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NEW APPROACH TOWARDS POSITION FIXING

ABSTRACT

Navigation aids indications, measured distances and bearings are random values governed by various distributions, normal characteristic is widely assumed. Navigation handbooks read that mean error of bearing taken with radar is within a given range, distance error is within certain percentage of an obtained value. It is also known that measurements taken to different landmarks can be subjectively evaluated therefore diversified. All the mentioned factors are to be taken into account once vessel position is being fixed. In order to include them into a calculation scheme one has to engage new ideas and use different approaches. Mathematical Theory of Evidence extended for fuzzy environment proved to be universal platform for wide variety of new solutions in navigation.

Keywords:
position fixing, Mathematical Theory of Evidence, fuzzy reasoning.

INTRODUCTION

Figure 1 shows traditional way of position fixing with three distances. Three circles intersect at three points in the vicinity of the actual ship position. Assuming measured distances as random variables, the true position is somewhere inside obtained triangle. It is up to navigator's knowledge and experience to estimate the fix. The more accurate the measured distances, the smaller is the triangle and thus the better is the estimation of the fixed position. Obviously an experienced navigator is able to assess acceptable dimensions of such a triangle. Intersection area, greater than an average, results in rejection of the fix. The most common approach to analytical way of position fixing exploits the least square adjustment method. One has to find a point for which expression $\sum_k w_k * \Delta_k^2$ reaches its minimum. Sum of weighted squared deflections Δ_k from the measured values is calculated. Weights w_k introduce credibility masses attributed to each of the distances. Traditional way of position fixing engages:

- available indications;
- characteristics of the measured values and type of distribution are not important, although normal distribution is widely assumed and exploited in the least square adjustment method;
- subjectively evaluated masses of credibility attributed to each of measurements included in analytical approach.

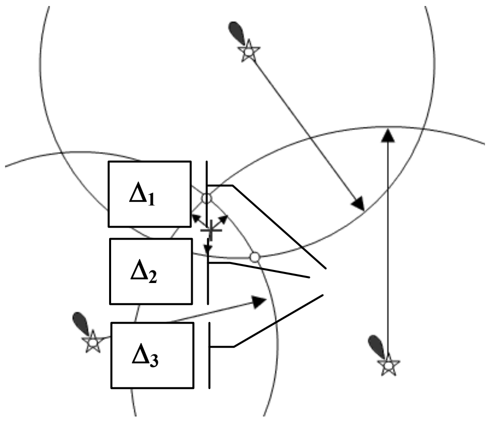

Fig. 1. Position fixing with three distances

The main disadvantage of traditional approach is the lack of inherited evaluation method of the obtained fix. The solution proposed herein is based on Mathematical Theory of Evidence (MTE) extended to fuzzy environment [8] is more flexible as it enables considering of the following:

- available indications;
- various characteristics of the measured values; kind of distribution is important and may affect final solution; empirical and theoretical distribution can be considered;
- accuracy of measured distances, including ability of engaged aids, distances' lengths and characteristic of the reference object;
- imprecision in accuracy estimation[1];
- subjectively evaluated masses of credibility attributed to each of measurement;
- inconsistencies of the computation process;
- fix adjustment in case of abnormal high inconsistency;
- evaluation of selected position quality; plausibility, belief and inconsistency values enable direct assessment of the fix.

[1] In navigation handbooks one can read that mean error attributed to measuring with particular aid is *x*, but reaching *y* (*y*>*x*) value is also possible.

In his previous papers [2, 3, 4] the author presented concept of engaging MTE extended for fuzzy environment to position fixing computation scheme. The Theory appeared to be flexible enough to be used for reasoning on the fix, provided various systems indications are available. Contrary to the traditional approach, it enables embracing knowledge and uncertainty into calculations. Knowledge regarding position fixing includes: characteristics of random distributions of measuring values as well as ambiguity and imprecision in obtained parameters of such distributions. Uncertainty can be expressed by subjectively evaluated masses of confidence attributed to each of observations.

MTE exploits: belief and plausibility measures, events, masses of evidence and belief structures. Latter used in navigation consist of fuzzy location vectors, which are specific events, and masses of evidence assigned to each of the vectors. Structures are to be upgraded related to each indicated position, measured distance and/or bearing.

POSITION FIXING AND MATHEMATICAL THEORY OF EVIDENCE

In order to make a fix using MTE one has to explore intersection area of a distances and/or bearings. Search space grid is used for exploration. Example of crossing area of a distance and two bearings with search space grid is shown in figure 2.

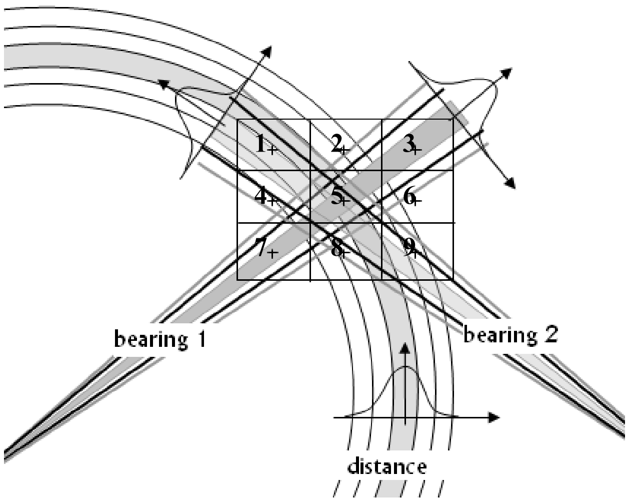

Fig. 2. Intersection area of a distance and two bearings with search space grid

Centers of search grid cells, marked with +, are to be located with respect to each measured values. In the vicinity of each measurement several ranges are established. Probability of the true distance or bearing within each range depends on distribution of indicated value. Assuming normal distribution, the probability can be calculated for standard deviation (σ_d) intervals selected at both sides of the obtained measure. Three ranges at each side can be established. Ranges marked with *a*, *b*, *c* are situated at the right side and *a'*, *b'*, *c'* at the left side of the indicated value (see figure 3). Probabilities of the true distance or bearing within the ranges are equal to: 0.34, 0.14 and 0.02 respectively. Probability that the true value is situated outside the third range is very close to zero. It should be also noted that distinguishing adjacent ranges *a* and *a'* is not very important. Furthermore, both ranges will be considered jointly as a single range with occurrence probability of 0.68.

As it was already mentioned, assuming standard deviation as a precise value is not justified. On the contrary, it is widely described as imprecise interval value $[\sigma^-_d, \sigma^+_d]$. In recent book [5] one can read that mean error of distance measured with radar variable range marker is within the range of ±1% to ±1.5%, and, for bearings, between ±1° and ±2.5°, provided medium class modern radar was used. Therefore, interval valued transition zones between proposed ranges are to be introduced. Width of the distance zone between ranges *a* and *b* is an interval $[d^-_g, d^+_g] = [d+0.01d, d+0.015d]$ and respective interval valued gap for bearings is $[\alpha^-_g, \alpha^+_g] = [\alpha+1, \alpha+2.5]$. Figure 3 presents separation zones as widening intervals. Their respective limits can be calculated as: $[d+n\sigma^-_d, d+n\sigma^+_d]$ for n = 1, 2 and 3.

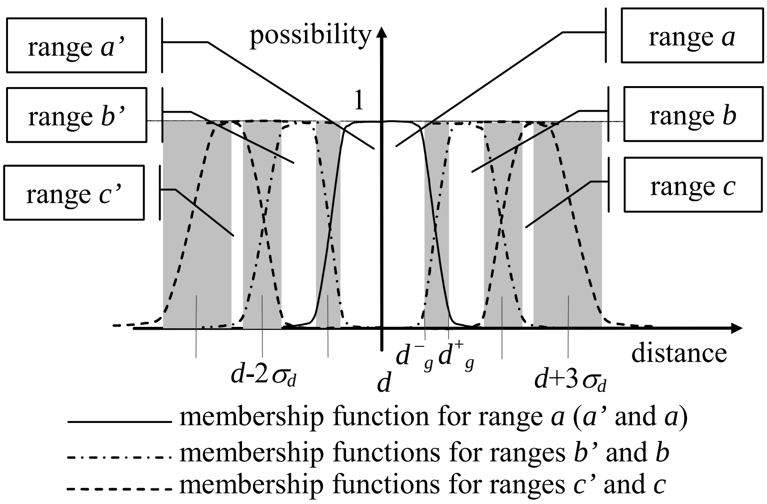

Fig. 3. Selected ranges in the vicinity of measured distance, their imprecise limits and membership functions

Any search space point may partially belong to a few ranges. In particular, point in the centre of a zone should be located with the same possibility of 0.5 within left and right range. In order to calculate locations within selected ranges a family of sigmoid membership functions is adopted [6]. The latter are diagrammed in figure 3 each of them consists of two parts. The left hand side sigmoid function is defined by formula 1 and the right by formula 2. Their intersection points are located at distances of: $-2.5\sigma_d$, $-1.5\sigma_d$, 0, $1.5\sigma_d$ and $2.5\sigma_d$ from measured value. N_i-values and way of calculation of c_i factor for ranges numbered from –3 (c') to 3 (c) are presented in table 1. Constant C was assumed to be equal to 0.05.

$$\mu_s^L(x) = \frac{1}{1+e^{-c_i \cdot (x-N_i)}}\ ; \tag{1}$$

$$\mu_s^R(x) = 1 - \frac{1}{1+e^{-c_i \cdot (x-N_i)}}\ . \tag{2}$$

Table 1. N_i-values and way of calculation of c_i factors for selected ranges

c' left border	$N_{-3} = d\text{-}3\sigma_i$	c_{-3}: $\mu_S^L(d-3\cdot\sigma_i^+) = C$	c' right border	$N_{-2} = d\text{-}2\sigma_i$	c_{-2}
b' left border	$N_{-2} = d\text{-}2\sigma_i$	c_{-2}: $\mu_S^L(d-2\cdot\sigma_i^+) = C$	b' right border	$N_{-1} = d\text{-}\sigma_i$	c_{-1}
a' left border	$N_{-1} = d\text{-}\sigma_i$	c_{-1}: $\mu_S^L(d-\sigma_i^+) = C$	a' right border	0	-
a right border	$N_1 = d\text{+}\sigma_i$	c_1: $\mu_S^R(d+\sigma_i^+) = C$	a left border	0	-
b right border	$N_2 = d\text{+}2\sigma_i$	c_2: $\mu_S^R(d+2\cdot\sigma_i^+) = C$	b left border	$N_1 = d\text{+}\sigma_i$	c_1
c right border	$N_3 = d\text{+}3\sigma_i$	c_3: $\mu_S^R(d+3\cdot\sigma_i^+) = C$	c left border	$N_2 = d\text{+}2\sigma_i$	c_2
considered system is symmetric; therefore: $c_i = c_{-i}$					

For example, in order to calculate membership within range b one has to use expression (3), which selects smaller value returned by left and right hand border functions related to this area. Figure 4 shows this function for $\sigma_d = 1$ and transition zones within ±20% deviation from mean error

$$\mu_S^b(x) = min\left(\frac{1}{1+e^{-c_1\cdot(x-d-\sigma_d)}},\quad 1-\frac{1}{1+e^{-c_2\cdot(x-d-2*\sigma_d)}}\right). \tag{3}$$

Example set of relative distances and their fuzzy locations in selected ranges are presented in table 2. Calculations are carried out with presented membership functions for which c_i takes following values: 14.5, 7.2 and 4.8.

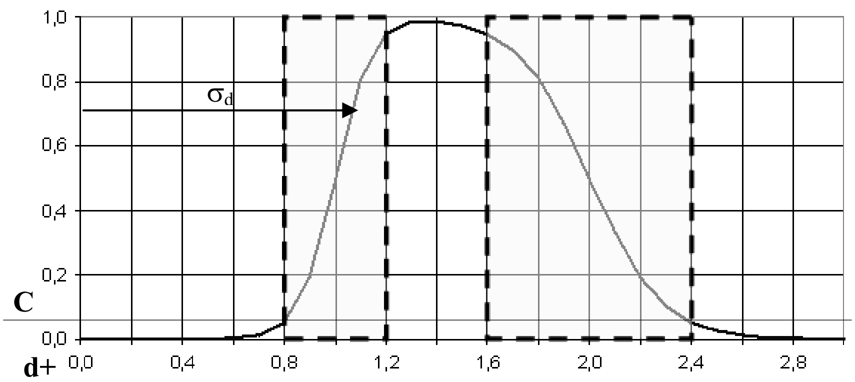

Fig. 4. Membership function for range *b*

Table 2. Example set of relative distances and their fuzzy location in selected ranges

	distances from measurement					
range	–2.20	–0.80	–0.20	0.30	0.85	2.20
c'	0.81	0.00	0.00	0.00	0.00	0.00
b'	0.19	0.05	0.00	0.00	0.00	0.00
a'	0.00	0.95	1.00	0.00	0.00	0.00
a	0.00	0.00	0.00	1.00	0.90	0.00
b	0.00	0.00	0.00	0.00	0.10	0.19
c	0.00	0.00	0.00	0.00	0.00	0.81

Table 3 presents fuzzy locations for the scheme shown in figure 2. Each of the grid cells centres is situated with reference to observed distance and bearings. Location factors are obtained using membership functions presented above. Nine search space points from figure 2 are located as described in table 3.

Table 3. Locations of the search space points regarding distance and bearings

point	reference to distance		reference to bearing 1		reference to bearing 2	
	location (evidence)	mass of evidence	location (evidence)	mass of evidence	location (evidence)	mass of evidence
1	0.6/*b'*, 0.4/*a'*	0.08, 0.14	0	0	0.5/*a*, 0.5/*b*	0.17, 0.07
2	0.1/*b*, 0.9/*c*	0.13, 0	0.2/*c'*	0	0	0
3	0	0	1/*a*	0.34	0	0
4	0.3/*c'*	0.006	0	0	0.7/*c'*, 0.3/*b'*	0.01, 0.04
5	1/*a'*	0.34	1/*a*	0.34	0.3/*a*, 0.7*b*	0.24, 0.10
6	0	0	0	0	0	0
7	0	0	1/*a*	0.34	0	0
8	0.9/*b'*, 0.1/*c'*	0.02, 0.01	0	0	0.9/*c'*	0.02
9	1/*c*	0.02	0	0	0.8/*b*, 0.2/*c*	0.11, 0.004

Point number 1 location with respect to the distance is encoded as: 0.6/*b'*, 0.4/*a'*, what means that particular point is situated within range *b'* with possibility of 0.6 and inside range *a'* with possibility level of 0.4. It reflects the term "fuzzy location", which says that any point may belong, to some extent, to any of the ranges. In case of a point located outside left border of range *c'* or right limit of *c* null value is used. Since probability that the true distance or bearing is that far away from the measured one is very close to zero, respective locations do not affect final solution.

Table 3 also contains calculated masses of evidence related to particular locations. Their values are obtained as a product of probability assigned to given range and level of membership within this range. For example location described by 0.6/*b'* has assigned mass of $0.14 \times 0.6 \approx 0.08$.

Table 4. Three belief structures for position fixing

		1	2	3	4	5	6	7	8	9	*m(..)*
distance	μ_{1a}	{0.4	0	0	0	1	0	0	0	0}	0.408
	$\mu_{1b'}$	{0.6	0	0	0	0	0	0	0.9	0}	0.084
	μ_{1b}	{0	0.1	0	0	0	0	0	0	0}	0.084
	$\mu_{1c'}$	{0	0	0	0.3	0	0	0	0.1	0}	0.012
	μ_{1c}	{0	0.9	0	0	0	0	0	0	1}	0.012
	μ_{1n}	{1	1	1	1	1	1	1	1	1}	0.40
bearing 1	μ_{2a}	{0	0	1	0	1	0	1	0	0}	0.544
	$\mu_{2b'}$	{0	0	0	0	0	0	0	0	0}	0.112
	μ_{2b}	{0	0	0	0	0	0	0	0	0}	0.112
	$\mu_{2c'}$	{0	0.2	0	0	0	0	0	0	0}	0.016
	μ_{2c}	{0	0	0	0	0	0	0	0	0}	0.016
	μ_{2n}	{1	1	1	1	1	1	1	1	1}	0.20
bearing 2	μ_{3a}	{0.5	0	0	0	0.3	0	0	0	0}	0.476
	$\mu_{3b'}$	{0	0	0	0.3	0	0	0	0	0}	0.098
	μ_{3b}	{0.5	0	0	0	0.7	0	0	0	0.8}	0.098
	$\mu_{3c'}$	{0	0	0	0.7	0	0	0	0.9	0}	0.014
	μ_{3c}	{0	0	0	0	0	0	0	0	0.2}	0.014
	μ_{3n}	{1	1	1	1	1	1	1	1	1}	0.30

Facts and knowledge regarding measured value enable creating belief structures. The latter is an assignment of masses of evidence to location vectors ($\mathrm{m} : 2^{\Omega} \rightarrow [0, 1]$), for which mass assigned to an empty set is zero and sum of all masses is one ($m(\varnothing) = 0$, $\sum_{A \subset 2^{\Omega}} m(A) = 1$). Assignments for which above constraints are not observed are pseudo belief structures and should be converted to their normal form [7].

Table 4 shows three belief structures related to position fixing with distance and two bearings. Encoded locations gathered in table 3 are major parts of these structures. Encoded facts are location vectors; their elements are degrees of search space points belonging to selected ranges with respect to each of the measured values. Since a ranges are treated jointly five location vectors are within single belief structure, each related to particular range.

Location vectors are supplemented with all one set, which expresses uncertainty. Mass attributed to this vector shows lack of confidence to a particular measurement. Thanks to this value all observations can be differentiated, possibly in subjective manner. All location vectors have assigned mass of confidence. Values are calculated based on probability attributed to particular range and on complement of uncertainty. Initial probabilities are multiplied by factors 1-m(μin). It should be noted that the sum of all masses within a single belief structure is to be equal to one.

Belief structures are subject of combination in order to obtain knowledge base enabling reasoning on the position of the ship. It is known that combination of belief structures increase their initial informative context. By taking several distances and/or bearings a navigator is supposed to be confident on true location of the ship.

COMBINING LOCATIONS VECTORS AND REASONING ON THE FIX

Combination table can be used to carry out association of two belief structures. Example of combination is presented in table 5. The table contains columns that include data from one of involved structure and rows with other sets of data. Each cell in the table contains product of engaged masses as well as indication of combined set.

Table 5. Combination of two belief structures related to measured distance and bearing

		distance					
		$m(\mu_{1a})$=0.408	$m(\mu_{1b'})$=0.084	$m(\mu_{1b})$ =0.084	$m(\mu_{1c'})$=0.012	$m(\mu_{1c}$=0.012)	$m(\mu_{1u})$=0.4
bearing 1	$m(\mu_{2a})$ 0.544	$m_{1-2}(\mu_{1a}\wedge\mu_{2a})$ = 0.222	$m_{1-2}(\mu_{1b'}\wedge\mu_{2a})$ = 0.046	$m_{1-2}(\mu_{1b}\wedge\mu_{2a})$ = 0.046	$m_{1-2}(\mu_{1c'}\wedge\mu_{2a})$ = 0.007	$m_{1-2}(\mu_{1c}\wedge\mu_{2a})$ = 0.007	$m_{1-2}(\mu_{2a})$ = 0.218
	$m(\mu_{2b'})$ 0.112	$m_{1-2}(\mu_{1a}\wedge\mu_{2b'})$= 0.046	$m_{1-2}(\mu_{1b'}\wedge\mu_{2b'})$ = 0.009	$m_{1-2}(\mu_{1b}\wedge\mu_{2b'})$ = 0.046	$m_{1-2}(\mu_{1c'}\wedge\mu_{2b'})$ = 0.001	$m_{1-2}(\mu_{1c}\wedge\mu_{2b'})$ = 0.001	$m_{1-2}(\mu_{2b'})$ = 0.045
	$m(\mu_{2b})$ 0.112	$m_{1-2}(\mu_{1a}\wedge\mu_{2b})$= 0.046	$m_{1-2}(\mu_{1b'}\wedge\mu_{2b})$ = 0.009	$m_{1-2}(\mu_{1b}\wedge\mu_{2b})$ = 0.046	$m_{1-2}(\mu_{1c'}\wedge\mu_{2b})$ = 0.001	$m_{1-2}(\mu_{1c}\wedge\mu_{2b})$ = 0.001	$m_{1-2}(\mu_{2b})$ = 0.045
	$m(\mu_{2c'})$ 0.016	$m_{1-2}(\mu_{1a}\wedge\mu_{2c'})$= 0,007	$m_{1-2}(\mu_{1b'}\wedge\mu_{2c'})$ = 0.001	$m_{1-2}(\mu_{1b}\wedge\mu_{2c'})$ = 0.001	$m_{1-2}(\mu_{1c'}\wedge\mu_{2c'})$ = 0.000	$m_{1-2}(\mu_{1c}\wedge\mu_{2c'})$ = 0.000	$m_{1-2}(\mu_{2c'})$ = 0.006
	$m(\mu_{2c})$ 0.016	$m_{1-2}(\mu_{1a}\wedge\mu_{2c})$= 0,007	$m_{1-2}(\mu_{1b'}\wedge\mu_{2c})$ = 0.001	$m_{1-2}(\mu_{1b}\wedge\mu_{2c})$ = 0.001	$m_{1-2}(\mu_{1c'}\wedge\mu_{2c})$ = 0.000	$m_{1-2}(\mu_{1c}\wedge\mu_{2c})$ = 0.000	$m_{1-2}(\mu_{2c})$ = 0.006
	$m(\mu_{2u})$ 0.20	$m(\mu_{1a})$= 0.082	$m_{1-2}(\mu_{1b'})$ = 0.017	$m_{1-2}(\mu_{1b})$ = 0.017	$m_{1-2}(\mu_{1c'})$ = 0.002	$m_{1-2}(\mu_{1c})$ = 0.002	$m_{1-2}(\mu_{2u})$ = 0.080

The result of combination contains initial locations vectors, placed in last row and last column since: $\mu_{ix} = \mu_{ix} \wedge \mu_u$. They receive modified masses as a result of association. Single step of combination reduces masses of initial locations vectors by the amount of uncertainty attributed to another measured distance or bearing. In case of multiple structures next steps are to be made, results of previous combination are associated with next structure until all items are exhausted.

Pseudo belief structures can be created as a result of association. The phenomenon occurs when non-zero mass is assigned to an empty or subnormal set. All null location vector result from combining ranges with empty intersection within the search area. It results from lack of support for given set of positions of the search space elements within another set of locations. Pseudo structures have some undesirable properties [7]. For this reason pseudo belief structure are to be converted to normal state, they are to be normalized. Two different approaches to the normalization process are used. Despite a few drawbacks, method known as Yager normalization can be used for nautical applications [3]. In the approach all grades of subnormal sets are increased by a complement of the highest one. At the same time masses assigned to the null sets, the occurrence is also called as inconsistency, increase uncertainty.

Combination of evidence encoded in two location vectors yields a new set $\mu_k(x_i)$ with mass equal to the product of masses involved $m(\mu_k(x_i))$. Therefore final belief structure consists of family of fuzzy locations sets $\{\mu_k(x_i)\}$ and collection of masses assigned to each of the sets $\{m(\mu_k(x_i))\}$. Given these data sets support for hypothesis represented by a set of $\mu_A(x_i)$ is sought. It is fundamental ability of MTE to reason on certain hypothesis based on relative ones. One can reason on true distance or ship position given measured values and knowledge on used aids and observed objects. Formula (4) defines support plausibility for proposition described by $\mu_A(x_i)$, which is embedded in collection of sets $\{\mu_k(x_i)\} = \{\mu_{1x} \wedge \mu_{2x} \wedge \ldots \wedge \mu_{nx}\}$: $x \in \{a, b, b', c, c'\}$ [1].

$$pl(\mu_A(x_i)) = \sum_{k=1}^{n} m(\mu_k(x_i)) \max_{x_i \in \Omega}(\mu_A(x_i) \wedge \mu_k(x_i)) \tag{4}$$

In position fixing distances and/or bearings to the observed objects, from each search space element, are examined. Search space point with highest plausibility is assumed to be position of the ship. Set that represent hypothesis on, for example, point number 2 as being the ship position is: $\mu_A(x_2) = \{0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\}$. Value of 1 appears at position indicated by point of interest number. Specificity of the reference set enable reducing formula (4) to simpler version (5) referring to l-th point from the search space:

$$pl(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) * \mu_k(x_l) \quad (5)$$

Formula (5) defines support plausibility for certain hypothesis embraced in given family of sets. It is quite often that belief of such hypothesis is also sought. Formula (6) expresses general case support belief:

$$bel(\mu_A(x_i)) = \sum_{k=1}^{n} m(\mu_k(x_i)) \min_{x_i \in \Omega}(\mu_A(x_i) \vee \neg\mu_k(x_i)). \quad (6)$$

Reduced version due to special kind of the reference fuzzy set is defined by formula (7). To calculate belief value one has to find minimum among complemented grades (¬) of given set, number of interest is to be omitted:

$$bel(x_l) = \sum_{k=1}^{n} m(\mu_k(x_i)) \min_{x_i \in \Omega;\, i \neq l}(\neg\mu_k(x_i)). \quad (7)$$

It should be noted that multiple point presence within given range causes that belief for each of them is zero. For this reason belief cannot be considered as primary factor in considered position fixing problem [3].

Table 6. Obtained plausibility and belief values

ssp	1	2	3	4	5	6	7	8	9
pl(..)	0.23	0.07	0.24	0.08	0.91	0.06	0.23	0.09	0.10
bel(..)	0.04	0.00	0.01	0.00	0.56	0.00	0.00	0.01	0.01
ssp stands for search space point number.									

Table 6 contains calculated support plausibility and belief for each cells centre as fixed position. Support is sought in combination result family of sets. Highest values are related with point 5, which is assumed to represent the ship position. Note that support for this point as indicating fixed position of the ship is considerably high. Closer look at figure 2 proves the proposition. Point 5 is located at intersection of two *a* ranges related to distance and one of bearings, additionally it is located very close to the same range with respect to the second bearing.

Obtained sum of uncertainty and inconsistency was about 0.5. Following Yager normalization concept these two values can be considered jointly. It is worth to emphasis that in presented example final uncertainty is equal to: $0.4 \times 0.2 \times 0.3 = 0.024$. Balance of the final value is due to empty sets that occurred during association. High inconsistency value compared to uncertainty indicates abnormality which can result from:

- larger than expected measurement errors;
- wrongly adjusted size and position of search space grid.

To reduce abnormality caused by size and position of the grid iterative algorithm was implemented. Size of the grid and its location are adjusted during iterations. Dimensions of the search area are decreased and its position randomly shifted around point with the highest metric. In order to reduce effects of large measurement errors standard deviations are proportionally increased, figure 5 shows results of this idea.

SUMMARY AND CONCLUSIONS

New method of position fixing in terrestrial navigation is proposed. The method uses MTE concept, which enables reasoning on position fixing based on measured distances and/or bearings. It was assumed that measured values are random ones with normal distribution. Knowledge on used aids and observed objects is included into combination scheme. The true distance or bearing is somewhere in the vicinity of the measured one. To define true distance location probabilities six ranges were introduced. Probability levels assigned to each range are calculated based on features of normal distribution. Standard deviation of the distribution is assumed to be within known range. Imprecise interval valued limits of ranges are adopted. Sigmoid membership function are introduced and used for establishing points of interest levels of locations within established ranges. Calculated locations are elements of fuzzy sets called location vectors. Vectors supplemented with the one expressing uncertainty compose one part of belief structure. Another part embraces masses of initial believes assigned to location vectors and uncertainty.

Complete belief structure is related to each of measured value. Mass assigned to uncertainty expresses subjective assessment of measuring conditions. One has to take into account: radar echo signature, height of objects, visibility and so on to include measurement evaluation. Fuzzy values such as poor, medium or good can be used instead of crisp figures. Imprecise masses values engage different way of calculation and will be discussed in a future paper.

Belief structures are combined. During association process search space points within common intersection region are selected. Result of association is to be explored for reasoning on the fix. All associated items are to be taken into account in order to select final solution. Two formulas enabling calculation of credibility and plausibility of propositions represented by fuzzy sets are available. The formulas were simplified due to unique property of the reference set.

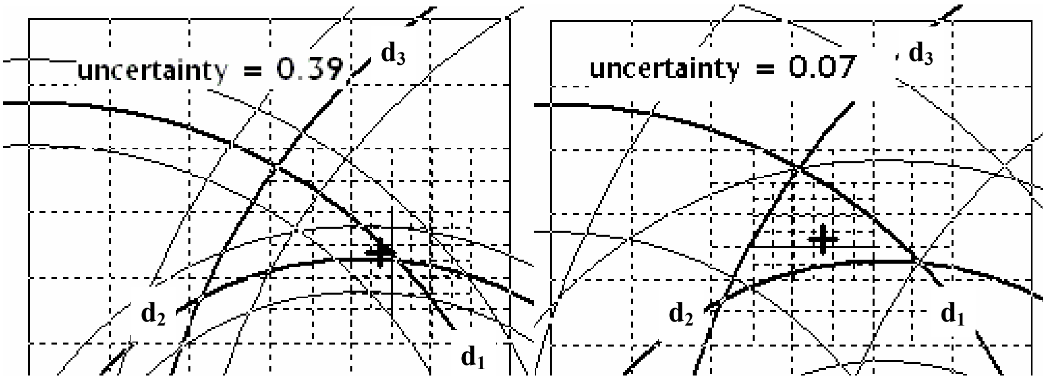

Fig. 5. Two versions of position fixing with three distances and different characteristics of accuracy

In case of high mass of inconsistency position adjustment is necessary. Figure 5 presents output of an application implementing combination scheme. Position fixings with three distances are shown for two different input data sets. Distances were treated as random values with normal distributions. Each of them is shown as three rings with radiuses equal respectively to: $d^-_i = d_i - 3\sigma_{di}$, d_i, $d^+_i = d_i + 3\sigma_{di}$. Figure also emphasis iterative concept of the algorithm. In consecutive iteration search area is reduced for the sake of obtaining required accuracy. Credibility attributed to each of the measured distances is such that least mass is assigned to the third value. Therefore at the left side of the figure point at the intersection of the best distances (1 & 2) is selected as final solution. For this case sum of uncertainty and inconsistency is equal to 0.39. The high value results from lack of overlapping of selected ranges. Overlapping degrees of the ranges can be increased by their widths adjustment. Effect of double increment of initial standard deviations is presented at the right side of the figure 5. Final uncertainty is significantly reduced and selected position shifted to the centre of three circles intersection area.

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