Paper

Recurrent Method for Blocking Probability Calculation in Switching Networks with Overflow Links

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Abstract—This article presents a new recurrent method for modelling multi-service switching networks with overflow links. In the proposed method, the blocking probability for a given stage of the switching network is determined on the basis of the characteristics of the preceding stage. A particular attention is given to a possibility of a considerable reduction of the internal blocking probability of the switching network that would result from an application of additional overflow links between neighbouring switches of the first stage of the network. The results of the analytical modelling of selected multi-service switching networks with overflow links in the first stage are compared with the results of the simulation experiments. The study confirms the accuracy of all the adopted theoretical assumptions in the proposed analytical model of the multi-service switching network.

Keywords—inter-section links, switching networks.

1. Introduction

Switching networks are essential for the efficient operation of many devices used in network nodes. The parameters of the switching network directly influence the operation of the network in terms of average connection speed and have an effect on the availability of resources for the network's users. Hence, a choice of the optimal switching network with regard to its efficiency and economy is absolutely vital for the effective operation of the network.

Switching networks can be generally divided into non-blocking and blocking networks [1]. Non-blocking networks are the most effective in terms of traffic efficiency as they eliminate the phenomenon of the internal blocking. The phenomenon involves the impossibility of setting up a connection in the switching network that has free available input and output. The structures of non-blocking networks, however, are very expensive since they require a great number of switching elements and advanced controlling algorithms. In this case, the increase in the traffic effectiveness of the network is disproportionally burdened economically as compared to costs of its construction. Blocking networks allow for a loss in part of offered traffic due to the occurrence of the internal blocking phenomenon. Even though this limitation is a hindrance, solutions based on blocking networks structures are viable and economically reasonable (on account of a lower number of switching elements) and are commonly applied in practice.

There are a number of methods for limiting the influence of the phenomenon of the internal blocking in blocking networks [2]. The three most important include: dynamic routing, call repacking and changes in the structure. The first two methods are based on the implementation of dedicated controlling algorithms that decide on the way connections are set up. However, these solutions do not require changes in the structure, they increase considerably the load of the controlling device and, in consequence, slow down the operation of the switching network. The third approach does not involve any interference into the controlling algorithms, but it requires a change in the structure of the switching network. One of the options that is characterized by a relatively small change in the structure is the application of overflow links. This option requires switches with increased number of inputs and outputs to be applied in a given stage of the network. Switching networks with overflow were implemented for the first time in production practice in the Pentaconta cross-bar exchange as early as the 1970s and the 1980s [3]. In Pentaconta switching networks, overflow links were used in the first stage. This solution eventually led to a decrease in the internal blocking probability by several per cent [4]. The possibility of the application of overflow links in switching networks of electronic automatic telephone exchanges were later analysed in many interesting works, including [5]–[7].

Present-day networks service multi-service traffic [8], [9]. The possibility of the application of overflow links in multi-stage switching networks is examined by simulation methods in [10], [11]. These studies have confirmed significant increase in the effectiveness of the switching network with overflow links measured by a decrease in, and in some cases a virtual elimination of, the phenomenon of the internal blocking.

Switching networks with single-service traffic and overflow links are analysed in [3]–[7]. In [4], to model these networks, overflow models were used [12], while to evaluate changes in the blocking probability depending on the capacity of overflow links the exponential function was used. Ref. [7] applies a modification to the effective availability method [13], [14] that was also used in many variants to model single-service blocking switching networks, e.g., in [15]–[18].

The effective availability method is based on a reduction of the blocking probability in a multi-stage network to the calculation of this probability in the single-stage system, i.e., in a non-full-availability group [19]. The accompanying assumption is that the non-full availability group has the same capacity as the output group of the switching network, and that the availability (the so-called effective availability) is determined on the basis of the structure of the switching network and offered traffic. Article [20] proposes an interesting variant of the effective availability method, the so-called recurrent method in which the blocking probability in stage x of a multi-stage switching network is determined on the basis of the blocking probability in stage $x-1$.

In [21]–[23], to analyse multi-service switching networks the effective availability method is used. The method involves an exchange of the multi-service switching network into the equivalent model of a single-service network. The effective availability for calls of a considered class is determined in the equivalent network. After determining effective availabilities for individual classes of calls, on the basis of appropriate models of groups with multi-service traffic [24], the internal and the external blocking probabilities in the switching network are determined. For the analysis of multi-stage multi-service switching networks, [25], [26] propose a recurrent method, the so-called SN-BPPRec method (Switching Networks – BPP – Recurrently). Effective availability methods are characterised by great accuracy, are of universal nature, and can be applied in modelling multi-service switching networks with any structure and any mixture of offered traffic.

In [27], to analyse multi-service switching networks with overflow links and the point-to-group selection, the approach proposed in one of the effective availability methods is used, i.e., the so-called PGBMT method (Point-to-Group Blocking with Multi-rate Traffic) [21]. Article [28] discusses a method for modelling multi-service switching networks with overflow links and the point-to-point selection. Ref. [29] proposes a method for modelling networks with overflow links that are offered Engset traffic. The present article aims at a modification of the recurrent method [25], [26], which will make it possible to model multi-service switching networks with overflow traffic and the point-to-group selection in a much easier way than in the approach proposed in [27], [29].

The structure of the article is as follows. Section 2 describes the structure and operation of a three-stage Clos network [30] with overflow links. Section 3 discusses the concept of the equivalent network and the assumptions for the recurrent method [26]. Section 4 proposes a modification to the recurrent methods and algorithms for modelling multi-service switching networks with overflow links. In Section 5, the results of the calculations are compared with the results of the simulations for selected switching networks. Section 6 sums up the article.

2. The Structure and Operation of the Switching Network with Overflow Links

Figure 1 shows the structure of a multi-service, three-stage Clos switching network. This network is composed of $k \times k$ symmetrical switches in each stage. All input, output and interstage links in the network have the same capacity of f allocation units, the so-called Basic Bandwidth Units (BBU) [31]. Output links of the switching network are grouped according to directions in such a way that each i-th output link of each switch of the last stage belongs to the i-th output direction. The network, composed of $k \times k$ symmetrical switches, has thus k output directions.

In the switching network presented in Fig. 1 overflow links in the first stage are used. When this is the case, the overflow links connect the additional output of a given switch with the additional input of a neighbouring switch of the first stage (Fig. 1). The output of the last switch is connected with the additional input of the first switch of the first stage. A relevant simulation study was performed in [11], [10] in regard to the introduction of overflow links to different stages of the network. The study proves that the introduction of a system of overflow links to the first stage of the network, as shown in Fig. 1, is the most effective. It was also verified in the course of the study that the application of overflow links with a twofold capacity (i.e., $2f$ BBU) is followed by a virtual elimination of the blocking phenomenon in inter-stage links and a stabilization of the internal blocking probability at the level of very low values as compared to the external blocking probability. In such circumstances, the switching network can be considered to be a quasi-non-blocking network. In the model proposed in the article it is assumed that the capacity of the overflow links is at least twice as high as the capacity of the inter-stage link.

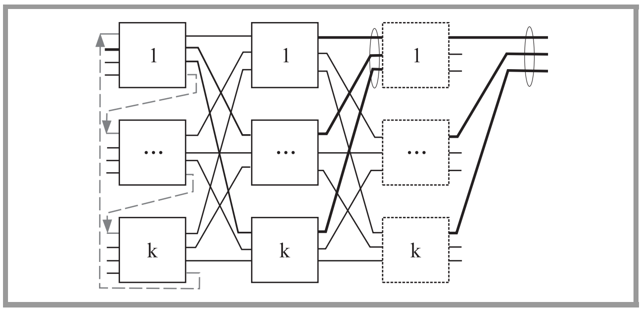

Fig. 1. The structure of a three-stage Clos switching network with overflow links.

The point-to-group selection has been used in the switching network. The algorithm for setting up a connection for this type of selection in the network with overflow links operates in the following way. After a new call appears in a given input link of the first stage switch, the controlling algorithm chooses a switch of the last stage that has an output link in the demanded direction that has free resources to set up the connection. In the next step, the controlling algorithm attempts to set up a connection between selected switches of external stages. If the connection cannot be set up, the controlling algorithm attempts to set up a connection between a selected switch of the last stage and such a switch of the first stage to which the considered switch of the first stage has access to via an overflow link (the assumption is that a given call can make use of only one overflow link that connects neighbouring switches of the first stage). If the attempt at setting up a connection between these switches fails to succeed, the controlling algorithm will choose another switch of the last stage that has a free output link in the demanded direction and will try to set up a connection with this switch directly or via an overflow link. This operation is repeated until the connection is successfully set up or all of the switches that have output links with free resources in a given direction are checked. If, after checking all switches, a connection still cannot be set up, the controlling algorithm rejects the call due to the internal blocking in the switching network. If all the output links of a given direction are busy, i.e., have no free resources, the controlling algorithm rejects the call due to the external blocking in the switching network.

3. Equivalent Network

In this section the concept of the equivalent network will be defined. The equivalent network is a single-service equivalent of a multi-service switching network for calls of a given traffic class. Then, the method for a determination of the effective availability parameter in the equivalent switching network will be discussed. This parameter forms the basis for the evaluation of the internal blocking probability in multi-service switching networks.

3.1. The Structure of the Equivalent Network

Assume that the switching network is offered multi-service traffic that is a mixture of a number of different traffic streams. In order to set up a connection of a given class, an appropriate number of BBUs is required, called the demand of a given class. Demands of different classes are different. The call stream of each of the class is a Poisson stream generated by an infinite number of traffic sources. The service time of calls of each of the class is described by the exponential distribution. It follows that all traffic streams are Erlang streams [8], [9].

The basis for most of multi-service models of switching networks is the notion of the equivalent switching network for the traffic stream of a given class [21]. This means that, for traffic of a given class, a fictitious model of the single-service network servicing fictitious traffic of that class is constructed. The equivalent network is thus a single-service network with the same structure as the multi-service network under consideration. In this network, each link has a capacity of 1 BBU, while each call demands 1 BBU to set up a connection. In networks carrying single-service traffic (with the demand of 1 BBU), the average load of a link (with the capacity of 1 BBU) determines at the same time the value of the blocking probability of the link. In multi-service models of switching networks, for a traffic stream of a given class i, the multi-service switching network is reduced to the equivalent network. Each link of this equivalent network is allocated the load $e(i)$, equal to the blocking probability for a stream of class i in an inter-stage link of the real multi-stage network. This probability can be determined on the basis of the recursive Kaufman-Roberts equations [32], [33] that determine the occupancy distribution in the multi-service full-availability group with the capacity equal to f BBUs:

$$n[P_n]_f = \sum_{i=1}^{M} A_i t_i [P_{n-t_i}]_f \,, \tag{1}$$

$$e(i) = \sum_{i=f-t_i+1}^{f} P[n]_f \,, \tag{2}$$

where:

- $[P_n]_f$ – the occupancy distribution (the probability of the occupancy of n BBUs) in the multi-service full-availability group with the capacity of f BBUs,
- A_i – the average traffic intensity of traffic of class i,
- t_i – call demand of class i, expressed in the number of BBUs,
- M – the number of traffic classes offered to the system,
- $e(i)$ – the blocking probability of calls of class i in the multi-service full-availability group.

3.2. Effective Availability in the Equivalent Network

Having determined the parameter $e(i)$, it is possible to evaluate the effective availability in the switching network for calls of class i. The parameter can be determined on the basis of the modified formula presented in [21] that was derived for the recurrent method [26]:

$$d_{e,z}(i) = d(i, V_z, \pi_z(i)) = [1 - \pi_z(i)]V_z + \pi_z(i)\eta Y_1(i) \,, \tag{3}$$

where:

- $d_{e,z}(i, Vz, \pi_z(i))$ – the notation of the effective availability in the form the function dependent on the parameters i, V_z, $\pi_z(i)$, respectively,
- $d_{e,z}(i)$ – effective availability for the stream of class i in a z-stage equivalent network,
- $\pi_z(i)$ – direct unavailability probability of the last stage switch for calls of class i, i.e., the probability that a connection between the selected first stage switch and the selected last z-stage switch cannot be executed. This probability is determined in many methods for the evaluation of the effective availability, such as in [20], [18]–[23], on the basis of the probability graph method [34],
- V_z – capacity of the output direction in the equivalent network. For the network shown in Fig. 1, this parameter is equal to the number of switches of the third stage:

$$V_z = k_z = k, \tag{4}$$

$Y_1(i)$ – the average fictitious traffic of class i, carried by the first-stage switch; in the considered case of the three-stage switching network, this parameter is equal to:

$$Y_1(i) = ke(i), \tag{5}$$

η – part of fictitious traffic of the first-stage switch that is carried by the considered direction. If we assume that traffic is offered to each of the directions with the same probability, then:

$$\eta = 1/k. \tag{6}$$

The parameter $d_{e,z}(i)$, which forms the basis for the determination of the internal blocking probability, is used in effective availability methods. The parameter determines the average number of switches of stage z that is available from an input of a single first-stage switch for calls of class i. The notion of availability determines those switches with which a connection via free inter-stage links can be set up (the first element of Eq. (3)), as well as those switches with which the first-stage switch has a connection in a given direction (the second element of Eq. (3)).

4. Recurrent Method for Modelling Switching Networks with Overflow Links

This section proposes a method for a determination of the internal, external and the total blocking probability in multi-service switching networks with overflow links. The basis for the proposed method is the SN-BPPRec method, proposed in [26], for multi-service switching networks without overflow links. The latter method will be subsequently modified and then used to analyse switching networks with overflow links. The modification of the method involves an introduction of a number of changes in the determination of the effective availability for subsequent stages of the switching network.

4.1. Internal Blocking Probability

The internal blocking phenomenon in the multi-service switching network occurs when a connection between given switches of the first and the last stage (i.e., between a given input and output link in a given direction) cannot be set up due to the lack of free resources in inter-stage links. In the SN-BPPRec method, the internal blocking probability $E_{\text{in}}(i)$ for calls of class i is determined in a recurrent way.

In the recurrent model, the switching network is considered as a set of sub-systems. The assumption is that each stage of the network is composed of one or more subsystems (Fig. 2). The unavailability probability for s-stage switch, for calls of class i, i.e., the probability $\pi_s(i)$ in a sub-system composed of s first stages of the switching network, can be interpreted as the probability of the occurrence of the blocking phenomenon $E_{s-1}(i)$ in a group of V_{s-1} inter-stage links that lead to one s-stage switch of the equivalent network:

$$\pi_s(i) = E_{s-1}(i). \tag{7}$$

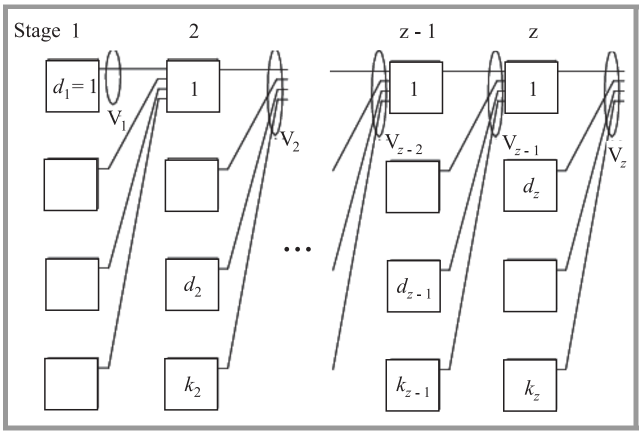

Fig. 2. Multi-stage switching network with indicated parameters used in the recurrent method.

Notice that, according to [16], the blocking probability in a group of inter-stage links that lead to one switch of the s stage, can be treated as the point-to-group blocking probability in a $(s-1)$-stage subsystem of a z-stage switching network. Therefore, the effective availability of the s-stage sub-system is the function of the point-to-group blocking probability of the $(s-1)$-stage sub-system of the switching network:

$$d_{e,s}(i) = d(i, V_s, \pi_s(i)) = d(i, V_s, E_{s-1}(i)), \tag{8}$$

where V_s is the capacity of the output group of the s-stage sub-system of the switching network, i.e., the link group that leads to one switch of the next stage.

In the SN-BPPRec method, the point-to-group blocking probability in each equivalent sub-system of the switching network is approximated by the blocking probability in the Erlang's Ideal Grading (EIG) [19]:

$$E_s(i) = \text{EIF}(A_s(i), V_s, d_{e,s}(i)), \tag{9}$$

where $\text{EIF}(A,V,d)$ determines the blocking probability in the Erlang's Ideal Grading with the capacity V, availability d and the intensity of offered traffic A. This probability is determined on the basis of the so-called EIF Formula (Erlangs Interconnection loss Formula) [19]:

$$\text{EIF}(A,V,d) = \frac{\sum\limits_{i=d}^{V} \binom{i}{d} \Big/ \binom{V}{d} (A^i/i!) \prod_{l=d}^{i-1} \left[1 - \binom{l}{d} \Big/ \binom{V}{d}\right]}{\sum\limits_{j=0}^{V} (A^j/j!) \prod_{l=d}^{j-1} \left[1 - \binom{j}{d} \Big/ \binom{V}{d}\right]} \tag{10}$$

Equations (7)–(9) determine the iterative algorithm for the calculation of the blocking probability in the equivalent switching network. The algorithm proceeds as long as the internal point-to-group blocking probability $E_{\text{in}}(i)$ in z-stage network is determined:

$$E_{\text{in}}(i) = E_z(i) = \text{EIF}(A_z(i), V_z, d_{e,z}(i)). \tag{11}$$

In Eqs. (9)–(11), the parameter $A_z(i)$ is the average intensity of traffic of class i offered to one output direction in the equivalent network with the capacity of V_z of output links.

The operation of the algorithm commences with a determination of the blocking probability $E_1(i)$, i.e., the blocking probability of a group of links that lead from one second-stage switch to one sub-system of the first stage (Fig. 2). The first-stage switch is the sub-system of the first stage of the equivalent switching network. This sub-system is a non-blocking system. Therefore, the following parameters can be adopted: $\pi_1(i) = 1$, $V_1 = 1$ i $d_{e,1} = 1$, because only one output of the first-stage switch leads to a given switch of the second stage. Hence, the parameter $E_1(i)$ can be determined in the following way:

$$E_1(i) = \text{EIF}(A_1(i), 1, 1), \tag{12}$$

where $A_1(i)$ is the average intensity of fictitious traffic of class i offered to one switch of the second stage from one sub-system of the first stage. One sub-system of the first stage (i.e., one switch of the first stage) is connected to only one switch of the second stage via one inter-stage link.

On the basis of Equations (1)–(2), we can determine fictitious traffic $e(i)$, carried by one inter-stage link of this type. Since the link of the equivalent network has the capacity of one BBU, then the relation between offered and carried traffic can be expressed as follows:

$$e(i) = a(i)[1 - E_1(a(i))], \tag{13}$$

where $E_1(a(i))$ denotes Erlang B Formula [8], [9] that determines the blocking probability in the group with the capacity 1 BBU which is offered traffic with the intensity $a(i)$. After elementary transformations in Eq. (13), we obtain:

$$A_1(i) = a(i) = \frac{e(i)}{1 - e(i)}. \tag{14}$$

The SN-BPPRec method adopts the assumption that traffic offered to the output group of stage $s\,(s > 1)$, composed of V_s links, is equal to:

$$A_s(i) = k_s\, a(i). \tag{15}$$

In the switching network presented in Fig. 1, the capacity of the output group in each of the stages is equal to k. The internal blocking probability in the switching network shown in Fig. 1 (without overflow links taken into consideration) can thus be determined in the three consecutive steps.

Step 1:

$$d_{e,1}(i) = 1, \tag{16}$$

$$E_1(i) = \text{EIF}(a(i), 1, 1), \tag{17}$$

Step 2:

$$d_{e,2}(i) = [1 - E_1(i)]k + E_1(i)\eta Y_1(i), \tag{18}$$

$$E_2(i) = \text{EIF}(ka(i), k, d_{e,2}(i)), \tag{19}$$

Step 3:

$$d_{e,3}(i) = [1 - E_2(i)]k + E_2(i)\eta Y_1(i), \tag{20}$$

$$E_3(i) = E_{\text{in}}(i) = \text{EIF}(ka(i), k, d_{e,3}(i)), \tag{21}$$

4.2. The External and the Total Blocking Probability

The phenomenon of the external blocking in the multi-service switching network occurs when all output links in a given direction have no free resources, i.e., the appropriate number of BBUs, to service a call of a given class. The external blocking probability $E_{\text{ex}}(i)$ in the multi-service switching network in the SN-BPPRec method is approximated by the blocking probability in the Limited Availability Group (LAG) [35]. The limited-availability group is a model of k identical links, each with the capacity of f BBU. The group can service a call of a given class only when there is a possibility of a service of this call in one (any) link of this group. The above definition of LAG reflects the operation of the output group of the multi-service switching network. The occupancy distribution and the blocking probability in LAG are described by the Eqs. (22)–(23):

$$n[P_n]_{kf} = \sum_{i=1}^{M} A_i t_i \sigma_i(n - t_i)[P_{n-t_i}]_{kf}, \tag{22}$$

$$E_{\text{ex}}(i) = \sum_{n=k(f-t_i+1)}^{kf} [P_n]_{kf}[1 - \sigma_i(n)], \tag{23}$$

where:

- $[P_n]_{kf}$ – occupancy distribution (probability of the occupancy of n BBU) in LAG with the capacity of kf BBU,
- A_i – the average intensity of traffic of class i offered to LAG. This is traffic offered to a given direction, i.e., to k output links of the considered switching network,
- $\sigma_i(n)$ – the conditional transition probability for the call stream of class i.

The conditional transition probability (state-passage probability) in the LAG model is the parameter that defines the probability of favourable combinations of occupancy in state n, i.e., such combinations that make it possible to service a call of a given class in at least one link that belongs to LAG. In the LAG model [35] the conditional transition probability is approximated on the basis of the following combinatorial formula:

$$\sigma_i(n) = \frac{F(kf-n,k,f,0)-F(kf-n,k,t_i-1,0)}{F(kf-n,k,f,0)}, \quad (24)$$

where $F(x,k,f,t)$ is the number of arrangements of x free BBUs in k links, each with the capacity of f BBU, with the assumption that initially each link was assigned t free BBUs:

$$F(x,k,f,t) = \sum_{r=0}^{\left\lfloor \frac{x-kt}{f-t+1} \right\rfloor} (-1)^r \times \binom{k}{r}\binom{x-k(t-1)-1-r(f-t+1)}{k-1}. \quad (25)$$

After determining the internal blocking probability $E_{\text{in}}(i)$ and the external blocking probability $E_{\text{ex}}(i)$, we are in position to determine the total blocking probability $E_{\text{tot}}(i)$ for calls of class i in the multi-service switching network. The probability $E_{\text{tot}}(i)$ is the sum of the probabilities $E_{\text{in}}(i)$ and $E_{\text{ex}}(i)$ that exclude simultaneity of the occurrence of the internal and the total blocking events:

$$E_{\text{tot}}(i) = E_{\text{ex}}(i) + E_{\text{in}}(i)[1-E_{\text{ex}}(i)]. \quad (26)$$

The SN-BPPRec method – presented above – is characterized by high accuracy in the evaluation of the total blocking probability [26] and is particularly useful in modelling switching networks with a high number of stages. The simulation study performed by the authors also indicated, however, significant errors in the evaluation of the internal blocking. Due to relatively low values of the internal blocking probability, as compared to values of the external blocking probability, this error cannot be easily discerned. The next section proposes a modification to the recurrent method for the calculations of the internal blocking probability that results in better accuracy in the evaluation of the internal blocking probability.

4.3. Modified Internal Blocking Evaluation Method

The modified version of the recurrent method adopts that the group of output links of a given stage is equal to the sum of availabilities of sub-systems of the preceding stage. If sub-systems of a given stage are identical, then the following can be written:

$$V_s = L_{s-1} d_{e,s-1}(i), \quad (27)$$

where L_s is the number of sub-systems in stage s. Note that in the Clos switching network (Fig. 3), only the first stage is composed of $L_1 = k_1$ sub-systems. The remaining stages include only one sub-system. Therefore, for the three-stage Clos network we have:

$$L_1 = k_1,\ L_2 = L_3 = 1. \quad (28)$$

The structure of the equivalent switching network that corresponds to the adopted assumptions is presented in Fig. 3. The effective availability of s-stage sub-system in the modified structure of the equivalent network is then the function of availability and the point-to-group blocking probability in $(s-1)$-stage sub-system of the equivalent switching network. In this case, for each $s > 1$, Eqs. (8)–(9) will be written as follows:

$$d_{e,s}(i) = d_{e,s}(i, V_s, E_{s-1}(i)), \quad (29)$$

$$E_s(i) = \text{EIF}(A_s(i), V_s, d_{e,s}(i)), \quad (30)$$

where:

$$V_s = L_{s-1} d_{e,s-1}(i), \quad (31)$$

$$A_s(i) = L_{s-1} d_{e,s-1}(i) a(i). \quad (32)$$

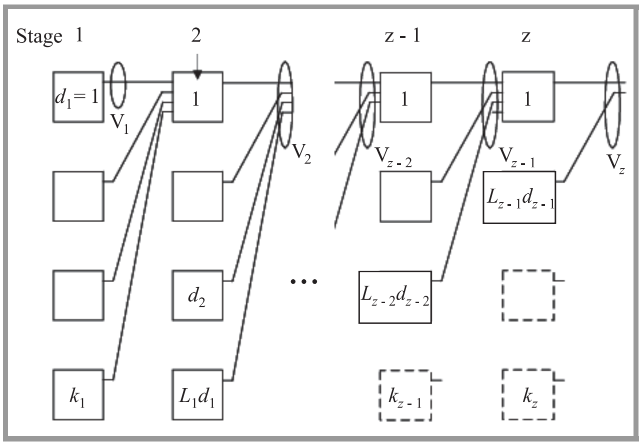

Fig. 3. Multi-stage switching network with indicated parameters used in the modified recurrent method.

For $s = 1$, the parameters V_1 and $d_{e,1}$ are determined exactly as in the recurrent method. The number of sub-systems in a given stage of the considered Clos network is determined by Eq. (28).

To sum up, the internal blocking probability in the switching network presented in Fig. 1 and determined by Eqs. (16)–(21) can be, in the case of the modified recurrent method, written in the following way:

Step 1:

$$d_{e,1}(i) = 1, \quad (33)$$

$$E_1(i) = \text{EIF}(a(i), 1, 1), \quad (34)$$

Step 2:

$$d_{e,2}(i) = [1-E_1(i)]k + E_1(i)\eta Y_1(i), \quad (35)$$

$$E_2(i) = \text{EIF}(ka(i), k, d_{e,2}(i)), \quad (36)$$

Step 3:

$$d_{e,3}(i) = [1 - E_2(i)]d_{e,2} + E_2(i)\eta Y_1(i), \quad (37)$$

$$E_3(i) = E_{\text{in}}(i)\text{EIF}(d_{e,2}a(i), d_{e,2}, d_{e,3}(i)). \quad (38)$$

4.4. The Internal Blocking Probability in Networks with Overflow Links

In order to decrease the internal blocking phenomenon, overflow links were introduced to the first stage of a three-stage Clos switching network. In this way, each neighbouring switches of the first stage are connected with each other by an overflow link (Fig. 4). After the introduction of overflow links we can thus assume that the availability to one switch of the second stage is equal to two links. In Fig. 4, these two links are marked with bold line. Access to the second link results from the introduction of the overflow link indicated in Fig. 4 by dotted line. One sub-system of the first stage can thus be treated as a system with the capacity and availability equal to two links. Therefore, for a switching network with overflow traffic, Step 1 (Eqs. (16)–(17)) for the recurrent method, and Eqs. (33)–(34) for the modified recurrent method, can be rewritten in the following way:

Step 1:

$$d_{e,1}(i) = 2, \quad (39)$$

$$E_1(i) = \text{EIF}(a(i), 2, 2), \quad (40)$$

The remaining steps of the algorithm, just as the equation determining the external and the total blocking probability, remain without any changes.

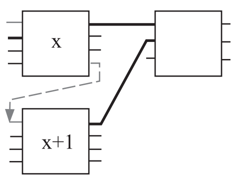

Fig. 4. Structure of links between switches of the first and second sections.

5. Numerical Results

The simulation study and the analytical calculations were performed for a three-stage Clos network. The network was composed of four 4×4 symmetrical switches in each stage. The capacity of the links in the network is 30 BBUs. The overflow links were introduced to the first stage of the network. The assumption was that the capacity of the overflow link was equal to 60 BBUs. The network was offered multi-rate traffic composed of 3 traffic classes that required 1 BBU, 2 BBUs and 6 BBUs, respectively. Traffic of all classes was offered in the following proportions: $A_1t_1 : A_2t_2 : A_3t_3 = 1 : 1 : 1$. To set up connections in the network, the point-to-group selection was used.

The simulation study was performed with a dedicated digital simulator based on the event scheduling method [36]. In the simulation experiments, the 95% confidence interval was determined, evaluated on the basis of the t-Student distribution for 10 series with 100,000 calls of the oldest class in each of the series. The analytical study was per-

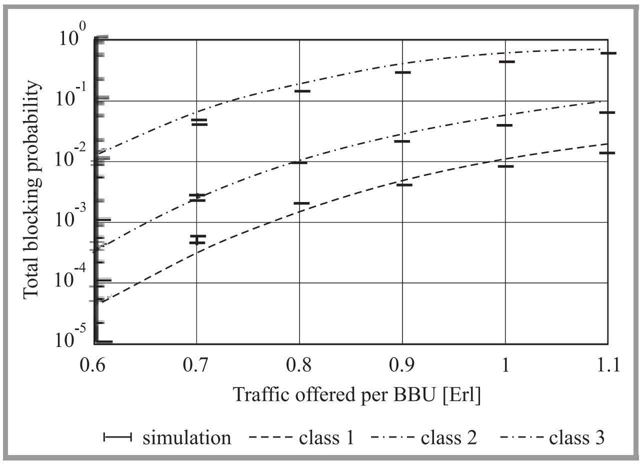

Fig. 5. Total blocking without overflow.

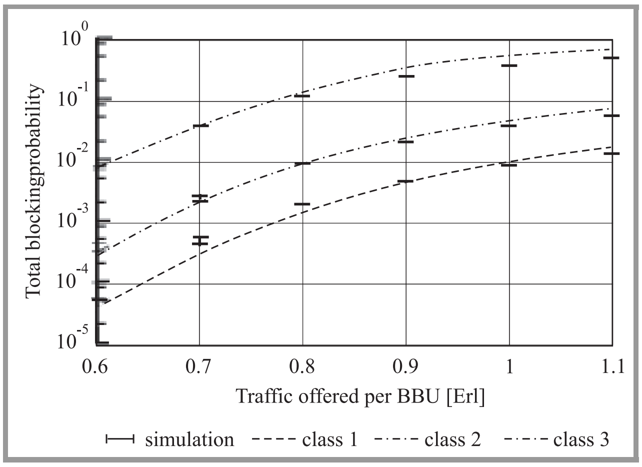

Fig. 6. Total blocking with overflow.

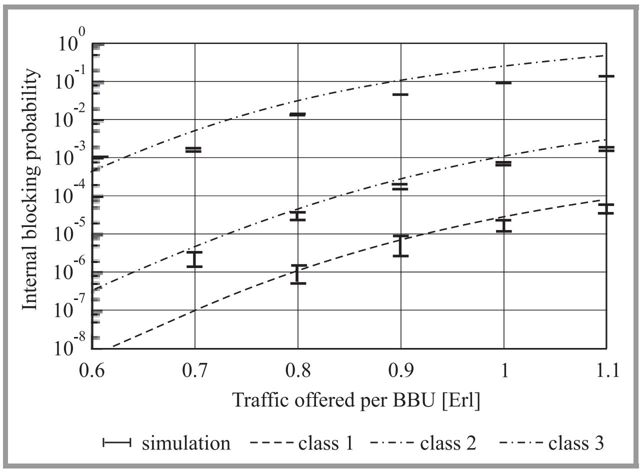

Fig. 7. Internal blocking with overflow links.

formed on the basis of the modified version of the recurrent method proposed in the article.

Figures 5 and 6 show a comparison of the results of the analytical calculations of the total blocking probability with the data obtained in the simulation experiments in a switching network without overflow links (Fig. 5) and in a network with overflow links (Fig. 6). The results confirm high accuracy of the proposed analytical method that is independent from the structure of the network, i.e., whether the system of overflow links was introduced or not. Figure 7 shows, in turn, the results of the analytical and the simulation modelling of the internal blocking probability in the switching network with overflow links. These results also confirm high accuracy of the proposed analytical method.

Figure 8 presents a percentage decrease in the value of the internal blocking probability after the introduction of the overflow links to the switching network for the analytical calculations (a) and in the case of the performed simulation experiments (b). On the basis of these graphs

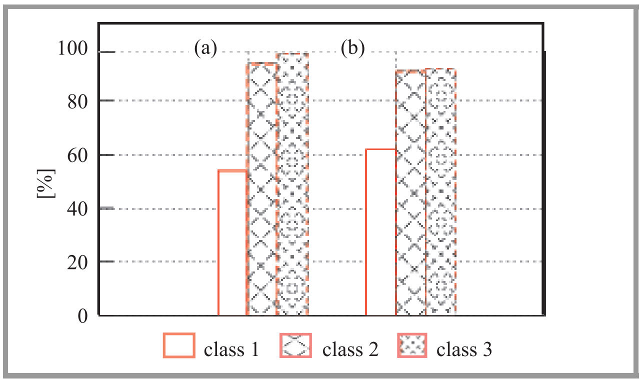

Fig. 8. Percentage decrease in the internal blocking probability.

it is possible to state that the introduction of overflow links results in a considerable percentage decrease in the internal blocking probability in the multi-service switching network. This fact indicates a need for further research into networks with overflow links as regards the possibility of the application of such systems in practice.

6. Conclusions

The article proposes a modified version of the recurrent SN-BPPRec method for analytical evaluation of the internal, external and the total blocking probability in multi-service switching networks with overflow links. The proposed method is characterised by high accuracy, both for modelling switching networks without overflow links and networks with overflow links. The conducted research study also indicates a possibility of the evaluation of the effectiveness of the system of overflow links in multi-service Clos switching networks. The network, in which each two neighbouring switches of the first stage are connected by overflow links, is characterised by a significant decrease in the value of the internal blocking probability. The obtained results indicate thus a need for further research on multi-service switching networks with overflow links that would be conducted within the context of a search for even more effective overflow systems, as well as, as regards of their future practical applications.

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