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ON SOME PROPERTIES OF QUASI-IDEAL CASCADES WITH LOSSES AT STAGES

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ABSTRACT

Some regularities of mass transfer in quasi-ideal cascades with losses at their stages are investigated. Simple relationships to evaluate the influence of losses on the integral characteristics of quasi-ideal cascades under the condition of the low factors of losses are obtained. The optimum parameters of a match-abundance ratio cascade, which is a special case of a quasi-ideal cascade, under the condition of the minimum total flow in a cascade are found.

Keywords: Quasi-ideal cascade, Multicomponent isotope mixture, Losses

INTRODUCTION

The concept of a quasi-ideal cascade has been proven to be extremely useful in analyzing the behavior of multi-stage installations for multicomponent isotope mixtures separation with large separation stage factors. The case when working substance losses are not observed at each cascade stage and the theory of quasi-ideal cascades with constant cut numbers of component flows have been reviewed, for example in [1-4]. The basic principles of the theory for multicomponent isotope mixture separation in such cascades with losses caused by decomposition of a working substance have been developed in [5].

In this paper, some regularities of mass transfer in quasi-ideal cascades with losses at their stages are investigated in order to find the optimum parameters of the process.

A MATHEMATICAL MODEL

Let us consider the separating cascade having a feed stream F with concentrations C_i^F, a product stream P with concentrations C_i^P, and a waste stream W with concentrations C_i^W, where $i = 1, 2, ..., m$, m is a component number in a separating isotope mixture (Fig. 1).

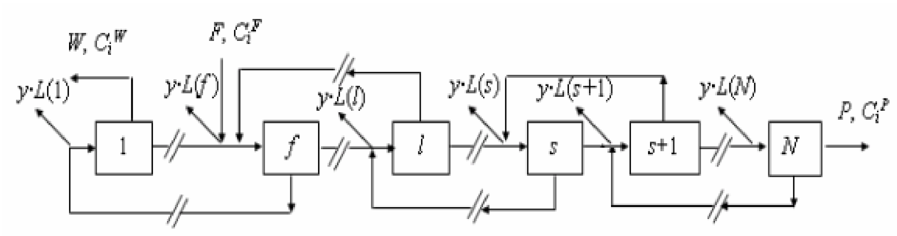

Fig. 1. Schematic drawing of a symmetrical cascade with losses at stages.

The definitions of the "product" and "waste" flows are conditional ones in the case of multicomponent isotope mixture separation. So, as a matter of convenience we shall assume that the "product" flow is taken from the end of the cascade where the lightest component of a separating mixture is enriched, and the "waste" one is extracted from the opposite end. In this manner, we number the stages of a quasi-ideal cascade in the order from the "waste" end (s=1) to the "product" one (s=N), and the mixture components are numbered in the order of increasing molecular masses. Also, we assume that the losses at each stage of the cascade are proportional to the input flow of a stage, i.e.:

$$\Delta L_s = yL(s) \tag{1}$$

where y is a factor of losses which is identical for all stages of the cascade [5].

The separating mixture entering each cascade stage as a feed flow L is divided into a product flow L' and a waste flow L'' with the component concentrations equal to C_i, C_i', C_i'', respectively. The values of the stage cuts $\theta(s)$, and the partial component flows $L_i(s), L_i^{'}(s), L_i^{''}(s)$ are defined by the formulae:

$$\theta(s) = \frac{L'(s)}{L(s)}, \qquad L_i(s) = C_i(s)L(s),$$
$$L_i^{'}(s) = C_i^{'}(s)L^{'}(s), \qquad L_i^{"}(s) = C_i^{"}(s)L^{"}(s), \quad i = 1,2,\cdots,m, \tag{2}$$

Moreover, to the relations (2) the following evident equalities have to be added $\sum_{i=1}^{m} C_i(s) = 1$, $\sum_{i=1}^{m} C_i^{'}(s) = 1$, $\sum_{i=1}^{m} C_i^{"}(s) = 1$

Thus, at the input of an arbitrary s-th stage, the balance equation for each component of the separating mixture can be written in the form:

$$L_i^{'}(s-1) + L_i^{"}(s+1) = (1+y)[L_i^{'}(s) + L_i^{"}(s)], \quad i = 1,2,\ldots,m\,. \tag{3}$$

Then we introduce the following relations [1]:

$$L_i^{"} = \frac{L_i^{'}}{g_i} \qquad L_i = \frac{g_i + 1}{g_i} L_i^{'} \tag{4}$$

where g_i are constant and equal to:

$$g_i = \frac{\alpha_{ik}(\beta_{ik} - 1)}{\alpha_{ik} - 1} \qquad i \neq k\,; \quad g_k = \frac{\beta_{ik} - 1}{\beta_{ik}(\alpha_{ik} - 1)} \tag{5}$$

Here, *k* is the number of a supporting component, α_{ik}, β_{ik} are the relative head and tail separation factors between *i*-th and *k*-th components, respectively, which do not depend on a stage number. Note that the g_k parameter is an invariant to the *i* component number because α_{ik}, β_{ik} are determined with regard to the supporting component the number of which is *k*. [1].

After that, the Eqs. (3) can be rewritten as follows:

$$L_i^{'}(s-1) + \frac{1}{g_i} L_i^{'}(s+1) = (1 + \frac{1}{g_i})(1+y)L_i^{'}(s), \quad i = 1,\, 2,\ldots,m\,, \tag{6}$$

The system of Eqs. (6) consists of the second-order finite-difference equations with constant coefficients for each component regarding the $L_i^{'}(s)$ functions. The boundary conditions are as follows:

$$\begin{cases} L_i^{'}(0) = L_i^{'}(N+1) = 0, \\ L_i^{'}(f-1) + \dfrac{1}{g_i} L_i^{'}(f+1) - (1+\dfrac{1}{g_i})(1+y)L_i^{'}(f) + FC_i^F = 0, \\ L_i^{'}(N) = PC_i^P, \\ L_i^{'}(1) = g_i(1)WC_i^W, \qquad i = 1, 2, \dots, m. \end{cases} \quad (7)$$

In addition, the solutions of the system (6) for the enriching and stripping sections of the cascade should coincide (for the stage) with the number $s = f$ where the feed flow enters. As a result, the fundamental solution for the system (6) will be expressed as:

$$L_i^{'}(s) = A_i\omega_{1i}^s + B_i\omega_{2i}^s \qquad i = 1, 2, \dots, m \quad (8)$$

where A_i, B_i are the constants which can be defined by using the boundary conditions (7), and ω_{1i}, ω_{2i} are the solutions of the square equations:

$$\omega_i^2 - (g_i + 1)(1 + y)\omega_i + g_i = 0 \quad (9)$$

that are written as:

$$\omega_{1i} = \frac{(g_i + 1)(1 + y) + \sqrt{((g_i + 1)(1 + y))^2 - 4g_i}}{2} \qquad i = 1, 2, \dots, m \quad (10)$$

$$\omega_{2i} = \frac{(g_i + 1)(1 + y) - \sqrt{((g_i + 1)(1 + y))^2 - 4g_i}}{2} \qquad i = 1, 2, \dots, m \quad (11)$$

When the boundary conditions (7) are applied, the solution of Eqs. (6) for both sections of the cascade can be obtained:
the stripping section

$$L_i^{'}(s) = WC_i^W \frac{\omega_{1i}\omega_{2i}}{\omega_{2i} - \omega_{1i}}\left(\omega_{2i}^s - \omega_{1i}^s\right) \quad s = 1, 2, \dots, f-1,\ i = 1, 2, \dots, m \quad (12)$$

$$WC_i^W = \frac{\omega_{2i}^{N+1-f} - \omega_{1i}^{N+1-f}}{\omega_{2i}^{N+1} - \omega_{1i}^{N+1}} FC_i^F \quad s = 1, 2, \dots, f-1,\ i = 1, 2, \dots, m \quad (13)$$

the enriching section

$$L_i^{'}(s) = PC_i^P \frac{\omega_{2i}^{s-N-1} - \omega_{1i}^{s-N-1}}{\omega_{2i}^{-1} - \omega_{1i}^{-1}} \qquad s = f,\ldots,N\,,\ i = 1, 2,\ldots,m \qquad (14)$$

$$PC_i^P = \frac{\omega_{2i}^{-f} - \omega_{1i}^{-f}}{\omega_{2i}^{-N-1} - \omega_{1i}^{-N-1}} FC_i^F \qquad s = f,\ldots,N\,,\ i = 1, 2,\ldots,m \qquad (15)$$

Taking into account the obvious condition $\sum_{j=1}^{m} C_j = 1$, from (13) and (15) the following relations can be obtained:

$$\frac{W}{F} = \sum_{j=1}^{m} \frac{\omega_{2j}^{N+1-f} - \omega_{1j}^{N+1-f}}{\omega_{2j}^{N+1} - \omega_{1j}^{N+1}} C_i^F \qquad (16)$$

$$\frac{P}{F} = \sum_{j=1}^{m} \frac{\omega_{2j}^{-f} - \omega_{1j}^{-f}}{\omega_{2j}^{-N-1} - \omega_{1j}^{-N-1}} C_i^F \qquad (17)$$

$$C_i^W = \frac{\omega_{2i}^{N+1-f} - \omega_{1i}^{N+1-f}}{\omega_{2i}^{N+1} - \omega_{1i}^{N+1}} C_i^F \Bigg/ \sum_{j=1}^{m} \frac{\omega_{2j}^{N+1-f} - \omega_{1j}^{N+1-f}}{\omega_{2j}^{N+1} - \omega_{1j}^{N+1}} C_i^F \qquad (18)$$

$$C_i^P = \frac{\omega_{2i}^{-f} - \omega_{1i}^{-f}}{\omega_{2i}^{-N-1} - \omega_{1i}^{-N-1}} C_i^F \Bigg/ \sum_{j=1}^{m} \frac{\omega_{2j}^{-f} - \omega_{1j}^{-f}}{\omega_{2j}^{-N-1} - \omega_{1j}^{-N-1}} C_i^F \qquad (19)$$

Furthermore, using the solutions (12), (14), the relations (2), (4) as well as the relation $L(s) = \sum_{i=1}^{m} L_i(s)$, it is possible to determine the distributions of flows $L(s)$, the concentrations $C_i(s)$ and the values of the stage cuts $\theta(s)$ over the quasi-ideal cascade with the losses at each stage.

For specified values of g_i, f, N, y the formulae (12) - (19) allow calculating parameters of a quasi-cascade with losses at its stages, i.e. determining the W/F, P/F ratios, the concentrations of components in the product and waste flows C_i^P, C_i^W, distributions of the flows $L(s)$

entering each stage of a cascade as well as the component concentrations $C_i(s)$ over the cascade.

The case of small losses at separation stages has to meet the condition as $y << 1$. Taking into account this condition and decomposing the square roots in the expressions (10) and (11) to a series, one can get the following result:

$$\omega_{1i} = 1 + y\frac{1+g_i}{1-g_i} \qquad i = 1, 2, \ldots, m \tag{20}$$

$$\omega_{2i} = g_i\left(1 - y\frac{1+g_i}{1-g_i}\right) \qquad i = 1, 2, \ldots, m \tag{21}$$

Taking into account that y<<1, the expressions (20) and (21) can be rewritten as:

$$\ln\omega_{1i} = y\frac{1+g_i}{1-g_i} \qquad i = 1, 2, \ldots, m \tag{22}$$

$$\ln\omega_{2i} = \ln g_i - y\frac{1+g_i}{1-g_i} \qquad i = 1, 2, \ldots, m \tag{23}$$

Therefore,

$$\omega_{1i}^s = \exp(y\frac{1+g_i}{1-g_i}s) \tag{24}$$

$$\omega_{2i}^s = g_i^s\exp(-y\frac{1+g_i}{1-g_i}s) \tag{25}$$

Substituting (24) and (25) into (16)-(17) gives:

$$\frac{W}{F} = \sum_{j=1}^{m}\frac{g_j^{N+1-f}\exp[-\tilde{y}_j(N+1-f)] - \exp[\tilde{y}_j(N+1-f)]}{g_j^{N+1}\exp[-\tilde{y}_j(N+1)] - \exp[\tilde{y}_j(N+1)]}C_i^F \tag{26}$$

$$\frac{P}{F} = \sum_{j=1}^{m}\frac{g_j^{-f}\exp(\tilde{y}_j f) - \exp(-\tilde{y}_j f)}{g_j^{-N-1}\exp[\tilde{y}_j(N+1)] - \exp[-\tilde{y}_j(N+1)]}C_i^F \tag{27}$$

$$C_i^W = \frac{\dfrac{g_i^{N+1-f}\exp[-\tilde{y}_i(N+1-f)]-\exp[\tilde{y}_i(N+1-f)]}{g_i^{N+1}\exp[-\tilde{y}_i(N+1)]-\exp[\tilde{y}_i(N+1)]}C_i^F}{\sum_{j=1}^{m}\dfrac{g_j^{N+1-f}\exp[-\tilde{y}_j(N+1-f)]-\exp[\tilde{y}_j(N+1-f)]}{g_j^{N+1}\exp[-\tilde{y}_j(N+1)]-\exp[\tilde{y}_j(N+1)]}C_j^F} \tag{28}$$

$$C_i^P = \frac{\dfrac{g_i^{-f}\exp[\tilde{y}_i f]-\exp[-\tilde{y}_i f]}{g_i^{-N-1}\exp[\tilde{y}_i(N+1)]-\exp[-\tilde{y}_i(N+1)]}C_i^F}{\sum_{j=1}^{m}\dfrac{g_j^{-f}\exp[\tilde{y}_j f]-\exp[-\tilde{y}_j f]}{g_j^{-N-1}\exp[\tilde{y}_j(N+1)]-\exp[-\tilde{y}_j(N+1)]}C_j^F} \tag{29}$$

where

$$\tilde{y}_i = y\frac{g_i+1}{1-g_i} \tag{30}$$

If the number of stages in a cascade is big enough, and for $g_i^f >> 1$, $g_i^n >> 1$ ($g_i > 1$), from the formula (29) one can obtain the approximation for the component concentrations enriching at the "light" ("product") end of the cascade as follows:

$$C_i^P = \frac{\exp\left[-y\dfrac{g_i+1}{g_i-1}(N+1-f)\right]C_i^F}{\sum_{g_j>1}\exp\left[-y\dfrac{g_j+1}{g_j-1}(N+1-f)\right]C_j^F} \tag{31}$$

Note that the relation (31) determining the maximum enrichment value for mixture components in the "product" flow from a cascade with losses at its stages is valid for *y*=0 and represents generalization of the formula obtained in [6]:

$$\left[C_i^P\right]_{\max} = \frac{C_i^F}{\sum_{g_j>1}C_j^F} \tag{32}$$

R-CASCADE AND CALCULATION CONDITIONS

There exists an unlimited number of quasi-ideal cascades intended to solve various separation problems. The type of a quasi-ideal cascade is defined by some additional conditions. So, for example, in the case of multi-component isotope mixture separation it is impossible to design a cascade with no mixing of flows of different concentrations for all components as in the case of a binary mixture (the so called ideal cascade). However, for quasi-ideal cascades, it is possible to ensure an equality of the abundance ratio $R_{nk} = C_n / C_k$ for a pair of chosen *n* and *k* components (the "target" and "supporting" ones, respectively) in the input flow at each cascade stage. Such a cascade is called a matched abundance ratio cascade or MARC [2]. If the no-mixing condition for a pair of components (n, k) is given, it means that for the isotope mixture of m components, it can be defined as $m(m-1)/2$ various quasi-ideal cascades.

In this paper, the problem of the optimum (in the sense of a minimum total flow in a cascade) MARC intended for a target component separation is investigated. The value of a parameter $M^* = (M_n + M_k)/2$ is used as a parameter of optimization, where M_n and M_k are molecular masses of the target and supporting components ($n, k = 1, 2, \ldots m, \quad n \neq k$), respectively. The value of M^* determines the flow distribution over a quasi-ideal cascade. To calculate the g_i constants the following relationship with *M** was used [4-5]:

$$g_i = q_0^{M^* - M_i} \tag{33}$$

where q_0 is a separation factor per unit mass difference at cascade stages.

The isotopic mixture under investigation is irradiated uranium from fuel assemblies with the composition as follows: ^{232}U - 1×10^{-9}, ^{234}U - 2×10^{-4}, ^{235}U - 8.3×10^{-3}, ^{236}U - 4.1×10^{-3}, ^{238}U - 0.9874 [7]. A separation factor per unit mass difference at cascade stages q_0 was taken equal to 1.1. The ^{235}U component was considered as a target component. Its concentration in the product (C_3^P) and waste (C_3^W) flows is equal to 3.5% and 0.2%, respectively.

RESULTS OF CALCULATION

We introduced the expression of $((\Sigma L)^* - \Sigma L)/\Sigma L$ which is a relative difference of a total flow (RDTF). The dependences of RDTF versus a value of dimensionless losses at cascade stages for different values of a parameter M^* are presented in Fig. 2. The indications of $(\Sigma L)^*$ and ΣL in RDTF are a total flow in MARC with and without losses, respectively. In accordance with the model applied in the theory of influence of working substance losses in a cascade on its separation characteristics, the dependences of RDTF versus losses are linear. Moreover, for the same value of a factor of losses, the RDTF has its maximum value when M^*=236.5, which corresponds to the no-mixing condition for the pair of the ^{235}U and ^{238}U components. In other words, for either M^*<236.5 (dependence 1) or M^*>236.5 (dependence 2), the RDTF is always smaller than that in the case of M^*=236.5 (dependence 3). One can observe this behavior more obviously from the dependence of RDTF versus a value of a parameter M^* shown in Fig. 3. It is explained by the "sensitivity" of a flow distribution in the optimum cascade to losses that play a role of a "parasitic" product flow.

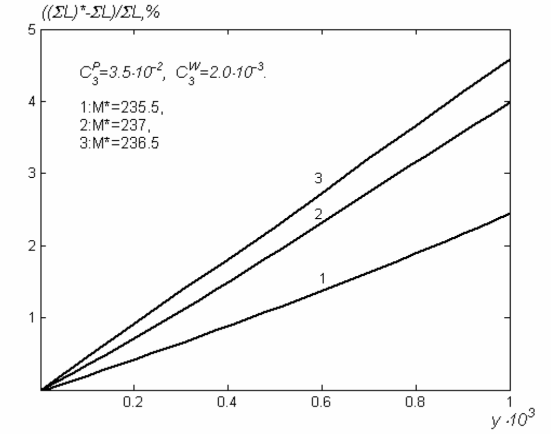

Fig. 2. Dependences of a relative difference of a total flow $((\Sigma L)^* - \Sigma L)/\Sigma L$ versus a factor of losses *y*.

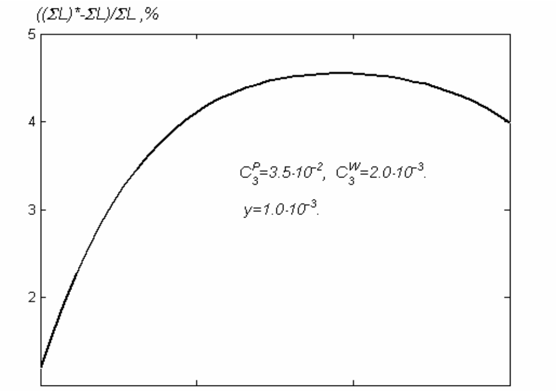

Fig. 3. Dependence of a relative difference of a total flow $((\Sigma L)^* - \Sigma L)/\Sigma L$ versus a value of a parameter M^*.

The results of a relative total flow $\Sigma L/P$ calculation in MARC versus the value of a parameter M^* for different factors of losses at cascade stages y are shown in Fig. 4. As it follows from the results obtained, when the concentrations of a target component in the product and waste flows are specified, there exists the optimum value of a parameter M^*_{opt}, corresponding to the minimum total flow that is equivalent to the minimum specific expenses for the production of the enriched product. Under the conditions reviewed in the paper, the optimum value of M^*_{opt} is equal to 236.5. Note that the presence of losses in a cascade does not practically influence the value of M^*_{opt} (see Fig. 4).

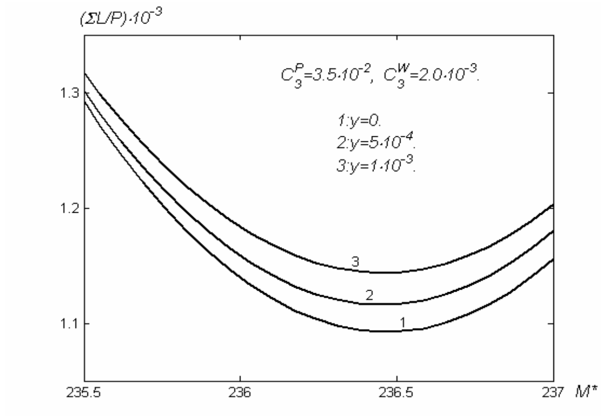

Fig. 4. Dependences of a relative total flow $\Sigma L/P$ in a cascade versus a value of a parameter M^* for different factors of losses y.

In other words, the optimum M^*_{opt} is not varied with the factor losses y. The losses can be interpreted as some additional product flow from the cascade, which has an effect only on the level of enrichment (or depletion) of a target component.

CONCLUSIONS

1. The simplified formulae to evaluate the influence of losses on the integral characteristics of quasi-ideal cascades under the condition of the low factors of losses are obtained.
2. For the specified values of the target component concentrations in product and waste flows from a quasi-ideal cascade, there is an optimum value of a parameter M^*_{opt} corresponding to the minimum total flow (or the minimal specific expenses for enriched product obtaining). The

presence of working substance losses in a cascade does not influence the value of M^*_{opt}.

3. The value of a relative difference of a total flow $((\Sigma L)^* - \Sigma L)/\Sigma L$ in MARC exhibits the maximum value when a parameter M^* reaches its optimum value.

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