Paper

Higher Order Cumulants for Identification and Equalization of Multicarrier Spreading Spectrum Systems

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Abstract—This paper describes two blind algorithms for multicarrier code division multiple access (MC-CDMA) system equalization. In order to identify, blindly, the impulse response of two practical selective frequency fading channels called broadband radio access network (BRAN A and BRAN E) normalized for MC-CDMA systems, we have used higher order cumulants (HOC) to build our algorithms. For that, we have focussed on the experimental channels to develop our blind algorithms able to simulate the measured data with high accuracy. The simulation results in noisy environment and for different signal to noise ratio (SNR) demonstrate that the proposed algorithms are able to estimate the impulse response of these channels blindly (i.e., without any information about the input), except that the input excitation is i.i.d. (identically and independent distributed) and non-Gaussian. In the part of MC-CDMA, we use the zero forcing and the minimum mean square error equalizers to perform our algorithms. The simulation results demonstrate the effectiveness of the proposed algorithms.

Keywords—blind identification and equalization, communication channels, higher order cumulants, MC-CDMA systems.

1. Introduction

Many important results [1]–[8] are established that blind identification of finite impulse response (FIR) single-input single-output (SISO) communication channels is possible only from the output second-order statistics, without using any restrictive assumption on the channel zeros, color of additive noise, channel order over-estimation errors, and without increasing the transmission rate of the data stream. Those algorithms have been termed transmitter-induced multistationary approaches. Some class of algorithms for blind channel identification are based on the iterative strategy.

The interest in higher order cumulants (HOC) or higher order statistics (HOS) is permanently growing in the last years. Principally finite impulse response system identification based on HOC of system output has received more attention. Tools that deal with problems related to either nonlinearity, non-Gaussianity, or non-minimum phase (NMP) systems are available, because they contain the phase information of the underlying linear system in contrast to second order statistics, and they are of great value in applications, such as radar, sonar, array processing, blind equalization, time delay estimation, data communication, image and speech processing and seismology [9]–[12].

Many algorithms have been proposed in the literature for the identification of FIR system using cumulants. These algorithms can be classified into three broad classes of solutions: closed form solutions [13], [14], [15], optimization based solutions [16], [17] and linear algebra solutions [18]–[25]. The linear algebra solutions have received great attention because they have "simpler" computation and are free of the problems of local extreme values that often occur in the optimization solution. Although, the closed-form solutions have similar features, they usually do not smooth out the noises caused from the observation and computation. Therefore, while these solutions are interesting from the theoretical point of view, they are not recommended for practical applications [26], [25]. The main goal of this investigation is to elaborate an accurate and efficient algorithm able to estimate the moving average (MA) (or FIR) parameters in noisy environment. So, we address the problem of estimating the parameters of a FIR system from the output observation when the system is excited by an unobservable independent identically distributed (i.i.d.) sequence. The proposed algorithms, based on third and fourth order cumulants, to estimate the parameters of MA process when the order is known, are presented. For validation purpose these method are used to search for a model able to describe the broadband radio access network (BRAN A and BRAN E) channels, represented by a FIR model.

In this paper we present two algorithms based on linear algebra solutions. These algorithms are based on third and fourth order cumulants. Our goal in this paper is to find a model able to represent the mobile channels without reference to the measures, standardized by the European Telecommunications Standards Institute (ETSI) for the "inside" indoor (BRAN A) or "outside" of an outdoor office (BRAN E) [27], [28]. Similarly, we perform the equalization, using the model developed a multicarrier multiple access division of codes (MC-CDMA) downlink [29], [30]. For this, we develop a "blind" algorithm able to simulate the measured data with high accuracy in noisy environment, and for different signal to noise ratio (SNR).

So, we have, principally, focussed on channel impulse response estimation. The considered channels are with non-minimum phase and selective frequency (i.e., normalized channels for MC-CDMA: BRAN A, BRAN E). In most wireless environments, there are many obstacles in the channels, such as buildings, mountains and walls between the transmitter and the receiver. Reflections from these obstacles cause many different propagation paths. This is called multipaths propagation or a multipath channel. The frequency impulse response of this channel, is not flat (ideal case) but comprising some hollows and bumps, due to the echoes and reflection between the transmitter and the receiver. Another problem encountered in communication is the synchronization between the transmitter and the receiver. To solve the problem of phase estimation we will use higher order cumulants (HOC) to test the robustness of those techniques if the channel is affected by noise. HOC are a fairly topic with many applications in system theory. The HOC are only applicable to non-Gaussian and non-linear process because the cumulants of a Gaussian process are identically zero [2], [21], [31]. Many real world applications are truly non-Gaussian [2], [4], [32]. Also, the Fourier transformation of HOC, which is termed higher order spectra (or polyspectra), provides an efficient tool for solving the problem of equalization technology used in communication. The major feature of HOC, from the point of view of equalization, is that the phase information of channels is present [7], [22], [23], [33]. Therefore they can be used to estimate the parameters of the channel model without any knowledge of the phase property (minimum phase – MP or non-minimum phase – NMP) of channel or the transmitted data (assuming a non-Gaussian distribution) [2], [4], [26].

In this paper, we propose two algorithms based on third and forth order cumulants. In order to test its efficiency, we have considered practical, i.e., measured, frequency-selective fading channel, called broadband radio access network, representing respectively the transmission in indoor and outdoor scenarios. These model radio channels are normalized by the ETSI in [27], [28]. Post-equalization at the receiver for downlink MC-CDMA systems in the form of single-user detection (SUD), i.e., transmission from the base station to the mobile systems, has been investigated by several authors, [34], [32], [35]. Recently, pre-equalization at the transmitter for downlink time division duplex (TDD), MC-CDMA has attained increased interest and has been investigated in details [30], [32], [34], [36], [37]. In this contribution, the novel concept of blind equalization is developed and investigated for downlink MC-CDMA systems. This paper shows that we can identify and equalize the MC-CDMA systems blindly. However, the classical equalization of MC-CDMA system assumes that the channel is known. The bit error rate (BER) performances of the downlink MC-CDMA systems, using blind BRAN A and BRAN E estimation, are shown and compared with the results obtained with the classical methods (in which, the channel parameters are assumed known).

1.1. Problem Statement

The output of a FIR channel, excited by an unobservable input sequences, i.i.d. zero-mean symbols with unit energy, across a selective channel with memory p and additive noise (Figure 1). The output time series is described by the following equation

$$r(k) = h_p x(k) + n(k)\,, \qquad (1)$$

where $h_p = (h(1), h(2), \ldots, h(p))$ represents the channel impulse response, $x(k)$ and $n(t)$ is the additive colored Gaussian noise with energy $E\{n^2(k)\} = \sigma^2$.

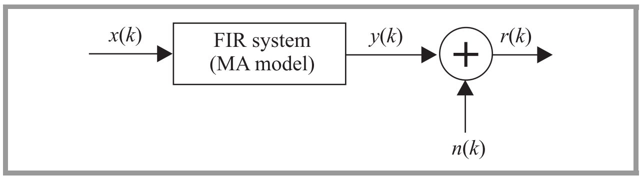

Fig. 1. Channel model.

The completely blind channel identification problem is to estimate h_p based only on the received signal $r(k)$ and without any knowledge of the energy of the transmitted data $x(k)$ nor the energy of noise.
The output of the channel is characterized by its impulse response $h(n)$, which we identify "blindly" its parameters, is given by the following equation

$$y(k) = \sum_{i=0}^{P} x(i)h(k-i); \quad r(k) = y(k) + n(k)\,. \qquad (2)$$

1.2. Proposed Algorithms

1.2.1. Algorithm Based on 3th Order Cumulant: Alg. 1

Hypothesis:

Let us suppose that:

- The additive noise $n(k)$ is Gaussian, colored or with symmetric distribution, zero mean, with variance σ^2, i.i.d. with the m^{th} order cumulants vanishes for $m > 2$.
- The noise $n(t)$ is independent of $x(k)$ and $y(k)$.
- The channel (FIR system) order p is supposed to be known and $h(0) = 1$.
- The system is causal, i.e., $h(i) = 0$ if $i < 0$.

The mth order cumulant of the output signal is given by the following equation [21], [38], [39]:

$$C_{my}(t_1,...,t_{m-1}) = \gamma_{mx} \sum_{i=-\infty}^{+\infty} h(i)h(i+t_1)...h(i+t_{m-1}), \quad (3)$$

where γ_{mx} represents the m^{th} order cumulants of the excitation signal $(x(k))$ at origin.
If $m=3$, Eq. (3) yield to

$$C_{3y}(t_1,t_2) = \gamma_{3x} \sum_{i=0}^{P} h(i)h(i+t_1)h(i+t_2), \quad (4)$$

the same, if $m=2$, Eq. (3) becomes

$$C_{2y}(t_1) = \sigma^2 \sum_{i=0}^{P} h(i)h(i+t_1). \quad (5)$$

The Fourier transform of Eqs. (4) and (5) gives us the bispectra and the spectrum respectively

$$S_{3y}(\omega_1,\omega_2) = \gamma_{3x} H(\omega_1)H(\omega_2)H(-\omega_1-\omega_2), \quad (6)$$

$$S_{2y}(\omega) = \sigma^2 H(\omega)H(-\omega). \quad (7)$$

If we suppose that $\omega = (\omega_1+\omega_2)$, Eq. (7) becomes

$$S_{2y}(\omega_1+\omega_2) = \sigma^2 H(\omega_1+\omega_2)H(-\omega_1-\omega_2), \quad (8)$$

then, from Eqs. (6) and (8) we obtain the following equation

$$H(\omega_1+\omega_2)S_{3y}(\omega_1+\omega_2) = \varepsilon H(\omega_1)H(\omega_2)S_{2y}(\omega_1+\omega_2), \quad (9)$$

where $\varepsilon = (\frac{\gamma_{3x}}{\sigma^2})$.
The inverse Fourier transform of Eq. (9) demonstrates that the 3rd order cumulants, the auto-correlation function (ACF) and the impulse response channel parameters are combined by the following equation

$$\sum_{i=0}^{P} h(i)C_{3y}(t_1-i,t_2-i) = \varepsilon \sum_{i=0}^{P} h(i)h(i+t_2-t_1)C_{2y}(t_1-i). \quad (10)$$

If we use the property of the ACF of the stationary process, such as $C_{2y}(t) \neq 0$ only for $(-p \leq t \leq p)$ and vanishes elsewhere. In addition, if we take $t_1=-p$, Eq. (10) takes the form

$$\sum_{i=0}^{P} h(i)C_{3y}(-p-i,t_2-i) = \varepsilon h(0)h(t_2+p)C_{2y}(-p), \quad (11)$$

else, if we suppose that $t_2=-p$, Eq. (11) will become

$$C_{3y}(-p,-p) = \varepsilon h(0)C_{2y}(-p). \quad (12)$$

Using Eqs. (11) and (12) we obtain the following relation

$$\sum_{i=0}^{P} h(i)C_{3y}(-p-i,t_2-i) = h(t_2+p), \quad (13)$$

else, if we suppose that the system is causal, i.e., $h(i)=0$ if $i<0$. So, for $t_2=-p,...,0$, the system of Eq. (13) can be written in matrix form as

$$\begin{pmatrix} C_{3y}(-p-1,-p-1) & \dots & C_{3y}(-2p,-2p) \\ C_{3y}(-p-1,-p)-\alpha & \dots & C_{3y}(-2p,-2p+1) \\ \vdots & \ddots & \vdots \\ C_{3y}(-p-1,1) & \dots & C_{3y}(-2p,-p)-\alpha \end{pmatrix} \begin{pmatrix} h(1) \\ h(2) \\ \vdots \\ h(p) \end{pmatrix} = \begin{pmatrix} 0 \\ -C_{3y}(-p,-p+1) \\ \vdots \\ -C_{3y}(-p,0) \end{pmatrix}, \quad (14)$$

where $\alpha = C_{3y}(-p,-p)$.

The above Eq. (14) can be written in compact form as

$$Mh_p = d_1, \quad (15)$$

where M is the matrix of size $(p+1)\times(p)$ elements, h_p is a column vector constituted by the unknown impulse response parameters $h(n): n=1,\dots,p$ and d is a column vector of size $(p+1)\times(1)$ as indicated in the Eq. (14). The least squares solution (*LS*) of the system of Eq. (15), permits blindly identification of the parameters $h(n)$ and without any "information" of the input selective channel. So, the solution will be written under the following form

$$h_p = (M^T M)^{-1} M^T d_1. \quad (16)$$

1.2.2. Algorithm Based on 4^{th} Order Cumulants: Alg. 2

From the Eq. (3), the m^{th} and n^{th} cumulants of the output signal, $\{y(n)\}$, and the coefficients $\{h(i)\}$, where $n>m$, are linked by the following relationship:

$$\sum_{j=0}^{p} h(j)C_{ny}(j+t_1,\dots,j+t_{m-1},t_m,\dots,t_{n-1}) = \frac{\gamma_{ne}}{\gamma_{me}} \sum_{i=0}^{p} h(i) \left[\prod_{k=m}^{n-1} h(i+t_k)\right] C_{my}(i+t_1,\dots,i+t_{m-1}). \quad (17)$$

If we take $n=4$ and $m=3$ into Eq. (17), we find the basic relationship developed in [40], [41]. If we take $n=3$ and $m=2$ into Eq. (16), we find the basic relationship of the algorithms developed in [2].
So, the equation proposed in [42] presents the relationship between different n^{th} cumulant slices of the output signal $\{y(n)\}$, as follows

$$\sum_{j=0}^{p} h(j)\left[\prod_{k=1}^{r} h(j+t_k)\right] C_{ny}(\beta_1,\dots,\beta_r,j+\alpha_1,\dots,\alpha_{n-r-1}) = \sum_{i=0}^{p} h(i)\left[\prod_{k=1}^{r} h(i+\beta_k)\right] C_{ny}(t_1,\dots,t_r,i+\alpha_1,\dots,i+\alpha_{n-r-1}), \quad (18)$$

where $1 \leq r \leq n-2$.

If we take $n=3$ we obtain that $r=1$, so the Eq. (17) will be

$$\sum_{j=0}^{p} h(j)h(j+t_1)C_{3y}(\beta_1, j+\alpha_1) = \sum_{i=0}^{p} h(i)h(i+\beta_1)C_{3y}(t_1, i+\alpha_1). \tag{19}$$

In the following, we develop an algorithm based only on 4^{th} order cumulants.

If we take $n=4$ into Eq. (18) we obtain the following equation:

$$\sum_{i=0}^{p} h(i)h(i+t_1)h(i+t_2)C_{4y}(\beta_1,\beta_2,i+\alpha_1) = \sum_{j=0}^{p} h(j)h(j+\beta_1)h(j+\beta_2)C_{4y}(t_1,t_2,j+\alpha_1), \tag{20}$$

if $t_1=t_2=p$ and $\beta_1=\beta_2=0$, Eq. (20) take the form:

$$h(0)h^2(p)C_{4y}(0,0,i+\alpha_1) = \sum_{j=0}^{p} h^3(j)C_{4y}(p,p,j+\alpha_1). \tag{21}$$

As the system is a FIR, and is supposed causal with an order p, so, the $j+\alpha_1$ will be necessarily into the interval $[0,p]$, this implies that the determination of the range of the parameter $\alpha_1$is obtained as follows: $0 \le j+\alpha_1 \le p \Rightarrow -j \le \alpha_1 \le q-j$, and we have $0 \le j \le p$. From these two inequations, we obtain:

$$-p \le \alpha_1 \le p. \tag{22}$$

Then, from the Eqs. (20) and (21) we obtain the following system of equations :

$$\begin{pmatrix} C_{4y}(p,p,-p) & \cdots & C_{4y}(p,p,0) \\ \vdots & \ddots & \vdots \\ C_{4y}(p,p,0) & \cdots & C_{4y}(p,p,p) \\ \vdots & \ddots & \vdots \\ C_{4y}(p,p,p) & \cdots & C_{4y}(p,p,2p) \end{pmatrix} \begin{pmatrix} h^3(0) \\ \vdots \\ h^3(i) \\ \vdots \\ h^3(p) \end{pmatrix} = h(0)h^2(p) \begin{pmatrix} C_{4y}(0,0,-p) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,p) \end{pmatrix} \tag{23}$$

and as we have assumed that $h(0)=1$, if, we consider that $h(p)\neq 0$ and the cumulant $C_{my}(t_1,\ldots,t_{m-1})=0$, if one of the variables $t_k>p$, where $k=1,\ldots,m-1$; the system of Eq. (23) will be written as follows:

$$\begin{pmatrix} 0 & \cdots & 0 & C_{4y}(p,p,0) \\ \vdots & \ddots & & \vdots \\ 0 & & & \\ C_{4y}(p,p,0) & \cdots & & C_{4y}(p,p,p) \\ \vdots & \ddots & & 0 \\ & & & \vdots \\ C_{4y}(p,p,p) & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{h^2(p)} \\ \vdots \\ \frac{h^3(i)}{h^2(p)} \\ \vdots \\ \frac{h^3(p)}{h^2(p)} \end{pmatrix} = \begin{pmatrix} C_{4y}(0,0,-p) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,p) \end{pmatrix}. \tag{24}$$

In more compact form, the system of Eq. (24) can be written in the following form:

$$Mb_{p_2} = d_2, \tag{25}$$

where M, b_q and d are defined in the system of Eq. 24). The least squares solution of the system of Eq. (25) is given by:

$$\hat{b}_{p_2} = (M^T M)^{-1} M^T d_2. \tag{26}$$

This solution give us an estimation of the quotient of the parameters $h^3(i)$ and $h^3(p)$, i.e., $b_{p_2}(i) = \widehat{\left(\frac{h^3(i)}{h^3(p)}\right)}$, $i=1,\ldots,p$. So, in order to obtain an estimation of the parameters $\hat{h}(i)$, $i=1,\ldots,p$ we proceed as follows:

- The parameters $h(i)$ for $i=1,\ldots,p-1$ are estimated from the estimated values $\hat{b}_p(i)$ using the following equation:

$$\hat{h}(i) = \mathrm{sign}\left[\hat{b}_{p_2}(i)(\hat{b}_{p_2}(p))^2\right]\left\{\mathrm{abs}(\hat{b}_{p_2}(i))(\hat{b}_{p_2}(p))^2\right\}^{\frac{1}{3}} \tag{27}$$

  where $\mathrm{sign}(x) = \begin{cases} 1, & \text{if } x>0; \\ 0, & \text{if } x=0; \\ -1, & \text{if } x<0. \end{cases}$ and $\mathrm{abs}(x)=|x|$ indicates the absolute value of x.

- The $\hat{h}(p)$ parameters is estimated as follows :

$$\hat{h}(p) = \frac{1}{2}\mathrm{sign}\left[\hat{b}_{p_2}(p)\right]\left\{\mathrm{abs}(\hat{b}_{p_2}(p)) + \left(\frac{1}{\hat{b}_{p_2}(1)}\right)^{\frac{1}{2}}\right\}. \tag{28}$$

2. Application: Identification and Equalization of MC-CDMA System

The principles of MC-CDMA [37] is that a single data symbol is transmitted at multiple narrow band subcarriers. Indeed, in MC-CDMA systems, spreading codes are applied in the frequency domain and transmitted over independent sub-carriers. However, multicarrier systems are very sensitive to synchronization errors such as symbol timing error, carrier frequency offset and phase noise. Synchronization errors cause loss of orthogonality among subcarriers and considerably degrade the performance especially when large number of subcarriers presents. There have been many approaches on synchronization algorithms in [30], [32]. In this part, we describe a blind equalization techniques for MC-CDMA systems using the algorithms (see Section 1.2) presented above.

2.1. MC-CDMA Transmitter

In the MC-CDMA modulator the complex symbol a_i of each user i is, first, multiplied by each chip $c_{i,k}$ of spreading code, and then applied to the modulator of multicarriers. Each subcarrier transmits an element of information multiply by a code chip of that subcarrier. We consider, for example, the case where the length L_c of spreading code is equal to the number N_p of subcarriers. The optimum space between two adjacent subcarriers is equal to inverse of duration T_c of chip of spreading code in order to guarantee the orthogonality between subcarriers. The MC-CDMA emitted signal is given by

$$x(t) = \frac{a_i}{\sqrt{N_p}} \sum_{q=0}^{N_{u-1}} \sum_{k=0}^{N_p-1} c_{i,k} e^{2jf_k t}, \tag{29}$$

where $f_k = f_0 + \frac{1}{T_c}$, N_u is the user number and N_p is the number of subcarriers.

We suppose that the channel is time invariant and it's impulse response is characterized by P paths of magnitudes β_p and phases θ_p. So the impulse response is given by

$$h(\tau) = \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} \delta(\tau - \tau_p).$$

The relationship between the emitted signal $s(t)$ and the received signal $r(t)$ is given by: $r(t) = h(t) * x(t) + n(t)$

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} \delta(\tau - \tau_p) x(t-\tau) d\tau + n(t) \\ &= \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} x(t - \tau_p) + n(t), \end{aligned} \tag{30}$$

where $n(t)$ is an additive white Gaussian noise (AWGN).

2.2. MC-CDMA Receiver

The downlink received MC-CDMA signal at the input receiver is given by the following equation

$$\begin{aligned} r(t) = \frac{1}{\sqrt{N_p}} \sum_{p=0}^{P-1} \sum_{k=0}^{N_p-1} \sum_{i=0}^{N_u-1} \times \\ \times \Re\left\{ \beta_p e^{j\theta} a_i c_{i,k} e^{2j\pi(f_0 + k/T_c)(t-\tau_p)} \right\} + n(t), \end{aligned} \tag{31}$$

The equalization goal, is to obtain a good estimation of the symbol a_i. At the reception, we demodulate the signal according the N_p subcarriers, and then we multiply the received sequence by the code of the user. Figure 2 explains the single user-detection principle.

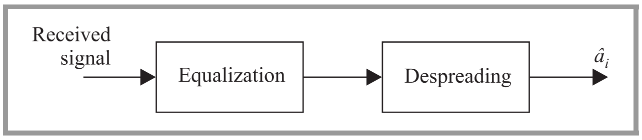

Fig. 2. Principle of the single user-detection.

After the equalization and the spreading operation, the estimation $\hat{a}_i$ of the emitted user symbol a_i, of the i^{th} user can be written by the following equation

$$\begin{aligned} \hat{a}_i &= \sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} \left(g_k h_k c_{q,k} a_q + g_k n_k \right) \\ &= \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 g_k h_k a_i}_{\mathrm{I}} + \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} g_k h_k a_q}_{\mathrm{II} \quad (q \neq j)} + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 g_k n_k}_{\mathrm{III}}, \end{aligned} \tag{32}$$

where the term I, II and III of Eq. (32) are, respectively, the desired signal (signal of the considered user), a multiple access interferences (signals of the others users) and the AWGN pondered by the equalization coefficient and by spreading code of the chip. We suppose that the user data are independents and the h_k are ponderated by the g_k equalization coefficient, are independent of the indices k.

3. Equalization for MC-CDMA

3.1. Zero Forcing (ZF)

The principle of the zero forcing technique is to reduce the effect of the fading and the interference while no enhancing effect of the noise on the decision of what data symbol was transmitted. Whenever there is a diversity scheme involved whether it may involve receiving multiple copies of a signal from time, frequency or antenna diversity, the field of classical diversity theory can be applied. These equalization techniques may be desirable for their simplicity as they involve simple multiplications with each copy of the signal. However, they may not be optimal in a channel with interference in the sense of minimizing the error under some criterion. However, the ZF cancels completely the distortions brought by the channel. The gain factor of the ZF equalizer, is given by the equation

$$g_k = \frac{1}{|h_k|}. \tag{33}$$

By that manner, each symbol is multiplied by a unit magnitude. So, the estimated received symbol, $\hat{a}_i$ of symbol a_i of the user i is described by:

$$\hat{a}_i = \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 a_i}_{\text{I}} + \underbrace{\sum_{q=0}^{N_u-1}\sum_{k=0}^{N_p-1} c_{i,k}c_{q,k}a_q}_{\text{II} \quad (q\neq i)} + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}\frac{1}{h_k}n_k}_{\text{III}} . \quad (34)$$

If we suppose that the spreading code are orthogonal, i.e.,

$$\sum_{k=0}^{N_p-1} c_{i,k}c_{q,k} = 0 \quad \forall i \neq q \quad (35)$$

Eq. (34) will become

$$\hat{a}_i = \sum_{k=0}^{N_p-1} c_{i,k}^2 a_i + \sum_{k=0}^{N_p-1} c_{i,k}\frac{1}{h_k}n_k . \quad (36)$$

Thus, the performance obtained using this detection technique is independent of the users number, if the spreading codes are orthogonal. But, if the h_k value is very weak, (great fading cases), the values g_k increase and the noise will be amplified (second term of Eq. (36)).

3.2. Minimum Mean Square Error, (MMSE)

The MMSE techniques combine the minimization of the multiple access interference and the maximization of signal to noise ratio. Thus as its name indicates, the MMSE techniques minimize the mean square error for each subcarrier k between the transmitted signal x_k and the output detection $g_k r_k$

$$\begin{aligned} \varepsilon[|\varepsilon|^2] &= \varepsilon[|x_k - g_k r_k|^2] \\ &= \varepsilon[(x_k - g_k h_k x_k - g_k n_k)(x_k^* - g_k^* h_k^* x_k^* - g_k^* n_k^*)] . \end{aligned} \quad (37)$$

The minimization of the function $\varepsilon[|\varepsilon|^2]$, gives us the optimal equalizer coefficient, under the minimization of the mean square error criterion, of each subcarrier as

$$g_k = \frac{h_k^*}{|h_k^*|^2 + \frac{1}{\gamma_k}} , \quad (38)$$

where $\gamma_k = \frac{E[|r_k h_k|^2]}{E|n_k|^2}$.

If the values h_k are small, the SNR for each subcarrier is minimal. So, the use of the MMSE criterion avoid the noise amplification. On the other hand, the greatest values of the h_k and g_k being inversely proportional, allows to restore orthogonality between users. So, the estimated received symbol, $\hat{a}_i$ of symbol a_i of the user i is described by

$$\begin{aligned} \hat{a}_i = & \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i}_{\text{I}} + \underbrace{\sum_{q=0}^{N_u-1}\sum_{k=0}^{N_p-1} c_{i,k}c_{q,k}\frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_q}_{\text{II} \quad (q\neq j)} \\ & + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k}_{\text{III}} . \end{aligned} \quad (39)$$

The same, if we suppose that the spreading code are orthogonal, i.e.,

$$\sum_{k=0}^{N_p-1} c_{i,k}c_{q,k} = 0 \quad \forall i \neq q \quad (40)$$

Eq. (39) will become

$$\hat{a}_i = \sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i + \sum_{k=0}^{N_p-1} c_{i,k} \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k . \quad (41)$$

4. Simulation

4.1. BRAN A Radio Channel

In this subsection we consider the BRAN A model representing the fading radio channels, the data corresponding to this model are measured in an indoor case for multicarrier code division multiple access (MC-CDMA) systems. The following equation describes the impulse response $h_A(n)$ of BRAN A radio channel

$$h_A(n) = \sum_{i=0}^{N_T} h_i \delta(n - \tau_i) , \quad (42)$$

where $\delta(n)$ is Dirac function, h_i the magnitude of the targets i, $N_T = 18$ the number of target and τ_i is the time delay (from the origin) of target i. In Table 1 we have summarized the values corresponding the BRAN A radio channel impulse response [27], [28].

Table 1
Delay and magnitudes of 18 targets
of BRAN A radio channel

Delay τ_i [ns]	Mag. C_i [dB]	Delay τ_i [ns]	Mag. C_i [dB]
0	0	90	−7.8
10	−0.9	110	−4.7
20	−1.7	140	−7.3
30	−2.6	170	−9.9
40	−3.5	200	−12.5
50	−4.3	240	−13.7
60	−5.2	290	−18
70	−6.1	340	−22.4
80	−6.9	390	−26.7

4.1.1. Blind Channel Impulse Response Estimation of BRAN A

Although, the BRAN A radio channel is constituted by $N_T = 18$ parameters and seeing that the latest parameters are very small. So, in order to estimate the parameters of BRAN A radio channel impulse response, using the maximum information obtained by calculating the cumulants function, we take the following procedure:

- We decompose the BRAN A radio channel impulse response into four subchannels as follows:

$$h(n) = \sum_{j=1}^{4} h_j(n); \qquad (43)$$

$$h_j(n) = \sum_{i=j}^{P_j} C_j \delta(n-\tau_j); \quad \sum_{j=1}^{4} P_j = N_T .$$

- We estimate the parameters of each subchannel, independently, using the proposed algorithms (Alg. 1 and Alg. 2).

- We add all subchannel parameters, to construct the full BRAN A radio channel impulse response.

In Fig. 3 we represent the estimation of the impulse response of BRAN A channel using the proposed algorithms in the case of $SNR = 16$ dB and data length $N = 2048$.

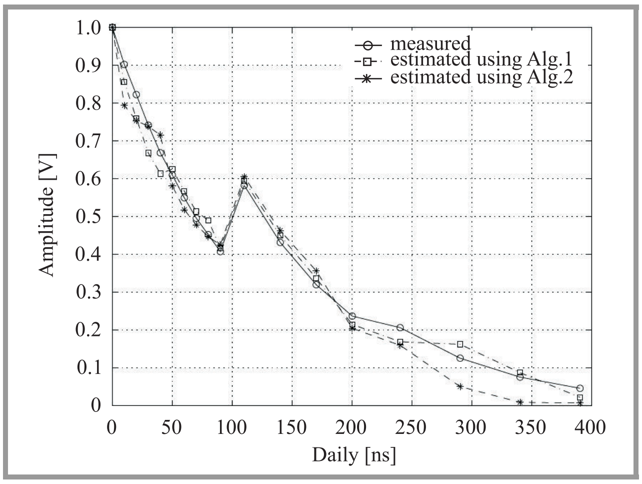

Fig. 3. Estimation of the BRAN A radio channel impulse response for $SNR = 16$ dB and data length $N = 2048$.

From Fig. 3, we can conclude that the algorithm (Alg. 1) gives good estimation for all parameters of BRAN A radio channel impulse response. If we observe the estimated values of BRAN A impulse response, using the algorithm (Alg. 2) shown in Fig. 3, we remark, approximately, the same results given by the Alg. 1 except the last four parameters. Concerning the estimation of BRAN A channel impulse response, for the data length $N = 2048$ and $SNR = 16$ dB, we have a minor difference between the estimated and the measured ones. This result is very interesting for the estimation of impulse response selective frequency channel impulse response in noisy environment. If the data sample increases, we remark that the noise is without influence on the BRAN A radio channel impulse response estimation.

4.2. BRAN E Radio Channel

We have considered in the previous subsection the BRAN A model representing the fading radio channels, where the data corresponding to this model are measured in a scenario of transmission in indoor environment. But in this subsection we consider the BRAN E model representing the fading radio channels, where the model parameters are measured in outdoor scenario. The Eq. (44) describes the impulse response of BRAN E radio channel.

$$h_E(n) = \sum_{i=0}^{N_T} h_i \delta(n-\tau_i), \qquad (44)$$

where $\delta(n)$ is Dirac function, h_i the magnitude of the target i, $N_T = 18$ the number of targets and τ_i is the delay of target i. In Table 2 we have represented the values corresponding to the BRAN E radio channel impulse response.

Table 2
Delay and magnitudes of 18 targets
of BRAN A channel

Delay τ_i [ns]	Mag. C_i [dB]	Delay τ_i [ns]	Mag. C_i [dB]
0	−4.9	320	0
10	−5.1	430	−1.9
20	−5.2	560	−2.8
40	−0.8	710	−5.4
70	−1.3	880	−7.3
100	−1.9	1070	−10.6
140	−0.3	1280	−13.4
190	−1.2	1510	−17.4
240	−2.1	1760	−20.9

4.2.1. Blind Channel Impulse Response Estimated of BRAN E

Seeing that the BRAN E radio channel is composed by $N_T = 18$ parameters and seeing that the latest parameters are very small. So, in order to estimate the parameters of the BRAN E radio channel impulse response with maximum information obtained by calculating the cumulants function, we take the same procedure used in BRAN A radio channel such as decomposing the BRAN E impulse response into four subchannels, and then we estimate each subchannel parameters using the proposed algorithms. This procedure gives a good estimation of the impulse response channel. The same, in time domain, we represent the BRAN E radio channel impulse response parameters (Fig. 4) for data length $N = 2048$ and for $SNR = 16$ dB.
From Fig. 4, we can conclude that the estimated BRAN E channel impulse response, using the algorithm Alg. 1, is very closed to the true one, for data length $N = 2048$ and $SNR = 16$ dB. But, the values given by the second algorithm (Alg. 2) have the same form comparing to those

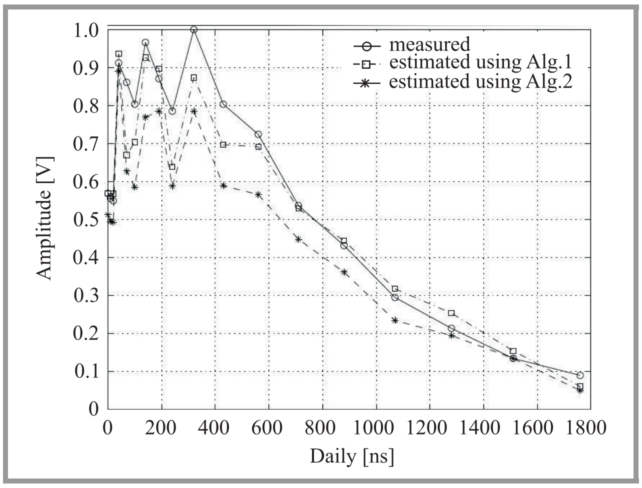

Fig. 4. Estimation of the BRAN E radio channel impulse response for an $SNR = 16$ dB and a data length $N = 2048$.

measured, with a light difference. This is because the BRAN E impulse response have more fluctuations comparing to BRAN A. This result is very interesting for the estimation of selective frequency channel impulse response in noisy environment.

5. MC-CDMA System Performance

In order to evaluate the performance of the MC-CDMA systems, using the proposed algorithms, we consider the BER, for the two equalizers ZF and MMSE, using measured and estimated (using the proposed algorithms) BRAN A and BRAN E channel impulse responses. The results are evaluated for different values of SNR.

5.1. ZF and MMSE Equalizers: Case of BRAN A Channel

In Fig. 5, we represent the BER for different SNR, using the measured and estimated BRAN A channel but the equalization is performed using the ZF equalizer. Figure 6 represents the BER for different SNR, using the measured and estimated BRAN A channel but the equalization is performed using the MMSE equalizer.

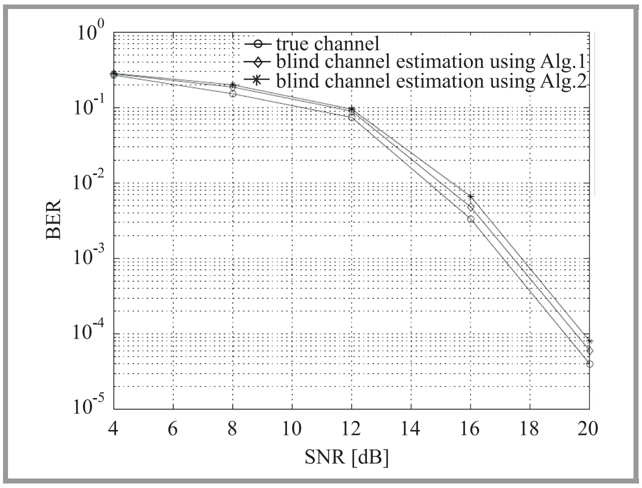

Fig. 5. BER of the estimated and measured BRAN A channel using the ZF equalizer.

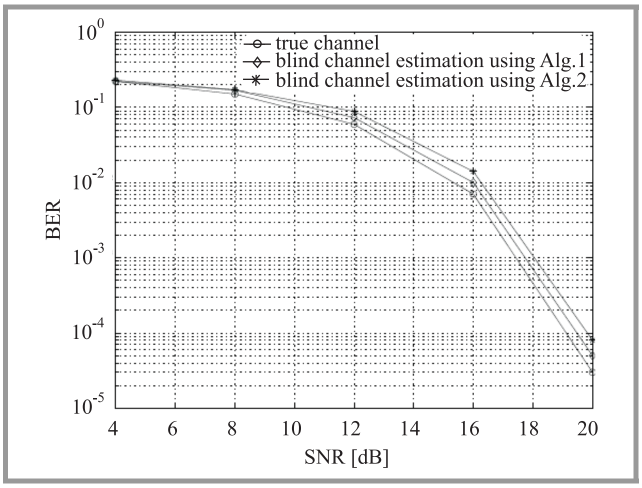

Fig. 6. BER of the estimated and measured BRAN A channel using the MMSE equalizer.

The BER simulation for different SNR, demonstrates that the estimated values by the first algorithm (Alg. 1) are more close to the measured value than those estimated by second algorithm (alg. 2). From Fig. 5, we conclude that: if the $SNR = 20$ dB we have a BER less than 10^{-4}, but using the MMSE equalizer we have only the BER less than 10^{-5}. This is because MMSE equalizer best than the ZF technique. In real case, and in abrupt channel, the proposed blind identification techniques can be useful as remarked in Figs. 5 and 6.

5.2. ZF and MMSE Equalizers: Case of BRAN E Channel

We represent in Fig. 7, the simulation results of BER estimation using the measured and blind estimated of the

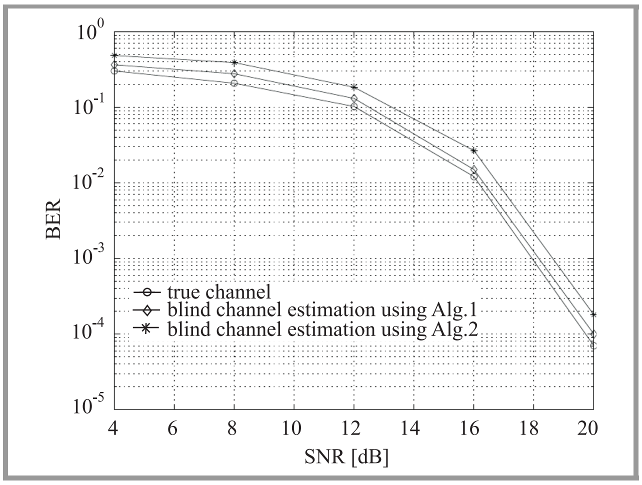

Fig. 7. BER of the estimated and measured BRAN E channel using ZF equalizer.

BRAN E channel impulse response. The equalization is performed using ZF equalizer. Figure 7 demonstrates clearly that the BER obtained using the estimated values by algorithms (Alg. 1 and alg. 2) for ZF equalization is like this obtained using measured values for ZF equalization. Both the two techniques give the 1 bit error if we receive 10^4 bits for a $SNR = 20$ dB.

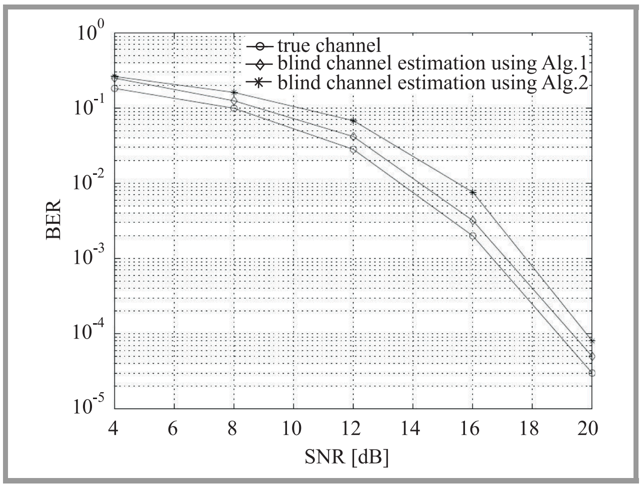

Fig. 8. BER of the estimated and measured BRAN E channel using MMSE equalizer.

In the same manner, we represent in Fig. 8 the simulation results of BER estimation using the measured and blind estimated of the BRAN E channel impulse response. The equalization is performed using MMSE equalizer. From Fig. 8, we observe that the blind MMSE equalization gives approximately the same results obtained using the measured BRAN E values for MMSE equalization. So, if the *SNR* values are superior to 20 dB, we observe that 1 bit error occurred when we receive 10^5 bits, but if the $SNR \geq 20$ dB we will obtain only one bit error occurred for 10^6 bits received.

6. Conclusion

In this paper we have presented two algorithms based on third and fourth order cumulants to identify the parameters of the impulse response of the frequency selective channel such as the experimental channels, BRAN A and BRAN E. The simulation results show the efficiency of these algorithms, mainly if the input data are sufficient. The magnitude of the impulse response is estimated with an acceptable precision in noisy environment, mainly, in the case of small number of samples. For the equalization of the MC-CDMA systems, we have obtained good results on bit error rate principally if we use the first algorithm (Alg. 1) based on third cumulants.

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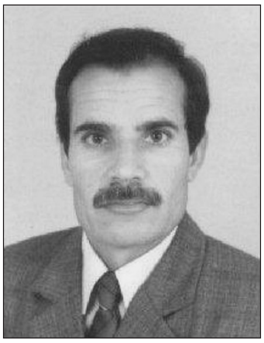

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