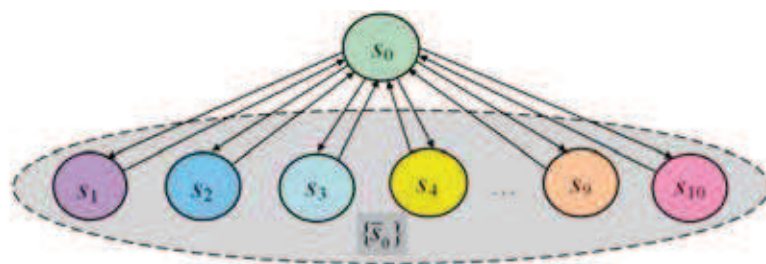


Fig. 3. A graph of changes in the states of a process $\{Y(t): t \in T\}$: s_0 - vehicle serviceability state; $\{\bar{s}_0\}$ - set of vehicle unserviceability states: $\{\bar{s}_0\} = \{s_1, s_2, s_3, \dots, s_{10}\}$, $s_i \in S(i = 1, \dots, 10)$ with the unserviceability state symbols to be interpreted as follows: s_1 - unserviceability of engine with fuel, lube oil, and coolant feeding systems; s_2 - unserviceability of clutch; s_3 - unserviceability of gearbox; s_4 - unserviceability of drive shaft; s_5 - unserviceability of driving axle; s_6 - unserviceability of steering and suspension system; s_7 - unserviceability of braking system; s_8 - unserviceability of electrical system; s_9 - unserviceability of body with chassis; s_{10} - unserviceability of measuring and monitoring equipment

The functional matrix $\mathbf{Q}(t)$ is a model of changes in the reliability states of the vehicle. The non-zero elements $Q_{ij}(t)$ of matrix $\mathbf{Q}(t)$ are the probabilities of transition in the said process from state s_i to state s_j ($s_i, s_j \in S$) within a time interval not exceeding t , defined as follows:

$$Q_{ij}(t) = P\{W(\tau_{n+1}) = s_j, \tau_{n+1} - \tau_n < t | W(\tau_n) = s_i\} = p_{ij}F_{ij}(t) \quad (8)$$

where:

p_{ij} - probability of a single-step transition in the homogenous Markov chain;

$$p_{ij} = P\{Y(\tau_{n+1}) = s_j | Y(\tau_n) = s_i\} = \lim_{t \rightarrow \infty} Q_{ij}(t);$$

$F_{ij}(t)$ - distribution function of a random variable T_{ij} representing the time of duration of state s_i of process $\{W(t): t \geq 0\}$ provided that the next state of the process is state s_j .

Therefore, matrix \mathbf{P} of the probabilities of transitions in the Markov chain adopted for this process will be as follows, according to functional matrix $\mathbf{Q}(t)$ (8) [1, 3, 4]:

$$\mathbf{P} = \begin{bmatrix} 0 & p_{01} & p_{02} & p_{03} & p_{04} & p_{05} & p_{06} & p_{07} & p_{08} & p_{09} & p_{0,10} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Process $\{W(t): t \geq 0\}$ is irreducible [1, 3, 4] and the expected values of random variables T_{ij} are finite and positive. Therefore, its boundary distribution

$$P_j = \lim_{t \rightarrow \infty} P_{ij}(t) = \lim_{t \rightarrow \infty} P\{W(t) = s_j\}, s_j \in S(j = 0, 1, \dots, 10) \quad (10)$$

has the form as follows:

$$P_j = \frac{\pi_j E(T_j)}{\sum_{k=0}^{10} \pi_k E(T_k)} \quad (11)$$

The probabilities $\pi_j (j = 0, 1, \dots, 10)$ in formula (11) are boundary probabilities of the Markov chain adopted for process $\{W(t): t \geq 0\}$ while $E(T_j)$ and $E(T_k)$ are the expected values of random variables T_j and T_k , respectively, which represent the time intervals during which the vehicle remains in states s_j and s_k respectively, regardless of the vehicle states to follow.

For the boundary distribution (11) to be determined, a system of equations must be solved where the said boundary probabilities $\pi_j (j = 0, 1, \dots, 10)$ the Markov chain adopted for the process and the matrix \mathbf{P} of the probabilities of transitions from state s_i to state s_j defined by formula (9) should be represented. The said system of equations would have the following form:

$$\left. \begin{aligned} & [\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}] = [\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}] \cdot \mathbf{P} \\ & \sum_{k=1}^{10} \pi_k = 1 \end{aligned} \right\} (12)$$

Having solved the above system of equations (12), we may obtain the following dependences, with making use of formula (11):

$$\left. \begin{aligned} P_0 &= \frac{E(T_0)}{E(T_0) + \sum_{k=0}^{10} p_{0k} E(T_k)}, P_1 = \frac{p_{01} E(T_1)}{E(T_0) + \sum_{k=0}^{10} p_{0k} E(T_k)}, \\ P_2 &= \frac{p_{02} E(T_2)}{E(T_0) + \sum_{k=0}^{10} p_{0k} E(T_k)}, \\ P_3 &= \frac{p_{03} E(T_3)}{E(T_0) + \sum_{k=0}^{10} p_{0k} E(T_k)}, \dots, P_{10} = \frac{p_{10} E(T_{10})}{E(T_0) + \sum_{k=0}^{10} p_{0k} E(T_k)} \end{aligned} \right\} (13)$$

The probability P_0 is the boundary probability that the vehicle will remain for a prolonged period (time interval) of operation (theoretically for $t \rightarrow \infty$) in state s_0 . This means that this probability determines the technical availability factor of the vehicle. The probabilities $P_j (j = 1, 2, \dots, 10)$ are the boundary probabilities of the occurrence of states $s_j \in S$ of the said vehicle at $t \rightarrow \infty$, i.e. the probabilities that the functional vehicle components (and thus, the vehicle as a whole due to its serial reliability structure) will be in a state of unserviceability.

An example of realisations of process $\{W(t): t \geq 0\}$, which depicts the occurrence of various vehicle reliability states during the vehicle operation period, has been presented in Fig. 4.

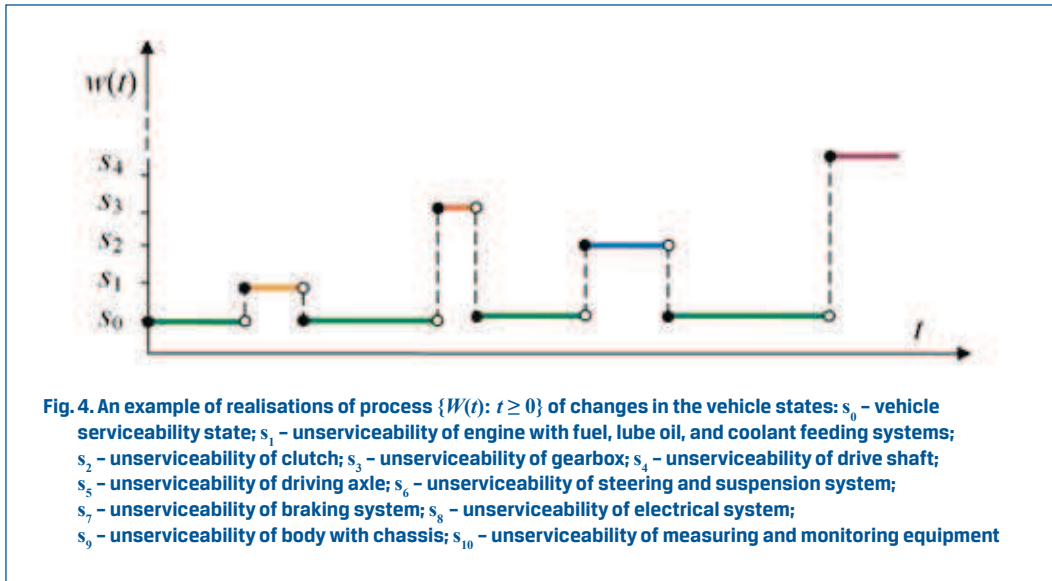
For the probability values $P_j (j = 1, 2, 3, \dots, 10)$ to be determined (of course, approximately), the p_{ij} and $E(T_j)$ values must be estimated.

The estimation of the probabilities p_{ij} and expected values $E(T_j)$ will be possible when a realisation $w(t)$ of process $\{W(t): t \geq 0\}$ is obtained for a sufficiently long test time interval, i.e. for $t \in [0, t_b]$, with the test time being $t_b \gg 0$. Only in such a case, the number n_{ij} ($i, j = 0, 1, \dots, 10; i \neq j$) of transitions of process $\{W(t): t \geq 0\}$ from state s_i to state s_j for a sufficiently long time may be determined.

The most reliable estimator of the probability of transition p_{ij} is the statistic [4]

$$\hat{P}_{ij} = \frac{N_{ij}}{\sum_j N_{ij}}, \quad i \neq j; \quad i, j = 0, 1, \dots, 10, \quad (14)$$

Its value, $\hat{p}_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$, is an estimate of the unknown probability of transition p_{ij} .



From the realisation $w(t)$ of the process $W(t)$, realisations $t_j^{(m)}$, $m. = 1, 2, \dots, n_{ij}$ of random variables T_j , may be derived. With the use of point estimation, $E(T_j)$ as the arithmetic mean of realisations $t_j^{(m)}$ may be easily estimated.

To obtain the information necessary for the estimating of the said probabilities, appropriate diagnosing systems (DGS) must be used for the vehicles, which in this case are diagnosed systems (DNS).

For the vehicle operation practice, the univariate distribution of process $\{W(t): t \geq 0\}$ of changes in the vehicle states is also important, where the process elements are functions $P_k(t)$ representing the probability that at the instant t (arbitrarily chosen), the process will be in the state $sk \in S$ ($k = 0, 1, \dots, 10$). This instantaneous distribution may be

calculated on the grounds of the initial distribution of process $\{W(t): t \geq 0\}$ and functions $P_{ij}(t)$ representing the probabilities of transition of the process from state s_i to state s_j ($s_i \in S, s_j \in S, i \neq j; i, j = 0, 1, \dots, 10$). To calculate these probabilities of transition, one must know the functions $F_{ij}(t)$, i.e. the distribution functions of random variables T_{ij} ($i \neq j; i, j = 0, 1, \dots, 10$). Therefore, appropriate reliability tests of vehicles and, simultaneously, functional components of these vehicles are necessary.

Obviously, the presented reliability description of any vehicle may be developed by adopting as many states of intermediate (partial) serviceability as necessary for the user of a specific vehicle to ensure reasonable operation of the vehicle.

Moreover, the following characteristics of semimarkovian processes may be of significant practical importance in the research on reliability of automotive vehicles [1, 2, 4]:

- Asymptotic distribution of the renewal process $\{V_{ij}(t): t \geq 0\}$ generated by the time intervals of return of the semimarkovian process (to state s_j achievable from state s_i), which, at an instant t , assumes a value equal to the number of "entries" of this process into state s_j ,
- Approximate distribution of the total time during which process $W(t)$ remained in state s_j provided that the preceding state was s_i ,
- Expected value $E(T_i)$ of the time T_i of duration of state s_i of process $W(t)$ regardless of the state into which the process turns at the instant $\tau_n + 1$,
- Variance $D^2(T_i)$ of the time T_i of duration of state s_i ,
- Expected value $E(T_{ij})$ of the time T_{ij} of duration of state s_i of process $W(t)$ provided that the next state is state s_j ,
- Variance $D^2(T_{ij})$ of the time T_{ij} of duration of the i^{th} state of process $W(t)$ provided that the next state is state s_j .

3. Final remarks and conclusions

The semimarkovian processes are increasingly often used to solve various problems related to the reliability, mass servicing, and diagnostics of equipment, e.g. automotive vehicles.

For these processes to be used in practice, the following two conditions must be fulfilled:

- Appropriate mathematical statistics must have been collected;
- A semimarkovian model of changes in the reliability state of a device, with a small number of states of the device and, simultaneously, with a simple (in mathematical terms) functional matrix, must have been prepared.

The latter condition is important for the calculation of the instantaneous distribution $P_k(t)$, $k = 0, 1, 2, \dots, 10$, of states of process $\{W(t): t \geq 0\}$ the vehicle reliability. Naturally, this distribution may be calculated on the grounds of the initial distribution of the process and the functions $P_{ij}(t)$. To calculate the probabilities $P_{ij}(t)$, a system of Volterra equations of the second kind (a system of convolution-type equations) should be solved [4], where the known quantities are functions $Q_{ij}(t)$, i.e. elements of the functional matrix $\mathbf{Q}(t)$ of the process (7). When the number of states of the process is small and the functional matrix of the process is simple then this system of equations may be solved with the use of operational

calculus methods, by employing the Laplace–Stieltjes transformation. However, if number of states of the process is large or if the functional matrix of the process (kernel of the process) is very complex then only an approximate solution of the system of equations can be obtained. Such a solution (in a numerical form) does not offer any possibility of determining the values of the probabilities that specific states would occur when the t values are high (theoretically, when $t \rightarrow \infty$). Thus, the numerical solution does not answer the following question, which is very important for the automotive vehicle operation practice: how do the probabilities of states of the semimarkovian process change when the t values are high? According to the theory of semimarkovian processes, these probabilities for ergodic semimarkovian processes approach with time certain precisely defined constant numbers. Such numbers are referred to as boundary probabilities of the states and a sequence of such numbers constitutes a boundary distribution of the process. This distribution makes it possible to determine the technical availability factor of the vehicle and the income or cost per unit of time of the vehicle operation. These quantities are criterion functions at the solving of problems related to optimisation of the vehicle operation process. Such a distribution is much easier to be calculated than the instantaneous distribution.

The semimarkovian (semi-Markov) process is used instead of the Markov process as a model of changes in the said vehicle reliability states at a specific time (at a specific instant) because it should be expected that the distribution of the random variable T_{ij} representing the time of duration of state s_i provided that the next state of the process is state s_j and the distribution of the random variable T_i representing the time of duration of state s_i of the vehicle regardless of the next state are free and concentrated in the set $R_+ = [0, +\infty)$. The application of the Markov process in this case would only be justified if an assumption might be made that the random variables T_{ij} and T_i have exponential distributions.

model presented may be of significant practical importance because of the possibilities of easy determining the estimators of the probabilities of transitions p_{ij} which are elements of the matrix \mathbf{P} (9), and easy estimating the expected values $E(T_j)$. It should be remembered here at the same time that the point estimation of the expected value $E(T_j)$ does not offer any possibility of determining the estimation accuracy. The estimation accuracy might be determined in the case of the interval estimation, where a confidence interval $[t_{d_j}^*, t_{g_j}^*]$ with random ends is defined, which is anticipated to include the unknown expected value $E(T_j)$ with a predetermined probability (confidence level) β .

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