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Tilted Transverse Isotropy

Introduction

In seismic prospecting, the transverse isotropy (TI) model, i.e. a model of thinly stratified (laminated) medium, is the most common one. The model has been proposed by Postma [11], and assumes that boundaries separating isotropic layers are parallel planes. In the case when the axis of symmetry is vertical, the model is referred to as the vertical transverse isotropy (VTI) model, while if the axis of symmetry and the vertical axis form an angle θ, this is a monoclinal medium described by the tilted transverse isotropy (TTI) model. This is a versatile model of anisotropy, which can be used to obtain formulas describing models with horizontal axes of symmetry, i.e. horizontal transverse isotropy (HTI) models as well as composed models [7].

Until now, studies of the mathematical description of the TTI medium [4, 10] dealt with one of two possible cases of monoclinal media (for every variant). This article presents descriptions of both cases for every variant of the model.

Basic relationships

Knowledge of transverse isotropy parameters is very useful for analysis of other types of anisotropy. Generally in the case of anisotropic media described by the models of transverse isotropy, elasticity matrices, which present the components of the elasticity tensor, depend on the spatial orientation of TI models.

Figure 1 presents a right-handed coordinate system *x, y, z* and a coordinate system of *x', y', z'* associated with dipping parallel strata which correspond with the TI medium. The spatial arrangement of the medium in relation to the coordinate system *x*, *y*, *z* is defined by an angle θ between planes *xy* and *x'y',* i.e. its inclination. The relation between the stress and strain tensors in the *x', y', z'* coordinate system are the same as in the VTI medium. In order to determine the principles of wave propagation in the *x, y, z* coordinate system, rotation of the *x', y', z'* system to the *x, y, z* system should be done using the matrix of cosines of angles between these two systems (angles are measured in the clockwise direction).

The geometrical situation presented in Figure 1 is described by the following matrix of direction cosines:

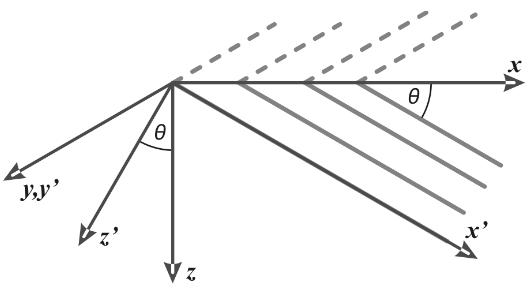

Fig. 1. Drawing of monoclinal strata dipping at an angle θ (between the *x*- and *x*'-axes) of transversely isotropic medium

$$
\begin{aligned}
r_{11} &= \cos(x,x') = \cos\theta \\
r_{12} &= \cos(x,y') = 0 \\
r_{13} &= \cos(x,z') = \cos(90^\circ + \theta) = -\sin\theta \\
r_{21} &= \cos(y,x') = 0 \\
r_{22} &= \cos(y,y') = 1 \\
r_{23} &= \cos(y,z') = 0 \\
r_{31} &= \cos(z,x') = \cos(270^\circ + \theta) = \sin\theta \\
r_{32} &= \cos(z,y') = 0 \\
r_{33} &= \cos(z,z') = \cos\theta
\end{aligned}
\qquad (1)
$$

By using Bond's law [2, 3, 13], we can derive the general relationship between the matrix D of elastic moduli, recorded in the x, y, z system, and the matrix C of known tensor elements in the x', y', z' system, where the $x'y'$ plane forms an angle θ with the xy plane:

$$D = R\,C\,R^T \tag{2}$$

where the R matrix is as follows:

$$R = \begin{vmatrix} r_{11}^2 & r_{12}^2 & r_{13}^2 & 2r_{12}r_{13} & 2r_{11}r_{13} & 2r_{11}r_{12} \\ r_{21}^2 & r_{22}^2 & r_{23}^2 & 2r_{22}r_{23} & 2r_{21}r_{23} & 2r_{21}r_{22} \\ r_{31}^2 & r_{32}^2 & r_{33}^2 & 2r_{32}r_{33} & 2r_{31}r_{33} & 2r_{31}r_{32} \\ r_{21}r_{31} & r_{22}r_{32} & r_{23}r_{33} & r_{22}r_{33}+r_{32}r_{23} & r_{23}r_{31}+r_{21}r_{33} & r_{21}r_{32}+r_{31}r_{22} \\ r_{11}r_{31} & r_{12}r_{32} & r_{13}r_{33} & r_{32}r_{13}+r_{12}r_{33} & r_{33}r_{11}+r_{13}r_{31} & r_{31}r_{12}+r_{11}r_{32} \\ r_{11}r_{21} & r_{12}r_{22} & r_{13}r_{23} & r_{12}r_{23}+r_{13}r_{22} & r_{13}r_{21}+r_{23}r_{11} & r_{11}r_{22}+r_{21}r_{12} \end{vmatrix} \tag{3}$$

while the matrix R^T is the transpose of the matrix R.

By using equation (1), we obtain Bond's matrix $R_{y(+)}$ (rotation around the y-axis, inclination oriented towards the positive x-axis):

$$R_{y(+)} = \begin{vmatrix} \cos^2\theta & 0 & \sin^2\theta & 0 & -\sin(2\theta) & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2\theta & 0 & \cos^2\theta & 0 & \sin(2\theta) & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & \sin\theta \\ \sin\theta\cos\theta & 0 & -\sin\theta\cos\theta & 0 & \cos(2\theta) & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & \cos\theta \end{vmatrix} \tag{4}$$

while the matrix C describing the relationship between stress and strain in the TI/VTI medium is as follows:

$$C = \begin{vmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{vmatrix} \tag{5}$$

Thus, by using formulas (4) and (5), we obtain from relationship (2) the elasticity matrix $D_{y(+)}$ (y indicates rotation around the y-axis, while (+) indicates the inclination of the plane).

$$D_{y(+)} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & 0 & d_{15} & 0 \\ d_{21} & d_{22} & d_{23} & 0 & d_{25} & 0 \\ d_{31} & d_{32} & d_{33} & 0 & d_{35} & 0 \\ 0 & 0 & 0 & d_{44} & 0 & d_{46} \\ d_{51} & d_{52} & d_{53} & 0 & d_{55} & 0 \\ 0 & 0 & 0 & d_{64} & 0 & d_{66} \end{vmatrix} \tag{6}$$

The elements of the matrix $D_{y(+)}$ are:

$$d_{11} = C_{11}\cos^4\theta + C_{33}\sin^4\theta + 2C_{13}\sin^2\theta\cos^2\theta + C_{44}\sin^2(2\theta)$$
$$d_{12} = d_{21} = C_{12}\cos^2\theta + C_{13}\sin^2\theta$$

$$
\begin{aligned}
&d_{13} = d_{31} = C_{13}\left(\sin^4\theta + \cos^4\theta\right) + \left(C_{11} + C_{33}\right)\sin^2\theta\cos^2\theta - C_{44}\sin^2(2\theta)\\
&d_{15} = d_{51} = \left[\left(C_{11} - C_{13}\right)\cos^2\theta + \left(C_{13} - C_{33}\right)\sin^2\theta\right]\sin\theta\cos\theta - C_{44}\sin(2\theta)\cos(2\theta)\\
&d_{22} = C_{11}\\
&d_{23} = d_{32} = C_{12}\sin^2\theta + C_{13}\cos^2\theta\\
&d_{25} = d_{52} = \left(C_{12} - C_{13}\right)\sin\theta\cos\theta\\
&d_{33} = C_{11}\sin^4\theta + C_{33}\cos^4\theta + 2C_{13}\sin^2\theta\cos^2\theta + C_{44}\sin^2(2\theta)\\
&d_{35} = d_{53} = \left[\left(C_{11} - C_{13}\right)\sin^2\theta + \left(C_{13} - C_{33}\right)\cos^2\theta\right]\sin\theta\cos\theta + C_{44}\cos(2\theta)\sin(2\theta)\\
&d_{44} = C_{44}\cos^2\theta + C_{66}\sin^2\theta\\
&d_{46} = d_{64} = \left(C_{66} - C_{44}\right)\sin\theta\cos\theta\\
&d_{55} = \left(C_{11} - 2C_{13} + C_{33}\right)\sin^2\theta\cos^2\theta + C_{44}\cos^2(2\theta)\\
&d_{66} = C_{44}\sin^2\theta + C_{66}\cos^2\theta
\end{aligned}
\tag{7}
$$

Let us now consider the same version of the TTI model, i.e. rotation around the y-axis, but in a situation where the $x'y'$ plane is dipping towards the negative x-axis (Figure 2). In such a case, the matrix of direction cosines is expressed as:

$$
r_{y(-)} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix} = \begin{vmatrix} -\cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & -\cos\theta \end{vmatrix} \tag{8}
$$

while Bond's matrix is expressed as:

$$
R_{y(-)} = \begin{vmatrix} \cos^2\theta & 0 & \sin^2\theta & 0 & \sin 2\theta & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \sin^2\theta & 0 & \cos^2\theta & 0 & -\sin 2\theta & 0 \\ 0 & 0 & 0 & -\cos\theta & 0 & \sin\theta \\ -\sin\theta\cos\theta & 0 & \sin\theta\cos\theta & 0 & \cos 2\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & 0 & -\cos\theta \end{vmatrix} \tag{9}
$$

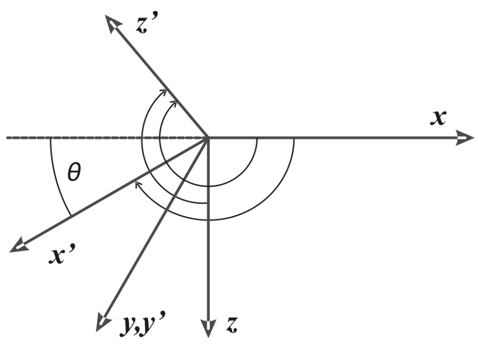

Fig. 2. Geometrical representation of coincidence of the y- and y'-axes. The x'-axis is inclined towards the strata inclination i.e. negative x-axis. Arrows indicate the clockwise direction of measuring angles

By using matrices (5) and (9), the elasticity matrix $D_{y(-)}$ is obtained and it represents a monoclinal medium inclined towards the negative x-axis:

$$
D_{y(-)} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & 0 & -d_{15} & 0 \\ d_{21} & d_{22} & d_{23} & 0 & -d_{25} & 0 \\ d_{31} & d_{32} & d_{33} & 0 & -d_{35} & 0 \\ 0 & 0 & 0 & d_{44} & 0 & -d_{46} \\ -d_{51} & -d_{52} & -d_{53} & 0 & d_{55} & 0 \\ 0 & 0 & 0 & -d_{64} & 0 & d_{66} \end{vmatrix} \tag{10}
$$

where elements are defined by equations (7). A comparison of matrices $D_{y(+)}$ and $D_{y(-)}$ shows that the matrices differ only by signs for elements d_{15}, d_{25}, d_{35}, d_{46} and for the symmetrical elements.

Let us now consider the second version of the TTI medium which is a result of rotating the isotropy plane around the *x*-axis. We will analyse the case where the lamination plane of the medium *x'y'* is oriented towards the positive *y*-axis and forms an angle θ with the *xy* recording surface (Figure 3).

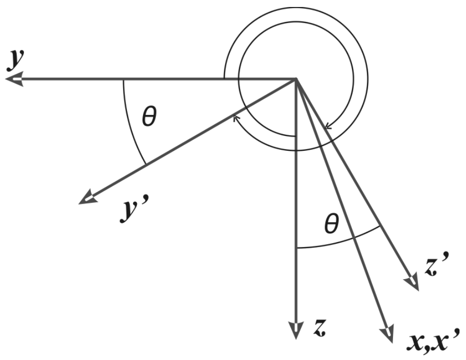

Fig. 3. Overlap of the *x*- and *x'*-axes. The *y'*-axis is oriented towards the positive *y*-axis and in the same direction as the inclination of the strata

In such a situation, the matrix of direction cosines is:

$$r_{x(+)} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{vmatrix} \tag{11}$$

while the elasticity matrix $D_{x(+)}$ calculated in a similar way as in the previous cases is:

$$D_{x(+)} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & d_{14} & 0 & 0 \\ d_{21} & d_{22} & d_{23} & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & d_{34} & 0 & 0 \\ d_{41} & d_{42} & d_{43} & d_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{55} & d_{56} \\ 0 & 0 & 0 & 0 & d_{65} & d_{66} \end{vmatrix} \tag{12}$$

where the elements of the matrix are:

$$\begin{aligned}
& d_{11} = C_{11} \\
& d_{12} = d_{21} = C_{12}\cos^2\theta + C_{13}\sin^2\theta \\
& d_{13} = d_{31} = C_{12}\sin^2\theta + C_{13}\cos^2\theta \\
& d_{14} = d_{41} = (C_{12} - C_{13})\sin\theta\cos\theta \\
& d_{22} = C_{11}\cos^4\theta + C_{33}\sin^4\theta + 2C_{13}\sin^2\theta + \cos^2\theta + C_{44}\sin^2(2\theta) \\
& d_{23} = d_{32} = (C_{11} + C_{33})\sin^2\theta\cos^2\theta + C_{13}(\cos^4\theta + \sin^4\theta) - C_{44}\sin^2(2\theta) \\
& d_{24} = d_{42} = (C_{11} - C_{13})\sin\theta\cos^3\theta + (C_{13} - C_{33})\cos\theta\sin^3\theta - C_{44}\sin(2\theta)\cos(2\theta) \\
& d_{33} = C_{11}\sin^4\theta + C_{33}\cos^4\theta + 2C_{13}\sin^2\theta\cos^2\theta + C_{44}\sin^2(2\theta) \\
& d_{34} = d_{43} = (C_{11} - C_{13})\sin^3\theta\cos\theta + (C_{13} - C_{33})\cos^3\theta\sin\theta + C_{44}\cos(2\theta)\sin(2\theta) \\
& d_{44} = (C_{11} - 2C_{13} + C_{33})\sin^2\theta\cos^2\theta + C_{44}\cos^2(2\theta) \\
& d_{55} = C_{44}\cos^2\theta + C_{66}\sin^2\theta \\
& d_{56} = d_{65} = (C_{66} - C_{44})\sin\theta\cos\theta \\
& d_{66} = C_{44}\sin^2\theta + C_{66}\cos^2\theta
\end{aligned} \tag{13}$$

Let us consider another case of rotating the isotropy plane around the *x*-axis where the dipping *y*'-axis is oriented towards the negative *y*-axis (Figure 4).

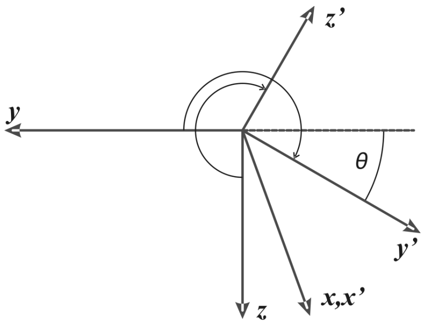

Fig. 4. The geometry of the *x*, *y*, *z* and *x*', *y*', *z*' coordinate systems. The *y*'-axis is oriented towards the strata inclination, i.e. the negative *y*-axis

In such a case the matrix of direction cosines is:

$$r_{x(-)} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -\cos\theta & -\sin\theta \\ 0 & \sin\theta & -\cos\theta \end{vmatrix} \tag{14}$$

From relationships (2) and (3), the elasticity matrix $D_{x(-)}$ is obtained:

$$D_{x(-)} = \begin{vmatrix} d_{11} & d_{12} & d_{13} & -d_{14} & 0 & 0 \\ d_{21} & d_{22} & d_{23} & -d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & -d_{34} & 0 & 0 \\ -d_{41} & -d_{42} & -d_{43} & d_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & d_{55} & -d_{56} \\ 0 & 0 & 0 & 0 & -d_{65} & -d_{66} \end{vmatrix} \tag{15}$$

where elements of the matrix are expressed by equations (13).

A comparison of matrices (12) and (15) shows that different results are obtained depending on the orientation of the inclined plane also in this version of a monoclinal medium.

The elasticity matrices D_y and D_x, composed of elements which are tensor components D_{ijkl} (using the shortened Voigt notation), are verified here both in terms of using the coordinate system *x*', *y*', *z*' oriented in any direction in relation to the coordinate system *x*, *y*, *z* and in terms of the method of calculating the tensor D_{ijkl}. The verification can be done using relationship [6]:

$$D_{ijkl} = r_{ii'} r_{jj'} r_{kk'} r_{ll'} C_{i'j'k'l'} \tag{16}$$

It gives the same results (7) and (13) for the elements of matrix *D* in both cases of the monoclinal medium, i.e. the two-dimensional wavefield recorded up-dip and recorded down-dip, as well as in the case when acquisition is carried out along the extent of the structure for both types of dipping strata.

By using the basic relationship between the tensor of stress T_{ij} and the tensor of strain E_{ik} (Hooke's law) the following matrix is obtained:

$$\begin{vmatrix} T_{11} \\ T_{22} \\ T_{33} \\ T_{23} \\ T_{13} \\ T_{12} \end{vmatrix} = D \begin{vmatrix} E_{xx} = U_{x,x} \\ E_{yy} = U_{y,y} = 0 \\ E_{zz} = U_{z,z} \\ 2E_{yz} = U_{y,z} \\ 2E_{xz} = U_{x,z} + U_{z,x} \\ 2E_{xy} = U_{y,x} \end{vmatrix} \tag{17}$$

The relationships between the components of the elasticity tensor and the derivatives of particle movement in the medium U_x, U_y and U_z are described in the two-dimensional case, i.e. the derivatives of the wavefield in relation to the y-axis – are equal to zero. Starting with the law of motion (ignoring external forces) for each component, the following equations are obtained:

$$T_{11,1} + T_{13,3} = \rho \frac{\partial^2 U_{1(x)}}{\partial t^2}$$
$$T_{21,1} + T_{23.3} = \rho \frac{\partial^2 U_{2(y)}}{\partial t^2}$$
$$T_{31,1} + T_{33,3} = \rho \frac{\partial^2 U_{3(z)}}{\partial t^2} \quad (18)$$

where ρ is density of strata and t is time.

Analysing the case of the TTI strata, where the symmetry axis is located on the xz plane, i.e. for matrix $D_{y(+,-)}$, in both cases we get the following wave equations:

$$d_{11}U_{x,xx} + d_{55}U_{x,zz} + 2(\pm)d_{15}U_{x,zx} + (\pm)d_{15}U_{z,xx} + (d_{13} + d_{55})U_{z,zx} + (\pm)d_{53}U_{z,zz} = \rho \frac{\partial^2 U_x}{\partial t^2} \quad (19)$$

$$d_{66}U_{y,xx} + d_{44}U_{y,zz} + 2(\pm)d_{46}U_{y,xz} = \rho \frac{\partial^2 U_y}{\partial t^2} \quad (20)$$

$$(\pm)d_{51}U_{x,xx} + (\pm)d_{35}U_{x,zz} + (d_{31} + d_{55})U_{x,xz} + d_{55}U_{z,xx} + d_{33}U_{z,zz} + 2(\pm)d_{35}U_{z,zx} = \rho \frac{\partial^2 U_z}{\partial t^2} \quad (21)$$

In formulas (19-21), the sign (+) refers to acquisition along the x-axis moving in the positive direction of the axis, i.e. down-dip (Figure 1), while the sign (–) refers to the up-dip direction.

The above relationships indicate that the cross-line displacement U_y is neither included in formula (19) nor in formula (21). The shear wave SH is described separately by equation (20).

In the case of low angles of inclination (θ), we can assume that the elements of the elasticity matrix $d_{15} = d_{51} \rightarrow 0, d_{53} = d_{35} \rightarrow 0$, $d_{46} \rightarrow 0$ and thus the influence of the dip directions of the isotropy plane on the wave equation can be ignored. From formulas (19-21) with $\theta = 0°$, the following wave equations for the VTI model with a vertical axis of symmetry are obtained:

$$C_{11}U_{x,xx} + C_{44}U_{x,zz} + (C_{13} + C_{44})U_{z,zx} = \rho \frac{\partial^2 U_x}{\partial t^2} \quad (22)$$

$$C_{66}U_{y,xx} + C_{44}U_{y,zz} = \rho \frac{\partial^2 U_y}{\partial t^2} \quad (23)$$

$$(C_{13} + C_{44})U_{x,xz} + C_{44}U_{z,xx} + C_{33}U_{z,zz} = \rho \frac{\partial^2 U_z}{\partial t^2} \quad (24)$$

while the following are obtained for the HTI model with the symmetry axis oriented parallel to the x-axis when the angle of inclination is $\theta = 90°$:

$$C_{33}U_{x,xx} + C_{44}U_{x,zz} + (C_{13} + C_{44})U_{z,zx} = \rho \frac{\partial^2 U_x}{\partial t^2} \quad (25)$$

$$C_{44}U_{y,xx} + C_{66}U_{y,zz} = \rho \frac{\partial^2 U_y}{\partial t^2} \quad (26)$$

$$(C_{13}+C_{44})U_{x,xz}+C_{44}U_{z,xx}+C_{11}U_{z,zz}=\rho\frac{\partial^2 U_z}{\partial t^2} \tag{27}$$

Let us now consider the second case of the TTI model, where the recording is carried out in the strike direction of the stratified medium, i.e. the symmetry axis is parallel to the y-axis. In that case, the following is obtained from equations (12), (15) and (18):

$$d_{11}U_{x,xx}+d_{55}U_{x,zz}+(d_{13}+d_{55})U_{z,xz}+[(\pm)d_{14}+(\pm)d_{56}]U_{y,xz}=\rho\frac{\partial^2 U_x}{\partial t^2} \tag{28}$$

$$d_{66}U_{y,xx}+d_{44}U_{y,zz}+[(\pm)d_{14}+(\pm)d_{65}]U_{x,xz}+(\pm)d_{65}U_{z,xx}+(\pm)d_{43}U_{z,zz}=\rho\frac{\partial^2 U_y}{\partial t^2} \tag{29}$$

$$d_{55}U_{z,xx}+d_{33}U_{z,zz}+(d_{13}+d_{55})U_{x,xz}+(\pm)d_{56}U_{y,xx}+(\pm)d_{34}U_{y,zz}=\rho\frac{\partial^2 U_z}{\partial t^2} \tag{30}$$

Formulas (28-30) imply that the direction of the isotropy plane inclined at an angle θ has a direct influence on the character of the wave equation. This influence disappears when the angle θ is relatively low, while in the extreme case, when the angle is $\theta = 0^{\circ}$, the expected wavefield equations for the VTI model, i.e. equations (22-24), are obtained.

It is easy to notice that in the case of the wavefield recorded along the extent of the structure (the x-axis) there is no separation of the longitudinal and shear SH waves which exist both in equations (28) and (30) despite the assumption of vanishing derivatives of movement $U_{i,y} = 0$.

In the case of vertical isotropy plane (or fractures, discontinuities) with the angle of $\theta = 90^{\circ}$, from (28-30) we obtain equations describing the wavefield in the HTI medium, i.e. with a horizontal axis of symmetry perpendicular to the recording direction along the x-axis:

$$C_{11}U_{x,xx}+C_{66}U_{x,zz}+(C_{12}+C_{66})U_{z,xz}=\rho\frac{\partial^2 U_x}{\partial t^2} \tag{31}$$

$$C_{44}U_{y,xx}+C_{44}U_{y,zz}=\rho\frac{\partial^2 U_y}{\partial t^2} \tag{32}$$

$$C_{11}U_{z,zz}+C_{66}U_{z,xx}+(C_{12}+C_{66})U_{x,xy}=\rho\frac{\partial^2 U_z}{\partial t^2} \tag{33}$$

The above equations indicate that the relationship for the shear SH wave is the separate formula (32), while there is no U_y component in equations (31) and (33).

The above motion equations for each type of the anisotropic media form a basis for calculations of dispersive equations. These are necessary for analysis of the propagation of all types of waves in the wavenumber-frequency domain. When equations (19) and (21) are used together with the Fourier transform ($x \to k_x$, $t \to \omega$), and elements d_{15}, d_{53}, d_{46} are ignored due to the low angle of inclination, the following matrix equation is expressed:

$$\begin{vmatrix} d_{11}k_x^2+d_{55}k_z^2-\rho\omega^2 & (d_{13}+d_{55})k_xk_z \\ (d_{13}+d_{55})k_xk_z & d_{33}k_z^2+d_{55}k_x^2-\rho\omega^2 \end{vmatrix} \begin{vmatrix} U_x \\ U_z \end{vmatrix} = 0 \tag{34}$$

where k_x and k_z are the horizontal and vertical wavenumbers, and ω is the angle frequency. When we assume that the velocity of the shear SV wave is zero for low angles θ, i.e. $d_{55} \approx 0$ similarly as for the VTI medium [1, 5], and we assume that:

$$\begin{aligned} d_{11} &\approx C_{11}\cos^4\theta \\ d_{33} &\approx C_{33}\cos^4\theta \\ d_{13} &\approx C_{13}\cos^4\theta \end{aligned} \tag{35}$$

the dispersive equation for the vertical wavenumber k_z^{TTI} in the TTI-type anisotropic medium is obtained:

$$k_z^{TTI} = \pm(\cos\theta)^{-1}\left[\frac{S_p^4\omega^4 - S_p^2\omega^2 q_{TTI}\cdot k_x^2}{S_p^2\omega^2 - \eta_{TTI}\cdot k_x^2}\right]^{1/2} \quad (36)$$

where:

$$q_{TTI} = q_{VTI}\cdot\cos^2\theta$$
$$\eta_{TTI} = \eta_{VTI}\cdot\cos^2\theta$$
$$S_p = (V_{pp})^{-1} \quad (37)$$

while V_{pp} is the velocity of longitudinal wave in the vertical direction, i.e. $V_{pp} = V_{p\perp}\cdot\cos\theta$, where $V_{p\perp}$ is the longitudinal wave velocity in the direction perpendicular to the stratification in the TTI medium, while $q_{VTI} = 1 + 2\varepsilon$, $\eta_{VTI} = 2(\varepsilon - \delta)$ are Thomsen's parameters [12] for the horizontally stratified medium (VTI) with the vertical axis of symmetry.

In a similar way, dispersive equations can be obtained for other orientations of the TI strata. Such equations are essential for solving dual-domain migration algorithms performed in the frequency-space and frequency-wavenumber domains [7, 8, 10].

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