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Measurements of absorbed heat flux and water-side heat transfer coefficient in water wall tubes

JAN TALER[1*]
DAWID TALER[2]
ANDRZEJ KOWAL[1]

[1] Cracow University of Technology, Department of Thermal Power Engineering, al. Jana Pawła II 37, 31-864 Cracow, Poland

[2] University of Science and Technology, Department of Energy Systems and Environment Protection, al. Mickiewicza 30, paw. B-3, 30-059 Cracow, Poland

Abstract The tubular type instrument (flux tube) was developed to identify boundary conditions in water wall tubes of steam boilers. The meter is constructed from a short length of eccentric tube containing four thermocouples on the fire side below the inner and outer surfaces of the tube. The fifth thermocouple is located at the rear of the tube on the casing side of the water-wall tube. The boundary conditions on the outer and inner surfaces of the water flux-tube are determined based on temperature measurements at the interior locations. Four K-type sheathed thermocouples of 1 mm in diameter, are inserted into holes, which are parallel to the tube axis. The non-linear least squares problem is solved numerically using the Levenberg-Marquardt method. The heat transfer conditions in adjacent boiler tubes have no impact on the temperature distribution in the flux tubes.

Keywords: Inverse heat conduction problem; Steam boiler; Heat flux meter; Slagging

*Corresponding author. E-mail address: taler@mech.pk.edu.pl

Nomenclature

a	–	inner radius of boiler tube and flux-tube, m
b	–	outer radius of flux-tube, m
Bi	–	Biot number, $\mathrm{Bi} = ha/k$
c	–	inner radius of boiler tube, m
e	–	eccentric, m
f_i	–	measured wall temperature at the i-th location, °C
$\mathbf{f}$	–	vector of measured wall temperatures, °C
h	–	heat transfer coefficient, W/(m^2 K)
$\mathbf{I_n}$	–	identity matrix,
$\mathbf{J_m}$	–	Jakobian matrix,
k	–	thermal conductivity, W/(m K)
m	–	number of temperature measurement points,
n	–	number of unknown parameters
r	–	radius, m
r_i	–	radial coordinate of the i-th thermocouple, m
t	–	pitch of water wall tubes, m
T	–	temperature, °C
$\mathbf{T_m}$	–	vector of computed temperatures,
u	–	ratio of the outside to the inside radius of the flux tube, $u(\varphi) = r_o/a$

Greek symbols

φ	–	angular coordinate, rad
φ_i	–	angular coordinate of the i-th thermocouple, rad
θ	–	temperature excess over the fluid temperature, $\theta = T - T_f$
ψ	–	view factor

Subscripts

i	–	number of temperature measurement point
o	–	outer flux tube

1 Introduction

The tubular type instruments (flux tube) [1–3] and other measuring devices [4] were developed to identify boundary conditions in water wall tubes of steam boilers. The meter is constructed from a short length of eccentric tube containing four thermocouples on the fire side below the inner and outer surfaces of the tube. The fifth thermocouple is located at the rear of the tube on the casing side of the water-wall tube (Fig. 1).

The boundary conditions on the outer and inner surfaces of the water flux-tube must then be determined from temperature measurements at the interior locations. Four K-type sheathed thermocouples, 1 mm in diameter, are inserted into holes, which are parallel to the tube axis. The thermal conduction effect at the hot junction is minimized as the thermocouples pass through the isothermal holes. The thermocouples are brought to the rear of the tube in the slot machined in the protecting pad. An austenitic cover plate with thickness of 3 mm welded to the tube is used to protect the thermocouples from the incident flame radiation. A K-type sheathed thermocouple with a pad is used to measure the temperature at the rear of the flux-tube. This temperature is almost the same as the water-steam temperature. A method for determining the fireside heat flux, heat transfer coefficient on the inner surface and temperature of water-steam mixture in water-wall tubes is developed. The unknown parameters are estimated based on the temperature measurements at few internal locations from the solution of the inverse heat conduction problem. The non-linear least squares problem is solved numerically using the Levenberg-Marquardt method. The diameter of the measuring tube can be larger than the water-wall tube diameter. The view factor defining the distribution of the heat flux on the measuring tube circumference was determined using exact analytical formulas and compared with the results obtained numerically using ANSYS software. The method developed can also be used for an assessment of scale deposition on the inner surfaces of the water-wall tubes or slagging on the fire side. The presented method is suitable for water walls made of bare tubes as well as for membrane water walls. The heat transfer conditions in adjacent boiler tubes have no impact on temperature distribution in the flux tubes.

2 Theory

At first, the temperature distribution at the cross section of the measuring tube will be determined, i.e. the direct problem will be solved. Linear direct heat conduction problem can be solved using analytical method. The temperature distribution will be calculated numerically using the finite element method (FEM). In order to show accuracy of a numerical approach, the results obtained from numerical and analytical methods will be compared.

The following assumptions have been made:

- thermal conductivity of the flux tube material is constant,
- heat transfer coefficient on the inner surface of the measuring tube does not vary on the tube circumference,
- rear side of the water-wall, including the measuring tube, is thermally insulated,

- diameter of the eccentric flux tube is larger than the diameter of the water-wall tubes,
- the outside surface of the measuring flux tube is irradiated by the plane flame surface, so the heat absorption on the tube fire side is non-uniform.

The temperature distribution in the eccentric heat flux tube is governed by heat conduction

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\,\frac{\partial\theta}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\varphi}\left(\frac{k}{r}\frac{\partial\theta}{\partial\varphi}\right)=0\,, \tag{1}$$

subject to the following boundary conditions

$$k\nabla\theta\cdot\mathbf{n}|_{r=r_o}=q_m\psi\left(\varphi\right)\,, \tag{2}$$

$$k\left.\frac{\partial\theta}{\partial r}\right|_{r=a}=h\theta|_{r=a}\,. \tag{3}$$

The cylindrical coordinate system is shown in Fig. 2. The left side of Eq. (2) can be transformed as follows (Fig. 2):

$$\begin{aligned}k\nabla\theta\cdot\mathbf{n}|_{r=r_o} &= (\mathbf{q}_r+\mathbf{q}_{rs})\cdot\mathbf{n}|_{r=r_o}= \\ &= \left.\left[k\frac{\partial T}{\partial r}\cos\left(\varphi_1-\varphi\right)+\frac{k}{r}\frac{\partial T}{\partial\varphi}\sin\left(\varphi_1-\varphi\right)\right]\right|_{r=r_o}.\end{aligned} \tag{4}$$

The second term in Eq. (4) can be neglected since it is very small and the boundary condition (2) simplifies to

$$k\left.\frac{\partial\theta}{\partial r}\right|_{r=r_o}=\frac{q_m\psi\left(\varphi\right)}{\cos\left(\varphi_1-\varphi\right)}\,. \tag{5}$$

Heat flux over the tube circumference can be approximated by the Fourier polynomial

$$\frac{q_m\psi\left(\varphi\right)}{\cos\left(\varphi_1-\varphi\right)}=q_0+\sum_{n=1}^{\infty}q_n\cos\left(n\varphi\right)\,, \tag{6}$$

where

$$q_0=\frac{1}{\pi}\int\limits_0^{\pi}\frac{q_m\psi\left(\varphi\right)}{\cos\left(\varphi_1-\varphi\right)}d\varphi\,,\quad q_n=\frac{2}{\pi}\int\limits_0^{\pi}\frac{q_m\psi\left(\varphi\right)}{\cos\left(\varphi_1-\varphi\right)}\cos\left(n\varphi\right)d\varphi\,,\quad n=1,\ldots \tag{7}$$

The boundary value problem (1)–(3) was solved using the separation of variables to give

$$\theta(r,\varphi) = A_0 + B_0 \ln r + \sum_{n=1}^{\infty} \left(C_n r^n + D_n r^{-n}\right) \cos n\varphi \,, \tag{8}$$

where

$$A_0 = \frac{q_0 r_o(\phi)}{k} \left(\frac{1}{\mathrm{Bi}} - \ln a \right) \,, \tag{9}$$

$$B_0 = \frac{q_0 r_o(\phi)}{k} \,, \tag{10}$$

$$C_n = \frac{q_n r_o(\varphi)}{k} \frac{\frac{1}{n} u^n (\mathrm{Bi} + n) \frac{1}{a^n}}{\mathrm{Bi}\,(u^{2n}+1) + n\,(u^{2n}-1)} \,, \tag{11}$$

$$D_n = -\frac{q_n r_o(\varphi)}{k} \frac{\frac{1}{n} u^n (\mathrm{Bi} - n)\, a^n}{\mathrm{Bi}\,(u^{2n}+1) + n\,(u^{2n}-1)} \,. \tag{12}$$

The ratio of the outer to inner radius of the eccentric flux tube: $u = u(\varphi) = r_o(\varphi)/a$ depends on the angle φ, since the outer radius of the tube flux

$$r_o = e\cos\varphi + \sqrt{b^2 - (e\sin\varphi)^2} \tag{13}$$

is the function of the angle φ .

Equation (8) can be used for the temperature calculation when all the boundary conditions are known. In the inverse heat conduction problem three parameters are to be determined, namely:

- absorbed heat flux referred to the projected furnace wall surface: $x_1 = q_m$,
- heat transfer coefficient on the inner surface of the boiler tube: $x_2 = h$,
- fluid bulk temperature: $x_3 = T_f$.

These parameters appear in boundary conditions (3) and (5) and will be determined based on the wall temperature measurements at m internal points (r_i, φ_i)

$$T(r_i, \varphi_i) = f_i \,, \quad i = 1, \ldots, m \,, \quad m \geq 3 \,. \tag{14}$$

In a general case, the unknown parameters: $x_1, \ldots, x_n$ are determined by minimizing the sum of squares

$$S = (\mathbf{f} - \mathbf{T_m})^T (\mathbf{f} - \mathbf{T_m}) \ , \tag{15}$$

where $\mathbf{f} = (f_1, \ldots, f_m)^T$ is the vector of measured temperatures, and $\mathbf{T}_m = (T_1, \ldots, T_m)^T$ the vector of computed temperatures $T_i = T(r_i,\varphi_i)$, $i = 1, \ldots, m$.

The parameters $x_1 \ldots x_n$, for which the sum (15) is minimum are determined using the Levenberg-Marquardt method [5]. The parameters, $\boldsymbol{x}$, are calculated by the following iteration

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta^{(k)}, \ \ k = 0, 1, \ldots \ , \tag{16}$$

where

$$\delta^{(k)} = \left[\left(\mathbf{J}_m^{(k)}\right)^T \mathbf{J}_m^{(k)} + \mu^{(k)}\mathbf{I}_n\right]^{-1} \left(\mathbf{J}_m^{(k)}\right)^T \left[\mathbf{f} - \mathbf{T}_m\left(\mathbf{x}^{(k)}\right)\right] \ . \tag{17}$$

The Jacobian $\mathbf{J}_m$ is given by

$$\mathbf{J}_m = \frac{\partial\, \mathbf{T}_m(\mathbf{x})}{\partial \mathbf{x}^T} = \left[\left(\frac{\partial\, T_i(\mathbf{x})}{\partial\, x_j}\right)\right]_{m\times n} i = 1, \ldots, m \quad j = 1, \ldots, n \ . \tag{18}$$

The symbol $\mathbf{I}_n$ denotes the identity matrix of $n \times n$ dimension, and $\mu^{(k)}$ the weight coefficient, which changes in accordance with the algorithm suggested by Levenberg and Marquardt. The upper index T denotes the transposed matrix. Temperature distribution $T(r,\varphi,\ \mathbf{x}^{(k)})$ is computed at each iteration step using Eq. (8). After a few iteration we obtain a convergent solution.

3 Algorithm verification and boiler tests

Firstly, a computational example will be presented. "Experimental data" are generated artifficially using the analytical formula (8).

Consider a water-wall tube with the following parameters (Fig. 1):

- outside radius: $b = 35$ mm,
- inside radius: $a = 25$ mm,
- pitch of the water-wall tubes: $t = 80$ mm,

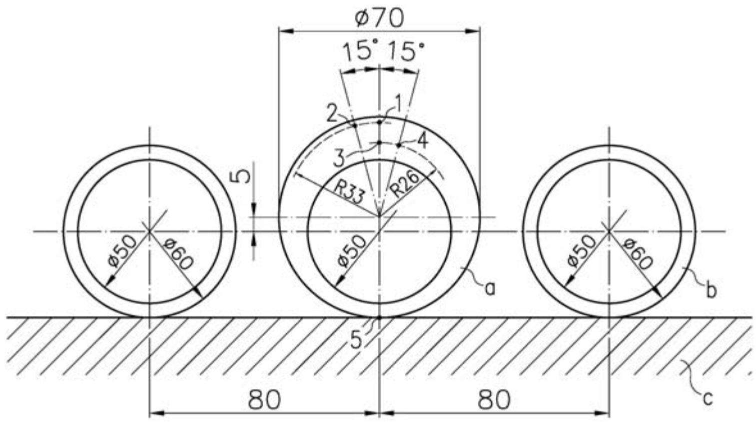

Figure 1. The heat flux tube placed between two water wall tubes, a – flux tube, b – water wall tube, c – thermal insulation.

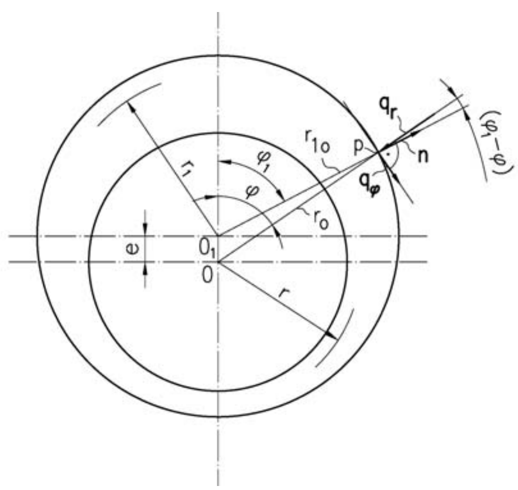

Figure 2. Approximation of the boundary condition on the outer tube surface.

- thermal conductivity: $k = 28.5$ W/(m K),
- absorbed heat flux: $q_m = 200000$ W/m^2,
- heat transfer coefficient: $h = 30000$ W/(m^2 K),
- fluid temperature: $T_f = 318$ °C.

The view factor distribution on the outer surface of water-wall tube was calculated analytically and numerically by means of the finite element

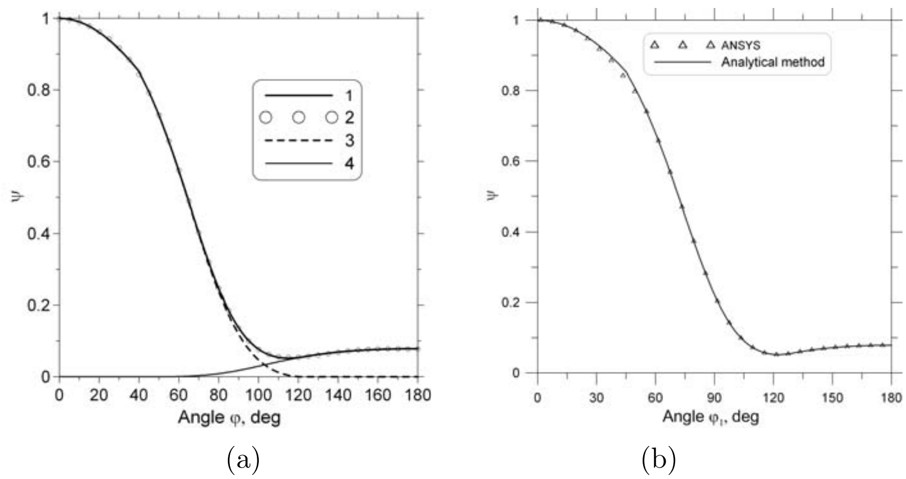

Figure 3. View factor associated with radiation heat exchange between elemental surface on the flux tube and flame: (a) 1 – total view factor accounting radiation from furnace and boiler setting, 2 – approximation by the Fourier polynomial of the seventh degree, 3 – exact view factor for furnace radiation, 4 – view factor from boiler setting; (b) comparison of total view factor calculated by exact and FEM method.

method. The changes of the view factor over the tube circumference are illustrated in Fig. 3a. Comparison of analytical and numerical results is presented in Fig. 3b. The agreement between the temperatures of the outer and inner tube surfaces which were both calculated analytically and numerically is also very good (Fig. 4). The small differences between the analytical and FEM solutions are caused by the approximate boundary condition (5). The temperature distribution in the flux tube cross section is shown in Fig. 5. The following input data is generated using Eq. (8): $f_1 = 438.24\,^{\circ}\mathrm{C}$, $f_2 = 434.79\,^{\circ}\mathrm{C}$, $f_3 = 383.52\,^{\circ}\mathrm{C}$, $f_4 = 380.90\,^{\circ}\mathrm{C}$, $f_5 = 321.58\,^{\circ}\mathrm{C}$.

The following values were obtained using the proposed method: $q_m^* = 200000.57\ \mathrm{W/m^2}$, $h^* = 30001.80\ \mathrm{W/(m^2K)}$, $T_f^* = 318.00\ ^{\circ}\mathrm{C}$. There is only a small difference between the estimated parameters and the input values. The highest temperature occurs at the crown of the flux-tube (Figs 4 and 5). The temperature of the inner surface of the flux tube is only a few degrees above the saturation temperature of the water-steam mixture. Since the heat flux on the rear side of the tube is small, the circumferential heat flow rate is significant. However, the rear surface thermocouple indicates temperatures of 2–4 °C above the saturation temperature. Therefore, the fifth thermocouple can be attached to the unheated side of the tube so as to measure the temperature of the water-steam mixture flowing through the flux tube.

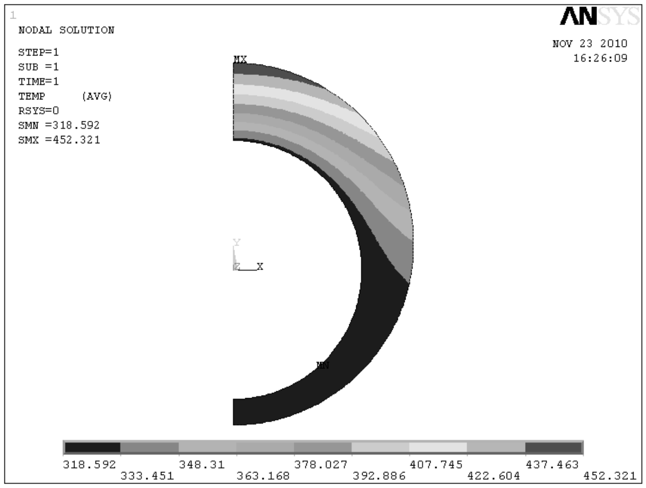

Figure 4. Computed temperature distribution in °C in the cross section of the heat flux tube; $q_m = 200000$ W/m^2, $h = 30000$ W/(m^2 K), $T_f = 318$ °C.

In the second example, experimental results will be presented. Measurements were conducted at a 50 MW pulverized coal fired boiler. The temperatures indicated by the flux tube at the height of 19.2 m are shown in Fig. 6. The heat flux tube is of 20 G low carbon steel with temperature dependent thermal conductivity

$$k\left(T\right) = 53.26 - 0.02376224T - 8.67133{\cdot}10^{-6}\,T^2, \tag{19}$$

where the temperature T is expressed in °C and thermal conductivity in W/(m K). The unknown parameters were determined for eight time points which are marked in Fig. 6.

The inverse analysis was performed assuming the constant thermal conductivity $k(\bar{T})$ which was obtained from Eq. (19) for the average temperature: $\bar{T} = (T_1 + T_2 + T_3 + T_4)\,/4$. The estimated parameters: heat flux q_m, heat transfer coefficient h, and the water-steam mixture T_f are depicted in Fig. 7.

The developed flux tube can work for a long time in the destructive high temperature atmosphere of a coal-fired boiler. Flux tubes can also be used as a local slag monitor to detect a build up of slag. The presence of the scale on the inner surface of the tube wall can also be detected.

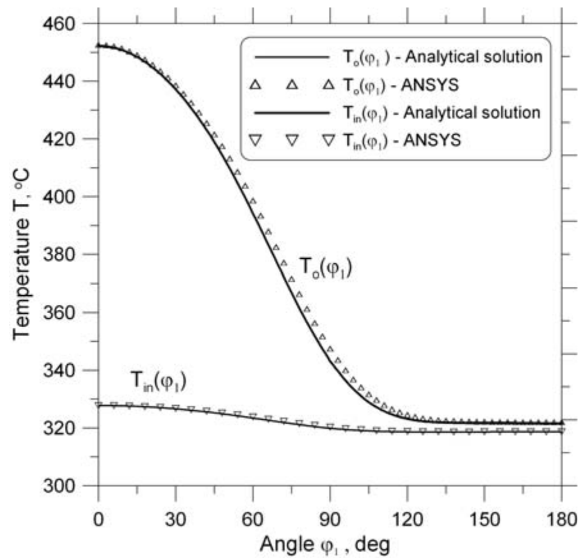

Figure 5. Temperature distribution on the inner and outer surface of the flux tube calculated by the analytical and finite element method.

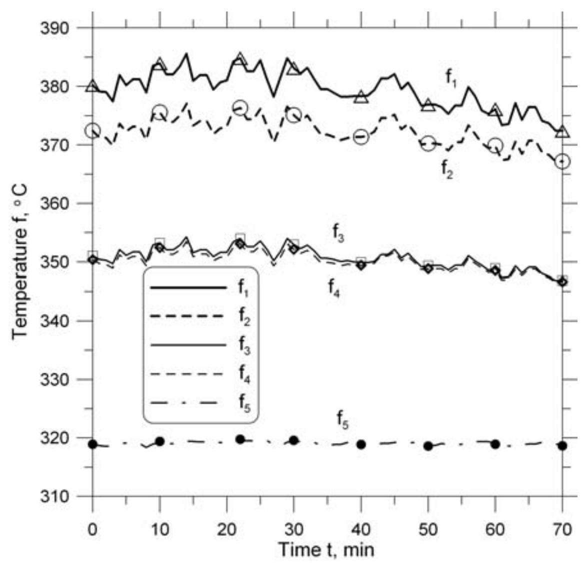

Figure 6. Measured flux tube temperatures; marks denote measured temperatures taken for the inverse analysis.

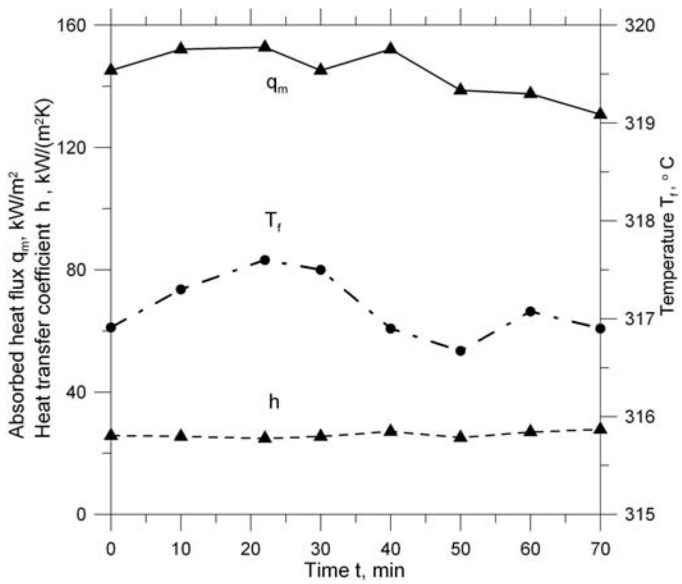

Figure 7. Estimated parameters: absorbed heat flux q_m, heat transfer coefficient h, and temperature of water-steam mixture T_f.

4 Conclusions

A new method for determining the heat flux absorbed by a furnace wall was developed. The measuring device is an eccentric tube. The ends of the four thermocouples are located at the fireside part of the tube and the fifth thermocouple is attached to the unheated rear surface of the tube. Using the temperature readings from the thermocouples and knowing the locations at which the termocouples are placed, the heat flux absorbed by the tube, the heat transfer coefficient on the inner surface, and the fluid temperature can be calculated. The meter presented in the paper has one particular advantage over the existing flux tubes to date.

The temperature distribution in the flux tube is not affected by the water wall tubes, since the flux tube is not connected to adjacent waterwall tubes with metal bars, referred to as membrane or webs. To determine the unknown parameters only the temperature distribution at the cross section of the flux tube must be analysed.

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