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Semi-Markov model of the operation process included in an utilization subsystem of the transport system

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Based on the identification of the detailed subsystem in a real urban transport system and the process of transport means operation utilized therein, a model of such process was built. For this purpose, crucial states of operation process utilized in a detailed subsystem were determined as well as possible transfers between those states. Based on this, an event-based model of the operation process of transport means included in the utilization subsystem was built, assuming that its model is the homogenous semi-Markov process. For operation data obtained after research conducted in a real transport system, values of unconditional periods of duration of process states, values of stationary distribution included in the Markov chain as well as values of probabilities of limit distribution of the semi-Markov process were determined. Based on this, an analysis of the transport means operation system in question was performed. This consideration in its entirety was presented based on the example of a chosen real transport means system – municipal bus transport system in a chosen urban complex. The semi-Markov model of transport means operation process utilized in the detailed subsystem presented in the article is the consecutive stage of the creation of the resultant model of operation process for the whole transport system. The resultant model will be a part of the decision-making model of shaping and evaluating the availability of transport system established as a part of a larger project.

1. Introduction

The operation of each system of the operation of technological objects, including transport systems, should be efficient. Obtaining high efficiency in operation of complex operation systems is possible when decisions made while controlling processes utilized in these systems are rational. In order to assure the appropriate running of decision-making process support tools are used, including all kinds of decision-making models, an important element of which is a mathematical model of technological object operation process (transport means). The creation of a mathematical model of technological object operation process makes it possible to perform an analysis of the process, which in turn constitutes the basis for evaluation and rational control of the system [4, 7, 8].

The process of the operation of transport means is a controlled process and, in general, it may be divided into the process of utilization (performed in the utilization subsystem) and repair process (introduced in the repair implementation subsystem). The goals of this paper include the description and analysis of the operation process of transport means utilized in municipal bus transport system in a chosen urban complex, with the help of event-based and mathematical models of this process. In the operation process the basic goal of transport system is obtained, at the same time generating profits due to the performance of the task.

Due to the random nature of the factors influencing the running and efficiency of the transport means operation process introduced in a complex system, most often in the process mathematical modeling of the operation process, stochastic processes are used. From among the random processes, both Markov and semi-Markov processes [1, 2, 3, 5, 6] are widely used in the modeling of technological objects. The implementation of model research while using the above-mentioned operation process models makes it possible to, on the one hand, analyze detailed problems connected with the operation of technological objects, and on the other hand, the relations between the designed number of model parameters. Using Markov processes for modeling of technological objects is possible only when random variables (periods of duration of staying in process states) characteristic of process states are defined by exponential distribution. For operation data obtained from the research of authentic transport systems, meeting the above mentioned condition is practically impossible. Because of that, in the creation of mathematical model of the operation process of transport means the theory pertaining to semi-Markov processes should be incorporated. Implementing semi-Markov processes makes it possible to create and analyze the mathematical model of the operation process in the case of the variables characteristic of the defined process states being distributions other than exponential.

2. Model of transport means operation process

2.1. Event-based operation process model

The model of operation process was created on the basis of the analysis of state space as well as operational events pertaining to technological objects (municipal buses) used in the analyzed authentic transport system. Due to the identification of the analyzed transport system and the multi-state process of technological object operation utilized in it, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation process states, shown in Figure 1.

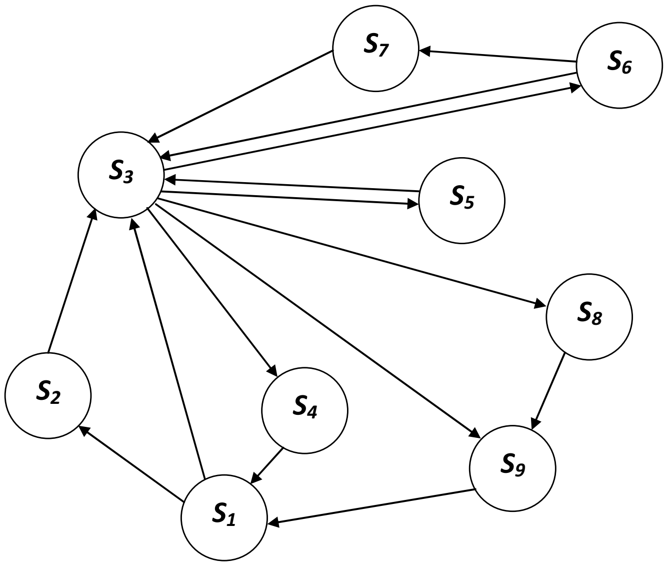

Fig. 1. Directed graph representing the transport means operation process:
S_1 – stopover at bus depot parking space, S_2 – repair at bus depot parking space, S_3 – carrying out of the transport goal, S_4 – fuel intake between transport peak hours, S_5 – repair by technical support unit without losing a ride, S_6 – repair by technical support unit with losing a ride, S_7 – awaiting the start of task realization after technical support repair, S_8 – emergency exit, S_9 – technical object repair at the efficiency implementation subsystem posts.

Rys. 1. Graf skierowany odwzorowania procesu eksploatacji środków transportu:
S_1 – postój na placu zajezdni autobusowej, S_2 – uzdatnianie na placu zajezdni autobusowej, S_3 – realizacja zadania przewozowego, S_4 – uzupełnianie paliwa między szczytami komunikacyjnymi, S_5 – uzdatnianie przez jednostkę pogotowia technicznego bez utraty kursu, S_6 – uzdatnianie przez jednostkę pogotowia technicznego z utratą kursu, S_7 – oczekiwanie na rozpoczęcie realizacji zadania po uzdatnianiu przez pogotowie techniczne, S_8 – zjazd awaryjny, S_9 – uzdatnianie obiektu technicznego na stanowiskach podsystemu zapewniania zdatności.

2.2. Mathematical model of the operation process

The mathematical model of transport means operation process implemented in the utilization subsystem of the analyzed transport system was built with the use of the semi-Markov processes theory. The model of transport means operation built with the use of Markov processes is a simplified model. In authentic conditions, assuming that periods of duration of technological objects performing at operation states have exponential distributions is unrealistic. The semi-Markovian $X(t)$ process is one, where periods of time between the changes of consecutive process states have arbitrary probability distributions and a transfer to the consecutive state depends on the current process state.

Using the semi-Markov processes in mathematical modeling of the operation process, the following assumptions were put forward:

- the modeled operation process has a finite number of states S_i, i = 1,2,…,9,
- if technological object at moment t is in state S_i, then $X(t)$ = i, where i = 1,2,…,9,
- the random process $X(t)$ being the mathematical model of the operation process is a homogenous process,
- at moment $t = 0$, the process finds is in state S_3 (the initial state is state S_3), i.e. $P\{X(0) = 3\} = 1$.

The homogenous semi-Markovian process is unequivocally defined when initial distribution and its kernel are given [1, 5, 6]. Form our assumptions and based on the directed graph shown in Figure 1, the initial distribution $p_i(0)$, i = 1,2,…,9 takes up the following form:

$$p_i(0) = \begin{cases} 1 & when \quad i = 3 \\ \\ 0 & when \quad i \neq 3 \end{cases} \tag{1}$$

where

$$p_i(0) = P\{X(0) = i\}, \quad i = 1,2,...,9 \tag{2}$$

whereas the kernel of process $Q(t)$ takes up the form:

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & Q_{13}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{23}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{34}(t) & Q_{35}(t) & Q_{36}(t) & 0 & Q_{38}(t) & Q_{39}(t) \\ Q_{41}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{53}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{63}(t) & 0 & 0 & 0 & Q_{67}(t) & 0 & 0 \\ 0 & 0 & Q_{73}(t) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t) \\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

where

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \leq t | X(\tau_n) = i\}, \quad i, j = 1,2,...,9 \tag{4}$$

means that the state of semi-Markovian process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where $\tau_1, \tau_2, ..., \tau_n, ...$ are arbitrary moments in time, so that $\tau_1 < \tau_2 < ... < \tau_n < ...$; as well as

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t) \tag{5}$$

where:

$$p_{ij} = \lim_{t \to \infty} p_{ij}(t) \tag{6}$$

p_{ij} – means that the conditional probability of transfer from state S_i to state S_j,

$$p_{ij}(t) = P\{X(t) = j | X(0) = i\} \tag{7}$$

as well as

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \le t | X(\tau_n) = i, X(\tau_{n+1}) = j\}, \quad i, j = 1,2,...,9 \tag{8}$$

is a distribution function of random variable Θ_{ij} signifying period of duration of state S_i, under the condition that the next state will be state S_j.

Limit probability p_i^* of staying in states of semi-Markov process were assigned on the basis of limit theorem for semi-Markovian processes [1, 5]:
If hidden Markov chain in semi-Markovian process with finite state S set and continuous type kernel contains one class of positive returning states such that for each state $i \in S, f_{ij} = 1$ and positive expected values $E(\Theta_i), i \in S$ are finite, limit probabilities

$$p_i^* = \lim_{t \to \infty} p_i(t) = \frac{\pi_i \cdot E(\Theta_i)}{\sum_{i \in S} \pi_i \cdot E(\Theta_i)} \tag{9}$$

exist where probabilities $\pi_i, i \in S$ constitute a stationary distribution of a hidden Markov chain, which fulfills the simultaneous linear equations

$$\sum_{i \in S} \pi_i \cdot p_{ij} = \pi_j, \quad j \in S, \quad \sum_{i \in S} \pi_i = 1 \tag{10}$$

In order to assign the values of limit probabilities p_i^* of staying in the states of semi-Markovian model of transport means operation, based on the directed graph shown in figure 1, the following were created: matrix *P* of the states change probabilities and matrix *Θ* of conditional periods of duration of the states in process *X(t)*:

$$P = \begin{bmatrix} 0 & p_{12} & p_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{34} & p_{35} & p_{36} & 0 & p_{38} & p_{39} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{63} & 0 & 0 & 0 & p_{67} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{11}$$

$$\Theta = \begin{bmatrix} 0 & \overline{\Theta}_{12} & \overline{\Theta}_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Theta}_{34} & \overline{\Theta}_{35} & \overline{\Theta}_{36} & 0 & \overline{\Theta}_{38} & \overline{\Theta}_{39} \\ \overline{\Theta}_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{53} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{63} & 0 & 0 & 0 & \overline{\Theta}_{67} & 0 & 0 \\ 0 & 0 & \overline{\Theta}_{73} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{\Theta}_{89} \\ \overline{\Theta}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

Based on the matrix of the states change probabilities $P = [p_{ij}]$ and on the matrix of average values $\Theta = \left[\overline{\Theta}_{ij}\right]$ of random variables Θ_{ij}, average values $\overline{\Theta_i}$ of non-conditional duration periods of process states were defined, according to the dependence

$$\overline{\Theta_i} = \sum_{i=1}^{9} p_{ij} \cdot \overline{\Theta}_{ij}, \quad i, j = 1,2,...,9 \quad . \tag{13}$$

Random variable Θ_{ij} means that the period of duration of technological object in i-th state, under the condition that the next state will be j-th state.

<u>Non-conditional periods of duration of process states:</u>

$$\overline{\Theta_1} = p_{12} \cdot \overline{\Theta}_{12} + p_{13} \cdot \overline{\Theta}_{13} \tag{14}$$

$$\overline{\Theta_2} = p_{23} \cdot \overline{\Theta}_{23} = \overline{\Theta}_{23} \tag{15}$$

$$\overline{\Theta_3} = p_{34} \cdot \overline{\Theta}_{34} + p_{35} \cdot \overline{\Theta}_{35} + p_{36} \cdot \overline{\Theta}_{36} + p_{38} \cdot \overline{\Theta}_{38} + p_{39} \cdot \overline{\Theta}_{39} \tag{16}$$

$$\overline{\Theta_4} = p_{41} \cdot \overline{\Theta}_{41} = \overline{\Theta}_{41} \tag{17}$$

$$\overline{\Theta_5} = p_{53} \cdot \overline{\Theta}_{53} = \overline{\Theta}_{53} \tag{18}$$

$$\overline{\Theta_6} = p_{63} \cdot \overline{\Theta}_{63} + p_{67} \cdot \overline{\Theta}_{67} \tag{19}$$

$$\overline{\Theta_7} = p_{73} \cdot \overline{\Theta}_{73} = \overline{\Theta}_{73} \tag{20}$$

$$\overline{\Theta_8} = p_{89} \cdot \overline{\Theta}_{89} = \overline{\Theta}_{89} \tag{21}$$

$$\overline{\Theta_9} = p_{91} \cdot \overline{\Theta}_{91} = \overline{\Theta}_{91} \tag{22}$$

Then, with the use of the MATHEMATICA software, stationary distribution of Markov chain hidden in the process and limit distribution of semi-Markov process were defined.

Stationary distribution hidden Markov chain:

$$\pi_1 = \frac{p_{34} + p_{38} + p_{39}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{23}$$

$$\pi_2 = \frac{p_{12}(p_{34} + p_{38} + p_{39})}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{24}$$

$$\pi_3 = \frac{1}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{25}$$

$$\pi_4 = \frac{p_{34}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{26}$$

$$\pi_5 = \frac{p_{35}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{27}$$

$$\pi_6 = \frac{p_{36}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{28}$$

$$\pi_7 = \frac{p_{36} \cdot p_{67}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{29}$$

$$\pi_8 = \frac{p_{38}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{30}$$

$$\pi_9 = \frac{p_{38} + p_{39}}{1 + [(2 + p_{12}) \cdot (p_{34} + p_{38} + p_{39})] + p_{35} + [p_{36} \cdot (1 + p_{67})] + p_{38}} \tag{31}$$

Limit distribution of semi-Markov process:

$$p_1^* = \frac{(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_1}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (32)$$

$$p_2^* = \frac{p_{12}\cdot(p_{34}+p_{38}+p_{39})\cdot\overline{\Theta_2}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (33)$$

$$p_3^* = \frac{\overline{\Theta_3}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (34)$$

$$p_4^* = \frac{p_{34}\cdot\overline{\Theta_4}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (35)$$

$$p_5^* = \frac{p_{35}\cdot\overline{\Theta_5}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (36)$$

$$p_6^* = \frac{p_{36}\cdot\overline{\Theta_6}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (37)$$

$$p_7^* = \frac{p_{36}\cdot p_{67}\cdot\overline{\Theta_7}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (38)$$

$$p_8^* = \frac{p_{38}\cdot\overline{\Theta_8}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (39)$$

$$p_9^* = \frac{(p_{38}+p_{39})\cdot\overline{\Theta_9}}{\left[(p_{34}+p_{38}+p_{39})\cdot\left(\overline{\Theta_1}+p_{12}\cdot\overline{\Theta_2}\right)\right]+\overline{\Theta_3}+p_{34}\cdot\overline{\Theta_4}+p_{35}\cdot\overline{\Theta_5}+\left[p_{36}\cdot\left(\overline{\Theta_6}+p_{67}\cdot\overline{\Theta_7}\right)\right]+p_{38}\cdot\overline{\Theta_8}+(p_{38}+p_{39})\cdot\overline{\Theta_9}} \quad (40)$$

Based on source data obtained from operation research performed in authentic municipal bus transport system, the values of the elements of matrix P (11) as well as matrix Θ (12) were estimated. Then matrices P and Θ are thus defined:

$$P = \begin{bmatrix} 0 & 0{,}028314 & 0{,}971686 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0{,}143773 & 0{,}088176 & 0{,}035203 & 0 & 0{,}037527 & 0{,}695321 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}102236 & 0 & 0 & 0 & 0{,}897764 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (41)$$

$$\Theta = \begin{bmatrix} 0 & 8,081 & 5,589 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,28 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 6,394 & 6,122 & 0 & 6,178 & 10,656 \\ 0,743 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,091 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0,975 & 0 & 0 & 0 & 1,002 & 0 & 0 \\ 0 & 0 & 0,442 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1,188 \\ 3,203 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (42)$$

In the matrix (42) the values of conditional duration periods of process $\overline{\Theta}_{ij}$ states were shown in [h].

For the data shown in matrices (41) and (42) values of non-conditional duration periods for process states, values of stationary distribution hidden in Markov chain process as well as values of limit distribution of semi-Markov process (limit probabilities values for technological object staying in process states) were defined. Results were shown in Tables 1 to 3.

Table 1. Values of non-conditional duration periods $\overline{\Theta}_i$ [h] of staying in the states of process *X(t)*.

Tablica 1. Wartości bezwarunkowych czasów $\overline{\Theta}_i$ [h] przebywania w stanach procesu *X(t)*.

$\overline{\Theta}_1 = 5,65956$	$\overline{\Theta}_2 = 0,28$	$\overline{\Theta}_3 = 8,85181$
$\overline{\Theta}_4 = 0,743$	$\overline{\Theta}_5 = 0,091$	$\overline{\Theta}_6 = 0,99924$
$\overline{\Theta}_7 = 0,442$	$\overline{\Theta}_8 = 1,188$	$\overline{\Theta}_9 = 3,203$

Table 2. Values of probabilities π_i of stationary distribution of hidden Markov chain.

Tablica 2. Wartości prawdopodobieństw π_i rozkładu stacjonarnego włożonego łańcucha Markowa.

$\pi_1 = 0,29510$	$\pi_2 = 0,00835$	$\pi_3 = 0,33663$
$\pi_4 = 0,04840$	$\pi_5 = 0,02968$	$\pi_6 = 0,01185$
$\pi_7 = 0,01064$	$\pi_8 = 0,01263$	$\pi_9 = 0,24670$

Table 3. Values of probabilities p_i^* of limit distribution of the semi-Markov process
Tablica 3. Wartości prawdopodobieństw p_i^* rozkładu granicznego procesu semi-Markowa

p_1^* = 0,30296	p_2^* = 0,00042	p_3^* = 0,54054
p_4^* = 0,00652	p_5^* = 0,00049	p_6^* = 0,00215
p_7^* = 0,00085	p_8^* = 0,00272	p_9^* = 0,14334

3. Conclusion

Based on the analysis of the defined values of probabilities π_i of the hidden Markov chain it may be determined that the highest probability of entry into the states of process pertains to the following states:

- S_3 carrying out of the task en route,
- S_1 awaiting the implementation of the task at the parking space of the bus depot,
- S_9 repair of technological object at repair implementation subsystem posts.

Resulting from the obtained values of probability π_i, more than 16% of objects implementing the task undergo damage en route. At the same time one may conclude that most of them are repaired (76.7%) by technical support units. The remaining (23.3%) objects damaged during the implementation of the transport task are moved to the bus depot – emergency exit on their own or towed. From among the repairs done by technical support units close to 71.5% are so-called minor repairs, mainly limited to regulation or exchange of small elements, performed without loss of a ride. The remaining repairs (28.5%) are done over a longer period of time than breaks between rides making it necessary to substitute technological objects damaged during the performance of the transport task by reserve objects. Resulting from the obtained probability value π_2, a small amount (less than 3%) of the technological objects awaiting the start of transport task are repaired at the bus depot parking space before the performance of the transport task. Repair of technological objects at the bus depot parking space is done during a break period, designed for checking and preparing of the transport means by the driver directly before the performance of the transport task.

Based on the non-conditional value of time periods $\overline{\Theta}_i$ as well as limit probabilities p_i^* defined for the semi-Markov process one may conclude that the states of the process in which a statistical technological object stays the longest are states S_3 (performance of the task), S_1 (awaiting the performance of the task at the parking space) – jointly over 84% of the operation time as well as S_9 (repair at the efficiency implementation subsystem posts) – more than 14% of the operation period. The remaining operation time pertains to technological object repair time by technical support units. The overall percentage of time spent on repairing technological objects by technical emergency units (p_5^*+p_6^*+p_8^*) in the total time of operation amounts to only a little more than 0.5 %. However, it is crucial due the possibility of appropriate performance of the transport task.

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Semimarkowski model procesu eksploatacji realizowanego w podsystemie wykonawczym systemu transportowego

S t r e s z c z e n i e

Na podstawie identyfikacji podsystemu wykonawczego w rzeczywistym systemie komunikacji miejskiej i realizowanego w nim procesu eksploatacji środków transportu został zbudowany model tego procesu. W tym celu wyznaczono istotne stany procesu eksploatacji realizowanego w podsystemie wykonawczym oraz możliwe przejścia między wyróżnionymi stanami. Na tej podstawie zbudowano zdarzeniowy model procesu eksploatacji środków transportu realizowanego w podsystemie wykonawczym, a następnie matematyczny model tego procesu, zakładając, że jego modelem jest jednorodny proces semi-Markowa. Dla danych eksploatacyjnych, uzyskanych z badań przeprowadzonych w rzeczywistym systemie transportowym, wyznaczono wartości bezwarunkowych czasów trwania stanów procesu, wartości rozkładu stacjonarnego włożonego w proces łańcucha Markowa oraz wartości prawdopodobieństw rozkładu granicznego procesu semi-Markowa. Na tej podstawie dokonano analizy rozpatrywanego procesu eksploatacji środków transportu. Całość rozważań przedstawiono na przykładzie wybranego rzeczywistego systemu eksploatacji środków transportu – systemu autobusowej komunikacji miejskiej w wybranej aglomeracji. Prezentowany w artykule semimarkowski model procesu eksploatacji środków transportu realizowanego w podsystemie wykonawczym, jest kolejnym etapem budowy modelu wynikowego procesu eksploatacji dla całego systemu transportowego. Model wynikowy stanowić będzie część składową, opracowywanego w ramach szerszego projektu, decyzyjnego modelu kształtowania i oceny gotowości systemu transportowego.