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PRECISE TRACKING POSITIONS OF A SUBMARINE VEHICLE

ABSTRACT

Using autonomous submarines for examination of sea bottom and underwater installations, also exploration of mineral deposits in shelf and hydrographical survey requires positioning of extended accuracy. The above implicates a necessity of developing a precise method of navigation and programme of working out the observational data. The commonly used radio navigation systems are useless in this case, as radio waves do not spread in water. Alternative inertial systems still prove inefficient accuracy. This is the reason why at present, more often, construction of under-water navigational systems on the model of radio navigational ones has been held forth. Another idea in this field is carrying out survey using various techniques and working out jointly the survey results on-board. Such a method allows improving accuracy of positioning.

Keywords:
underwater navigation, precise tracking.

INTRODUCTION

Basic assignments performed nowadays with a use of autonomous crewless submarine vehicles are as follows:

— hydro engineering works, such as examination of underwater structures or supporting processes of installation thereof;
— laying or inspection of underwater cable pipelines;
— exploration of sea bottom natural resources;
— verification of dredging results;
— natural environment protection;
— supporting hydrographical works;
— counteracting criminal usage of submarines;
— investigation and rescue;
— oceanographic, hydrological, biological, geological research.

As survey equipment for the above application, the most commonly the various types of sonar, television or magnetometers are used. Oftentimes precision of surveys (thus accuracy thereof), carried out using the above mentioned meters, seems to be unsatisfactory, in addition, strengthened by difficult conditions of propagation in water. However, positioning of a vehicle with extended precision is a must. Unfortunately it is not achievable neither by usage of inertial equipment, nor by any type of systems of positioning in sonar technique.

Contemporary level of ship engineering and expansion of sea area exploration needs enable to apply submarine vehicles in question for operations connected with all of the above mentioned works to be carried out at optional depths. However, a possibility of precise setting out submarine's location in water depth became fundamental barrier for extending of such sort of vehicles' usage. Nevertheless the authors assumed that there was feasible to improve the accuracy by working out appropriate combinations of measurements, taken using various techniques.

METHODS OF ESTIMATION IN UNDERWATER NAVIGATION

We assume that in working out of observational data in underwater navigation systems there will be applied advanced methods of estimation, mainly the estimation with the least squares method and *M*-estimation as well. Application of the above methods should be connected with survey and geometrical conditions of navigational tasks, and thereby accommodated to detailed, scientific and technical problems of underwater navigation. Such problems comprise:

— submarine's displacing and precise recording its dislocations;
— construction of underwater navigational system for fixing positions and a possibility of carrying out precise underwater survey of water depth;
— precise fixing positions of underwater structures which have to be included into an underwater navigational system or an object, position of which will be necessary for a given navigational task.

The first of the above mentioned issues is the essential technical problem. It is connected with quality of the submarine's equipment on-board and adequacy of autopilot's quality to the submarine's dynamics. It is reduced to using sensitive sensors of acceleration or speeds and directions. Up-to-date inertial systems, although expensive, are characterized with properties close to the expected ones. Due to the above fact there was assumed that in this work they would not be taken into consideration.

The second problem is strictly connected with knowledge of the sea bottom image and a possibility of acquiring information on characteristic elements of sea bottom in correlation to information on coordinates. The first part of this problem can be solved with a use of up-to-date results of bathymetric measurements. On that basis, applying comparative navigation methods, it is feasible to determine roughly a region of submarine's floating, for example with commonly known neural network apparatus. The second part of this problem is of technical and hydrological character, connected with a type of equipment used for carrying out navigational observations and an influence of sea depth on a quality of the observations acquired.

The third problem is of typically analytical nature, solvable if applying present-day methods of mathematical statistics, mainly the theory of estimation. And so, precision of navigation depends on knowledge of exact coordinates of objects' location in sea depth, quality of the possessed information and the system's geometry. Therefore in this part the modern methods of estimation, applied successfully, among the others, in geodesy, may pull their weight [1, 9, 10].

To discuss some suggestions involved, in reference to the navigational tasks, let's assume that $\mathbf{x} \in \Re^n$ is a vector of measured volumes, and $\mathbf{X} \in \Re^m$ a vector of the object's coordinates (submarine's position vector), whereat $n > m$. Let's also assume that vectors x and X are joined with the observational equations system:

$$\mathbf{x} = \mathbf{F(X)} \;\; \rightarrow \;\; \hat{\mathbf{x}} = \mathbf{F}(\hat{\mathbf{X}}), \tag{1}$$

where:
$$\hat{\mathbf{x}} = \widetilde{\mathbf{x}} + \mathbf{v} ;$$

$\hat{\mathbf{x}}$ — measured volumes' estimator;

$\widetilde{\mathbf{x}}$ — measurements results vector;

$\mathbf{v}$ — corrections vector (real measurement errors estimator);

$\hat{\mathbf{X}}$ — coordinates vector's estimator $\mathbf{F(\circ)}$ — vector function).

Moreover, if $rank[\partial_{\mathbf{X}} \mathbf{F(X)}] = m$, then the object's position equalizing problem can be formulated in the following general form:

$$\left. \begin{array}{l} \mathbf{x} + \mathbf{v} = \mathbf{F}(\hat{\mathbf{X}}) \\ \mathbf{C}_x = \sigma_0^2 \mathbf{Q}_x = \sigma_0^2 \mathbf{P}_x^{-1} \\ \varphi(\hat{\mathbf{X}}) = \mathbf{v}^T \mathbf{P}_x \mathbf{v} = \min \end{array} \right\} \;\; \rightarrow \;\; \hat{\mathbf{X}}, \; \mathbf{Q}_{\hat{X}} , \tag{2}$$

where: $\mathbf{C}_x$ — covariance matrix of measurement results vector $\widetilde{\mathbf{x}}$;

$\mathbf{Q}_x$ — cofactors' matrix (initial assessment of matrixes $\mathbf{C}_x$);

$\mathbf{P}_x = \mathbf{Q}_x^{-1}$ — weights matrix;

σ_0^2 — unknown variance coefficient (in general $\hat{\sigma}_0^2 = \mathbf{v}^T \mathbf{P}_x \mathbf{v} / (n - m)$).

A solution of the problem (2) is such a position estimator $\hat{\mathbf{X}}$, that:

$$\min_{\mathbf{X}} [\mathbf{F}(\mathbf{X}) - \mathbf{x}]^T \mathbf{P}_x [\mathbf{F}(\mathbf{X}) - \mathbf{x}] = \mathbf{v}^T \mathbf{P} \mathbf{v}, \quad \mathbf{v} = \mathbf{v}(\hat{\mathbf{X}}) = \mathbf{F}(\hat{\mathbf{X}}) - \mathbf{x}$$

and cofactors matrix:

$$\mathbf{Q}_{\hat{X}} = \left([\partial_{\mathbf{X}} \mathbf{F}(\hat{\mathbf{X}})]^T \mathbf{P}_x \partial_{\mathbf{X}} \mathbf{F}(\hat{\mathbf{X}})\right)^{-1}.$$

For some applications there is a necessity to complement the already existing and equalized geometrical navigational structure with new measurements or new measurements and new points. The structure developed in this way can be equalized 'from the beginning' as a whole or, (what is more reasonable operation), to take advantage of equalizing results obtained before [2, 5, 6, 9, 10]. With such operation there complies the following, general equalization problem:

— step (*i*)

$$\left.\begin{aligned} \mathbf{x}_{(i)} + \mathbf{v}_{(i)} &= \mathbf{F}_{(i)}(\hat{\mathbf{X}}_{(i)}) \\ \mathbf{C}_{x(i)} &= \sigma_0^2 \mathbf{Q}_{x(i)} = \sigma_0^2 \mathbf{P}_{x(i)}^{-1} \\ \varphi(\hat{\mathbf{X}}_{(i)}) &= \mathbf{v}_{(i)}^T \mathbf{P}_{x(i)} \mathbf{v}_{(i)} = \min \end{aligned}\right\} \rightarrow \hat{\mathbf{X}}_{(i)}, \mathbf{Q}_{\hat{X}(i)}$$

— step (*i*+1)   (3)

$$\begin{aligned} \mathbf{x}_{(i+1)} + \mathbf{v}_{(i+1)} &= \mathbf{F}_{(i+1)}(\hat{\mathbf{X}}_{(i+1)}) \\ \boldsymbol{\lambda}_{(i+1)} &= \hat{\mathbf{X}}_{(i+1)} - \hat{\mathbf{X}}_{(i)} \\ \mathbf{C}_{x(i+1)} &= \sigma_0^2 \mathbf{Q}_{x(i+1)} \\ \mathbf{C}_{\hat{X}(i)} &= \sigma_0^2 \mathbf{Q}_{\hat{X}(i)} \\ \varphi(\hat{\mathbf{X}}_{(i+1)}) &= \mathbf{v}_{(i+1)}^T \mathbf{P}_{x(i+1)} \mathbf{v}_{(i+1)} + \boldsymbol{\lambda}_{(i+1)}^T \mathbf{Q}_{\hat{X}(i)} \boldsymbol{\lambda}_{(i+1)} = \min \end{aligned}$$

($i = 0, 1, 2, \dots$), whereat

$$\mathbf{Q}_{\hat{X}(i)} = \left([\partial_{\mathbf{X}} \mathbf{F}(\hat{\mathbf{X}}_{(i)})]^T \mathbf{P}_{x(i)} \partial_{\mathbf{X}} \mathbf{F}(\hat{\mathbf{X}}_{(i)})\right)^{-1}.$$

In underwater navigation (although not only in this field) there may appear a necessity of identification of incorrect navigational signs. The problem can be solved applying the free estimation principle. Classical estimation consists in (the most generally) optimal fitting in the tracked out points (the object's positions) into the existing, geometrical observational system (represented by points of adjustment Z^∇, of coordinates' vector $\mathbf{X}^\nabla$). However, in free estimation, it is assumed that the whole measurement structure is internally coherent, but 'non attached' to the coordinates system, what causes, that except of proper coordinates $\mathbf{X}$ there are determined also increments on top of the adjustment points Z^∇. In such circumstances the geometrical navigational structure under equalization implicates degrees of freedom in relation to the accepted coordinates system, and thereby:

$$\left(rank\,[\partial_{\mathbf{X},\mathbf{X}^\Diamond}\mathbf{F}(\mathbf{X},\mathbf{X}^\nabla)] = r\right) < m$$

(the above problem had been discussed in details in the work by [10]). Therefore, apart from a classical equalization criterion $\varphi(\hat{\mathbf{X}}) = \mathbf{v}^T\mathbf{P}_x\mathbf{v} = \min$, it is necessary to apply an additional optimization criterion, relating to all the points coordinates increments, thus:

$$\psi(\hat{\mathbf{X}},\hat{\mathbf{X}}^\nabla) = (\hat{\mathbf{X}} - \mathbf{X}^0)^T\mathbf{P}_{X^0}(\hat{\mathbf{X}} - \mathbf{X}^0) + (\hat{\mathbf{X}}^\nabla - \mathbf{X}^\Diamond)^T\mathbf{P}_{X^\nabla}(\hat{\mathbf{X}}^\nabla - \mathbf{X}^\nabla) =$$
$$= \varsigma^T\mathbf{P}_{X^0}\varsigma + \varsigma_\nabla^T\mathbf{P}_{X^\nabla}\varsigma_\nabla = \min \tag{4}$$

where: $\mathbf{X}^0$ — vector of the object's approximate coordinates of weight matrix;

$\mathbf{P}_{X^0}$, $\mathbf{P}_{X^\nabla} = \mathbf{Q}_{X^\nabla}^{-1}$ — the weight matrix of the adjustment points' coordinates Z^∇ (navigational signs).

Moreover:

$$\varsigma = \hat{\mathbf{X}} - \mathbf{X}^0, \qquad \varsigma_\nabla = \hat{\mathbf{X}}^\nabla - \mathbf{X}^\nabla.$$

So, free equalization may consist in solving of the following problem:

$$\left.\begin{array}{l} \mathbf{x} + \mathbf{v} = \mathbf{F}(\hat{\mathbf{X}},\hat{\mathbf{X}}^\nabla) \\[4pt] \mathbf{C}_x = \sigma_0^2\mathbf{Q}_x = \sigma_0^2\mathbf{P}_x^{-1} \\[4pt] \mathbf{C}_{X^\nabla} = \sigma_0^2\mathbf{Q}_{X^\nabla} = \sigma_0^2\mathbf{P}_{X^\nabla}^{-1} \\[4pt] \varphi(\hat{\mathbf{X}},\hat{\mathbf{X}}^\nabla) = \mathbf{v}^T\mathbf{P}_x\mathbf{v} = \min \\[4pt] \psi(\hat{\mathbf{X}},\hat{\mathbf{X}}^\nabla) = \varsigma^T\mathbf{P}_{X^0}\varsigma + \varsigma_\nabla^T\mathbf{P}_{X^\nabla}\varsigma_\nabla = \min \end{array}\right\} \tag{5}$$

The obtained values of the difference $\varsigma_\nabla = \hat{\mathbf{X}}^\nabla - \mathbf{X}^\nabla$ (or its function) may underlie indicating an incorrect sign (or any of its coordinates). The simplest procedure in this range can be checking up on weather for each element of vector ς_∇ there occurs $[\varsigma_\nabla]_i \in \varDelta_{X^\nabla}$, where $\varDelta_{X^\nabla}$ is an admissible interval for random coordinates of the points Z^∇.

It may happen that in hard conditions of survey (as for example in considerable sea depth), some results of measurement are inadmissible values' errors biased. Such errors are called gross errors; the observations biased therewith — the observations outlying. In such situation the above presented methods of equalization should be replaced with robust methods. Using those methods in navigation had been described in the paper [9], and then developed in the works [3, 4, 7]. In general, applying the general rules of *M*-estimation, making the tasks (2) and (3) robust, can be achieved replacing the weight matrixes $\mathbf{P}_x = Diag(\mathbf{P}_x)$, with such a equivalent matrix $\widehat{\mathbf{P}}_x$, that:

$$\widehat{\mathbf{P}}_x = \mathbf{T}(\mathbf{v})\mathbf{P}_x, \;\; rank\{[\partial_{\mathbf{X}}\mathbf{F}(\hat{\mathbf{X}})]^T \mathbf{P}_x \partial_{\mathbf{X}}\mathbf{F}(\hat{\mathbf{X}})\} = m, \tag{6}$$

where $\mathbf{T}(\mathbf{v})$ is a diagonal matrix of attenuation, including elements $\forall ij : 0 \leq [\mathbf{T}(\mathbf{v})]_{ii} \leq 1$.

In free estimation, apart from model (6) there may also be applied a model:

$$\widehat{\mathbf{P}}_{X^\nabla} = \mathbf{T}(\hat{\mathbf{X}}^\nabla - \mathbf{X}^\nabla)\mathbf{P}_{X^\nabla} \tag{7}$$

(simplifying $\mathbf{P}_{X^\nabla} = Diag(\mathbf{P}_{X^\nabla})$), allowing in the process of tracking position of the object (coordinates $\hat{\mathbf{X}}$) to 'ignore' outlying adjustment points, which belong to Z^∇.

SIMULATION RESEARCH OF SUBMARINE'S NAVIGATION

Research on simulation of underwater navigation was carried out by stages:

1. The first stage referred to preparation of input data. It was necessary to obtain digital model of sea bottom, which was an environment within which a submarine would move.

2. The next stage included working out a programme of operation of on-board sonar systems, simulation of generating sonograms, also recording and processing these images.
3. The subsequent stage covered tracking/determining positions through comparison of survey images and the sea bottom map.
4. The last stage comprised the problems of equalizing survey results.

Stage of preparation of sea bottom images spatial distribution

For this work's purpose there have been prepared the following images of sea bottom: a water area block, reflecting the Gulf of Gdańsk, also some test images of water areas, characteristic for their differentiation of depth. There has been assumed random-deterministic character of on-board operation of an echo-sounder of the following proceeding:

1. Simulating 'perfect' parameters of water environment.
2. Generating randomly interfered environment parameters.
3. Simulating 'ideal' operation of hydro-acoustic equipment.
4. Generating interfered (random) parameters of hydro-acoustic equipment operation.
5. Generating deterministic values of parameters of receiver's receiving block's operation.
6. Generating interfered values of parameters of receiver's receiving block's operation.

Stage of fixing positions observed, applying the images comparison method

In this stage there has been carried out a submarine task's simulation; considering the task as the submarine's passing along a set route, whereat the route recording was performed through comparison of the recorded bottom image and the numerical bottom model, basing on various algorithms of images similarity. There have been tested the artificial neural networks as well, with approximation of a location of the survey image in relation to a location of images included in the studying sequence.

It has been assumed that in this stage the submarine had passed along the set trajectory and collected up-to-date information about the sea bottom configuration and objects found on the bottom.
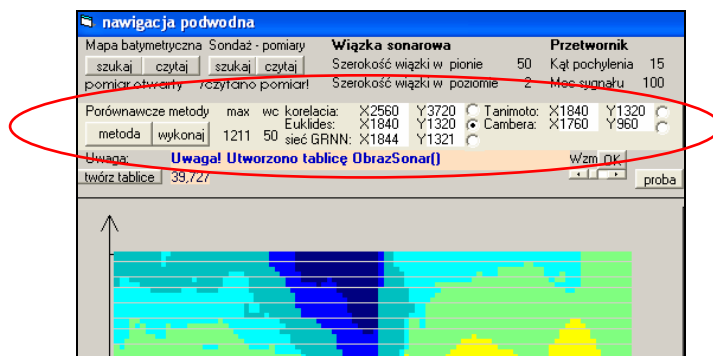
Fig. 1. The fragment of 'underwater navigation' program interface, displaying the object
for fixing the position observed, applying the comparative methods

Stage of fixing the positions, applying the robust equalization method

The software for solving problems of survey results robust equalization has been worked out and tested. It was assumed that the recorded sea bottom image is biased with the observed submarine position's error, resulting from the comparative navigation block's operation quality. Gross errors or random errors may occur. It was also assumed that the position, obtained as a result of comparing images (treated as approximated position), would serve for finding horizontal bearings and distances (NR and D) to objects located on the bottom. The objects had been identified in advance by the system's operator. Their position is precisely defined and known. Such an approach allowed equalization of erroneous results (gross errors) of bearings and distances measurements, also equalization of the observed position, defined on their basis.

The worked out simulator for carrying out underwater navigation was subject to some simplifications and constraints. Already in the time of working up a conception to create the computer application it had been accepted that firstly, the program in issue should be used for verification of the framed hypotheses and assumptions, concerning applying the comparative navigation methods for positioning submarine vehicles. The general and essential simplification is, that the program processes simulated data or real data prepared in post-processing. To work with the program the operator has to be familiar with a sailing area and skilled in identification of visual objects located on the spatial bottom model. In a result of digitalization of bathymetric map of the Gulf of Gdańsk water area (The North Port region), a spatial model of real sea bottom fragment was worked out; the fragment bounded with parallels and meridians (fig. 3).
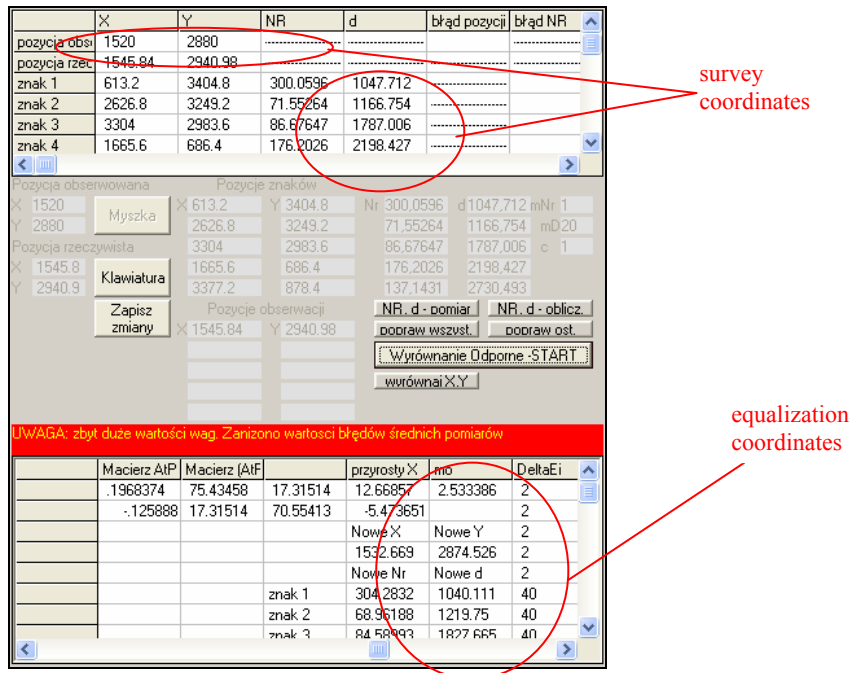
Fig. 2. Interface of robust equalization of navigational parameters survey results
('underwater navigation' program)

$\varphi 1=54o23,0'N$; $\varphi 2=54o26,0'N$; $\lambda 1=018o44,0'E$; $\lambda 1=018o49,18'E$

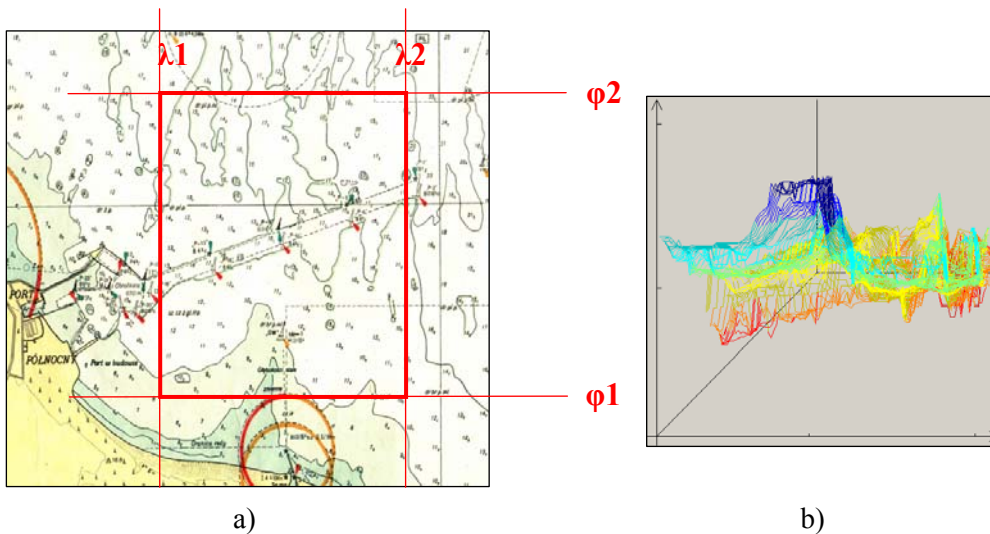a)   b)

Fig. 3. Real water area assigned for research — a) and the model of its bottom — b)

The submarine vehicle floats only above the read in bottom. It may travel along the investigated water area boundaries, or along a selected trajectory, defined with the position at start, course and speed. The vehicle carries out observations and takes measurements with a multi-beam sounder. The operator can store the measurements in the software memory, process them and use to compare with the sea bottom map.
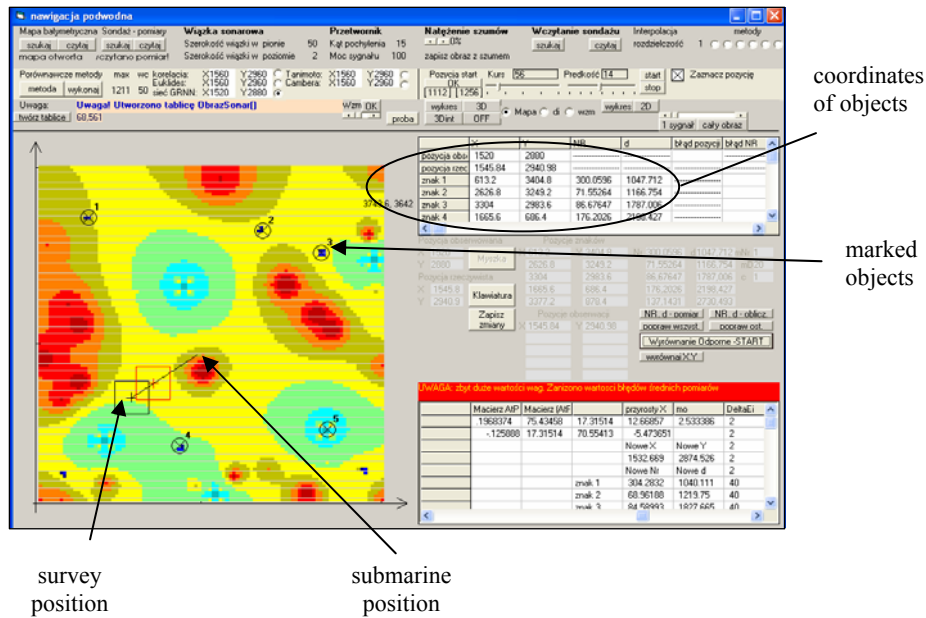

Fig. 4. Interface for the 'underwater navigation' software operator (user)

Tests results

It was assumed that the research works would be carried out on some water areas. Every one of them will differ from each other with bottom's configuration character. Models of very diversified and characteristic bottoms will be among of them, also those, of more flat bottom's surface. To equalize the results of surveys and setting out, on the bottom there will be placed distinguishing objects. Their positions will be known by the system's operator.

The system's operator will pass several times the route above each water area bottom, controlling the course and speed of the submarine. When manoeuvring, one has to remember that better results of applying the comparative methods are achieved, wherever the depth change gradient is very high.

The system has to enable setting out horizontal bearings and distances to characteristic objects situated on the bottom and indicated by the operator. A scope of information obtained in this way should provide adequate superabundance — a number of observations necessary to carry out equalization and accuracy assessment.

The successive tests included as follows:

Test No. 1: Determining the position above the 'flat' bottom (low depth's gradient); uniform distribution of marked objects on the bottom at the left and right side of the vehicle; the vehicle speed 5–8 knots; steady and changeable courses.

Test No. 2: Determining the position above the 'flat' bottom (low depth's gradient); distribution of marked objects on the bottom only at one side of the vehicle; the vehicle speed 5–8 knots; steady and changeable courses.

Test No. 3: Determining the position above the bottom of high depth's gradient; uniform distribution of marked objects on the bottom at the left and right side of the vehicle; the vehicle speed 5–8 knots; steady/changeable courses.

Test No. 4: Determining the position above the bottom of high depth's gradient; distribution of marked objects on the bottom only at one side of the vehicle; the vehicle speed 5–8 knots; steady and changeable courses.

Test No. 5: Determining the position in short time intervals (positions under observation were located close to each other). Stable positions of the marked objects on the bottom.
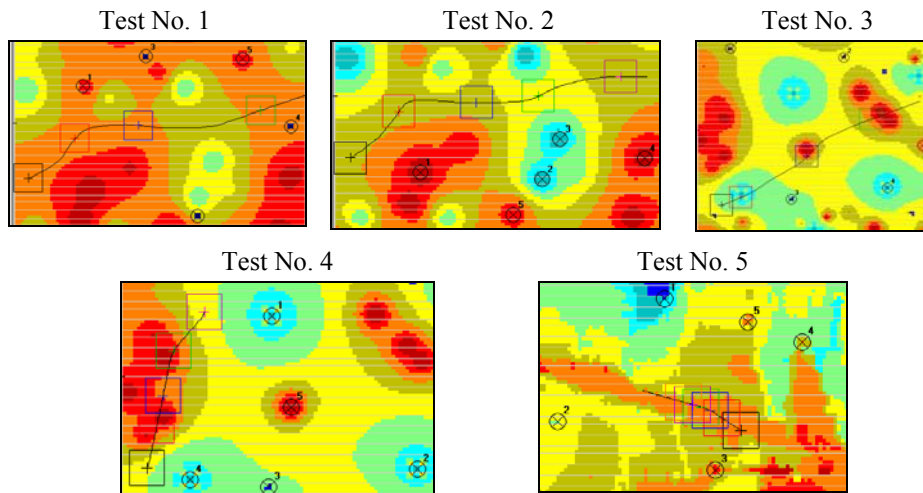

Fig. 5. Example of the submarine vehicle's passing route while carrying out the tests

ANALYSIS OF ACCURACY

The simulator has not been yet worked out comprehensively, tested and verified in many aspects as a system of underwater navigation. Thus it cannot be operated in real conditions and in real time. It is however an excellent researching tool, developable and modifiable. At this stage of the research works it enables to verify the set down theses, and generalized results of the carried out simulations allow working out conclusions concerning accuracy of the submarine vehicle's navigation on the grounds of the presented system. Anyhow one should be aware that it is necessary to carry out a series of statistic tests basing on maritime survey or taking advantage of real measurement images.

As a result of the carried out simulation tests there have been achieved the presented below accuracy characteristics.
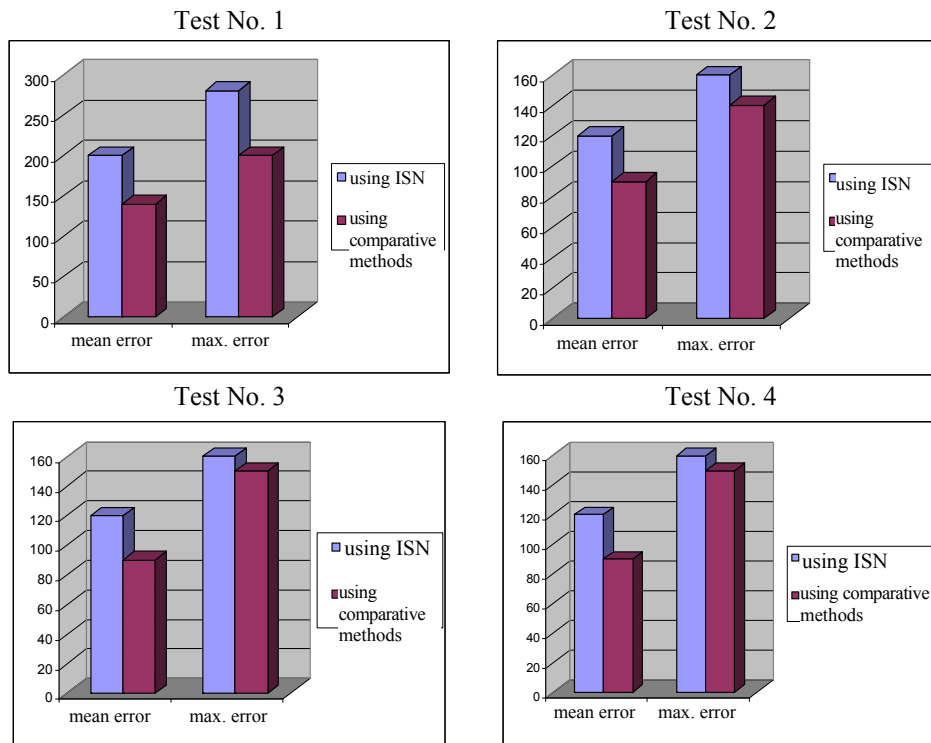

Fig. 6. Mean and maximal errors of the position observed before
and after equalization occurred in the first four tests

Test 5

The mean error of determining the position under observation — 200 m. Applying robust equalization allowed reducing the position's error by about 50 m.
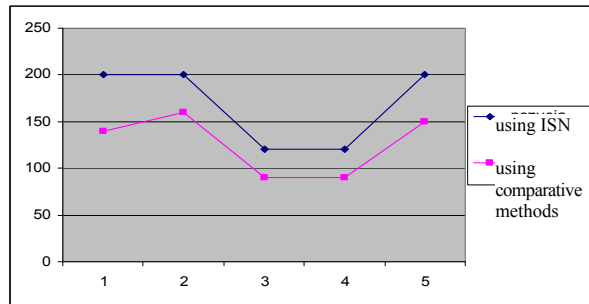
Fig. 7. Mean and maximal errors of the position observed before
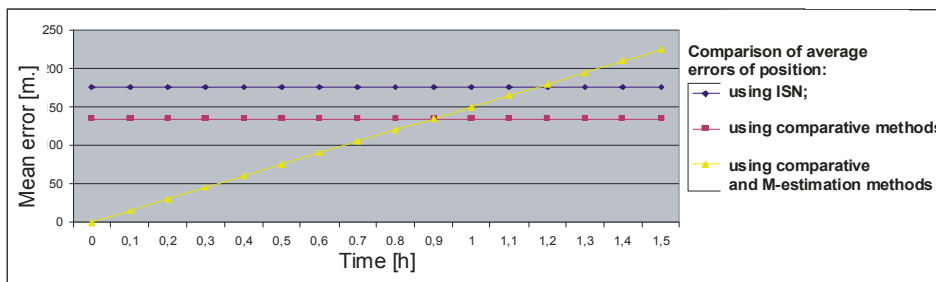and after equalization, which occurred in all the tests

Fig. 8. Comparison of the mean errors of the positions, obtained with a use
of the worked out method, and the errors of fixing positions by INS System

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