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ON ONE ANALYTICAL SOLUTION OF THE MASS TRANSFER PROBLEM IN MULTISTAGE INSTALLATIONS WITH CONSTANT CUT NUMBERS

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ABSTRACT

Separation of binary isotope mixtures in multistage installations consisting of separation elements with constant cut numbers and separation factors is considered. The analytical solution for component concentration distributions over a cascade with constant cut numbers is obtained. Major mass transfer regularities in such a kind of cascades are reviewed.

Keywords: Multicomponent isotope mixture; Separation cascade; Cut number; Mass transfer regularities

INTRODUCTION

The theory of a two-component isotope separation cascade dealing with constant cut numbers was developed by Turkin [1,2], Apelblat and Ilamed-Lehrer [3,4]. In their papers, an enriching section of a separation cascade was considered only. The present paper is devoted to the theory of a binary separation cascade with constant cut numbers containing both enriching and stripping parts of the cascade. A considerable interest in this type of cascades is caused by the following reasons. First, use of stages with identical cut numbers in a cascade is expedient and convenient from the technological point of view. Second, a reduction of an interstage flow to the cascade ends as it takes place in an ideal cascade can be provided in this

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kind of installations. It allows to make approximation of flow distributions in a cascade closer to optimal ones. Third, in a cascade with constant cut numbers, the flow does not depend on a composition of the separating mixture. In this case, the distribution of a mass flow in a cascade is the function of a cut number, a coordinate of the feed flow input, and values of the overall separation factor and product flow.

For the above reasons one more is to determine the distributions of component concentrations over a cascade and to find the best value of a cut number defining minimum of a total flow in a cascade with constant cut numbers at separation stages.

DEFINITIONS

We will consider a two-component isotope mixture with concentrations equal to *c* and 1-*c*. A separation element with one input flow (a feed flow) and two output flows (so-called up and down flows) is shown in Fig. 1. The primes (') and ('') indicate the up and down flows and there is no prime for the feed one. The overall separation factor is defined as

$$q = \frac{c'}{1-c'} / \frac{c''}{1-c''} \quad (1)$$

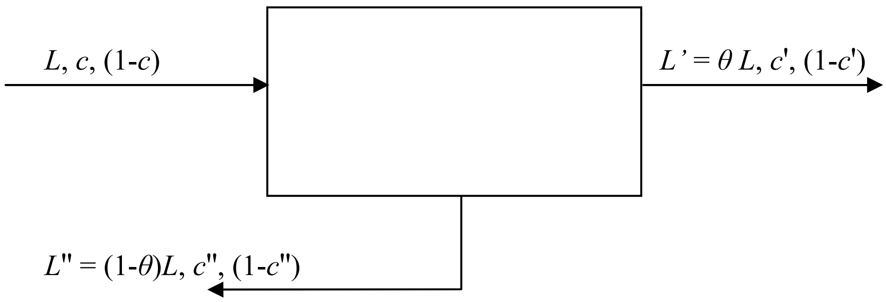

Fig. 1. Schematic drawing of separation element.

Note that the value of the overall separation factor *q* depends on the method of separation and characteristics of a mixture to be separated. Assume that *q* is not a function of a cut number $\theta = L'/L$ and concentrations [5]. The relationship including concentrations entering and leaving a cascade stage is written usually in the form of the component balance over a separation element:

$$c = \theta c' + (1-\theta)c'' \quad (2)$$

The flow entering the stage, which can consist of one separation element or a few of them connected in parallel, is equivalent to the sum of the feed flows entering the stage elements. A typical connection between the stages of a symmetrical cascade is presented in Fig. 2.

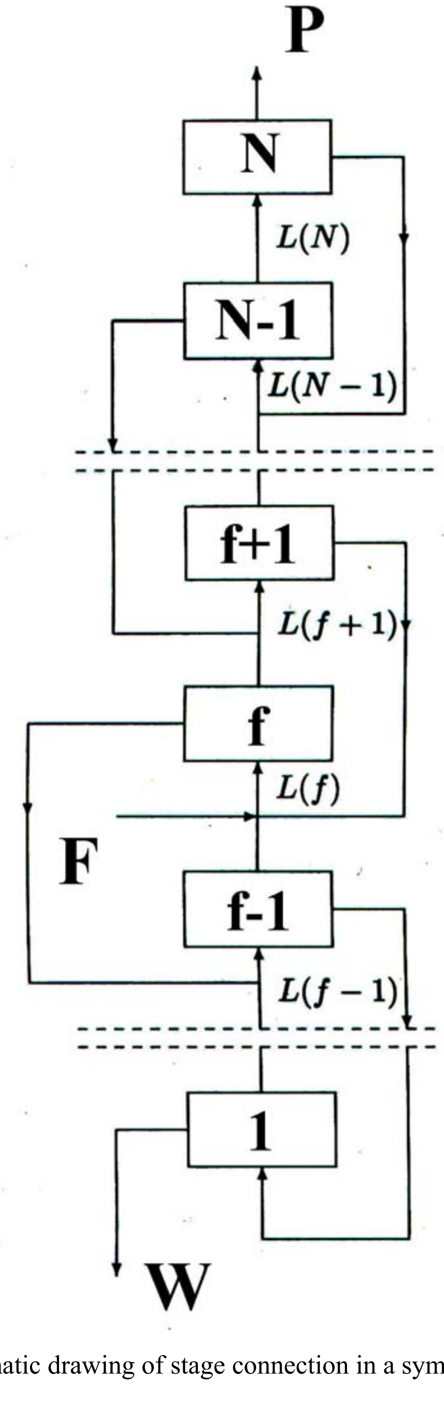

Fig. 2. Schematic drawing of stage connection in a symmetrical cascade.

Here, P, W and F are the product, waste, and feed flows, respectively. c_P, c_W and c_F are concentrations of a key-component in these flows. $L(s)$ is a flow entering the stage with a number s. Concentrations of a key-component entering and leaving a stage are $c(s)$, $c'(s)$ and $c''(s)$, respectively. Flows of mixture components which correspond to these concentrations are written as follows: $G(s) = L(s)c(s)$, $G'(s) = L'(s)c'(s)$ and $G''(s) = L''(s)c''(s)$. N is total number of stages in a cascade, and f is a stage number where a feed flow enters. The stages are numbered from 1 to N. Moreover, the case of small concentrations (c<<1) is considered.

MASS TRANSFER REGULARITIES

The expressions for flows in the separation cascade with constant cut numbers have been obtained in [6]. They are written as follows:
for the product flow

$$P = \frac{1-\rho^{f}}{1-\rho^{N+1}} F \tag{3}$$

for the enriching section of the cascade ($s = f,\ldots, N$)

$$L(s) = \frac{1+\rho}{1-\rho} \frac{(1-\rho^{f})(1-\rho^{N-s+1})}{1-\rho^{N+1}} F \tag{4}$$

for the waste flow

$$W = \frac{1-\rho^{N-f+1}}{1-\rho^{N+1}} \rho^{f} F \tag{5}$$

for stripping section of the cascade (s - 1,…,f-1):

$$L(s) = \frac{1+\rho}{1-\rho} \frac{(1-\rho^{s})(1-\rho^{N-f+1})}{1-\rho^{N+1}} \rho^{f-s} F \tag{6}$$

The expression for the total flow in the cascade is also important for practical calculations:

$$\frac{\sum_{s=1}^{N} L(s)}{P} = \frac{1+\rho}{1-\rho}\left[\left(N-f+1-\frac{\rho}{\rho-1}\left(\rho^{N-f+1}-1\right)\right)+\frac{1-\rho^{N-f+1}}{1-\rho^{f}}\rho^{f}\left(\frac{1-\rho^{1-f}}{\rho-1}-\left(f-1\right)\right)\right] \tag{7}$$

The expressions for a component flow $G = Lc$ can be obtained applying the approach described in [6]. For this purpose let us write the law of mass conservation at any flow junction point of the cascade that is given as:

$$G'(s\text{-}1)+G''(s+1)\text{-}G(s) = 0 \tag{8}$$

or using the expression for the flow L Eq. (7) can be rewritten as:

$$\theta L(s\text{-}1)c'(s\text{-}1) + (1\text{-}\theta)L(s+1)c''(s+1)\text{-}L(s)c(s)=0 \tag{9}$$

From Eqs. (1) and (2), on condition that $c << 1$, the expressions for concentrations c' and c'' are obtained:

$$c' = \frac{qc}{q\theta + (1-\theta)}, \quad c'' = \frac{c}{q\theta + (1-\theta)} \tag{10}$$

Using the definition $\rho = (1\text{-}\theta)/\theta$ and substituting (9) to (8), the governing equation for separation cascade with constant cut numbers can be derived:

$$G(s+1) - (1 + \frac{1}{\rho/q})G(s) + (\frac{1}{\rho/q})G(s-1) = 0 \tag{11}$$

The boundary conditions for Eq. (10) are written as:

$$G(0) = G(N+1) = 0, \tag{12}$$

$$G(f+1) - (1+\frac{1}{\rho/q})G(f) + (\frac{1}{\rho/q})G(f-1) + (1+\frac{1}{\rho/q})Fc_{\mathrm{F}} = 0 \tag{13}$$

$$Pc_{\mathrm{P}} = \frac{1}{1+\rho/q}G(N) \tag{14}$$

$$Wc_{\mathrm{W}} = \frac{\rho/q}{1+\rho/q}G(1) \tag{15}$$

Evidently, to solve the formulated boundary problem it is necessary to consider the material balance equation for the cascade installation as a whole:

$$Pc_{\mathrm{P}} + Wc_{\mathrm{W}} = Fc_{\mathrm{F}}\ . \tag{16}$$

The general solution of Eq. (5) can be written as [7]:

$$G(s) = A\varpi_1^S + B\varpi_2^S \tag{17}$$

where ω_1 and ω_2 are the roots of the characteristic Eq. below:

$$\varpi^2 - (1 + \frac{1}{\rho / q})\varpi + \frac{1}{\rho / q} = 0 \tag{18}$$

which are equal to $\omega_1 = \dfrac{1}{\rho / q}$ and $\omega_2 = 1$, respectively. (19)

The constants A and B can be found from Eqs. (11)-(15). As a result, the solution of the problem may be presented as follows:
for the product flow:

$$Pc_\mathrm{P} = \frac{1-(\rho / q)^f}{1-(\rho / q)^{N+1}} Fc_\mathrm{F} \tag{20}$$

for the enriching section of the cascade (s = f,…,N):

$$G(s) = \frac{1+\rho / q}{1-\rho / q} \frac{\left(1-(\rho / q)^f\right)\left(1-(\rho / q)^{N-s+1}\right)}{1-(\rho / q)^{N+1}} Fc_\mathrm{F} \tag{21}$$

for the waste flow:

$$Wc_\mathrm{W} = \frac{1-(\rho / q)^{N-f+1}}{1-(\rho / q)^{N+1}} (\rho / q)^f Fc_\mathrm{F} \tag{22}$$

for the stripping section of a cascade: (1, 2,…, f-1):

$$G(s) = \frac{1+\rho / q}{1-\rho / q} \frac{\left(1-(\rho / q)^s\right)\left(1-(\rho / q)^{N-f+1}\right)}{1-(\rho / q)^{N+1}} \left(\rho / q\right)^{f-s} Fc_\mathrm{F} \tag{23}$$

From the definition of the component flow ($G(s) = L(s)c(s)$), the expressions for distributions of concentrations over the cascade were obtained. Dividing Eqs. (19)-(22) by Eqs. (3)-(6), respectively, one will get the following expressions for the component concentrations:
for the product concentration:

$$c_\mathrm{P} = \frac{1-(\rho / q)^f}{1-(\rho / q)^{N+1}} \frac{1-\rho^{N+1}}{1-\rho^f} c_\mathrm{F} \tag{24}$$

for the enriching section of the cascade ($s = f$,..., N):

$$c(s)=\frac{1+\rho/q}{1-\rho/q}\,\frac{1-\rho}{1+\rho}\,\frac{\left(1-(\rho/q)^{f}\right)\left(1-(\rho/q)^{N-s+1}\right)}{1-(\rho/q)^{N+1}}\,\frac{1-\rho^{N+1}}{\left(1-\rho^{f}\right)\left(1-\rho^{N-s+1}\right)}\,c_{\mathrm{F}} \quad (25)$$

for the waste concentration:

$$c_{\mathrm{W}}=\frac{1-(\rho/q)^{N-f+1}}{1-(\rho/q)^{N+1}}\,\frac{1-\rho^{N+1}}{1-\rho^{N-f+1}}\,\frac{1}{q^{f}}\,c_{\mathrm{F}} \quad (26)$$

for the stripping section of the cascade (s = 1, 2,.., N-1):

$$c(s)=\frac{1+\rho/q}{1-\rho/q}\,\frac{1-\rho}{1+\rho}\,\frac{\left(1-(\rho/q)^{s}\right)\left(1-(\rho/q)^{N-f+1}\right)}{1-(\rho/q)^{N+1}}\,\frac{1-\rho^{N+1}}{\left(1-\rho^{s}\right)\left(1-\rho^{N-f+1}\right)}\left(1/q\right)^{f-s}c_{\mathrm{F}} \quad (27)$$

RESULTS AND DISCUSSION

The calculations made above allowed to obtain the dependences that define the separation process in the cascade with constant cut numbers.

In Fig. 3, the dependence of the total flow defined as $\sum L/P$ versus the cut number θ is shown. The parameters of the cascade N = 10, f = 3 were used in all calculations which results are presented below.

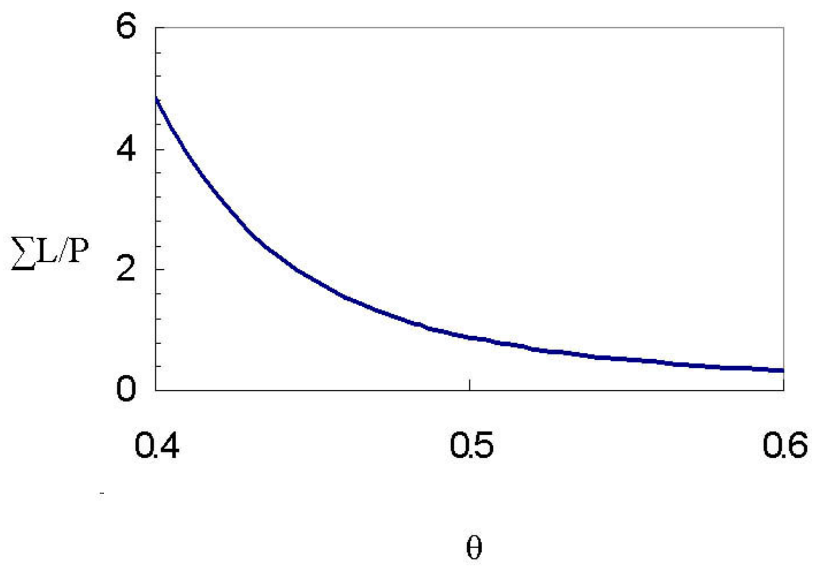

Fig. 3. Total flow of multistage installation versus a cut number.

The curve in Fig. 3 demonstrates that a decrease in the total flow value can be reached by means of an increase of a cut number θ. However, what is essential, it also entails some decrease of the key-component product concentration c_p. In Fig.4, the dependence of the relative key-component concentration c_p/c_F versus the cut number θ for the value of q = 1.1 is presented. Note that in the cases when q = 1.3 and 1.5, the corresponding dependences are very similar to that indicated in Fig. 4 differing in a concentration axis scale. An increase of θ in the range from 0.4 to 0.6 leads to a decrease of the total flow with the factor 5 and practically brings the key-component concentration c_P to nothing. In the above three cases considered, the values of the key-component concentrations in the product flow for θ = 0.6 differ slightly from c_F. The same concentration values calculated for θ = 0.4 exceed c_F in 1.7; 4 and 7 times for the cases when q = 1.1; 1.3 and 1.5, respectively.

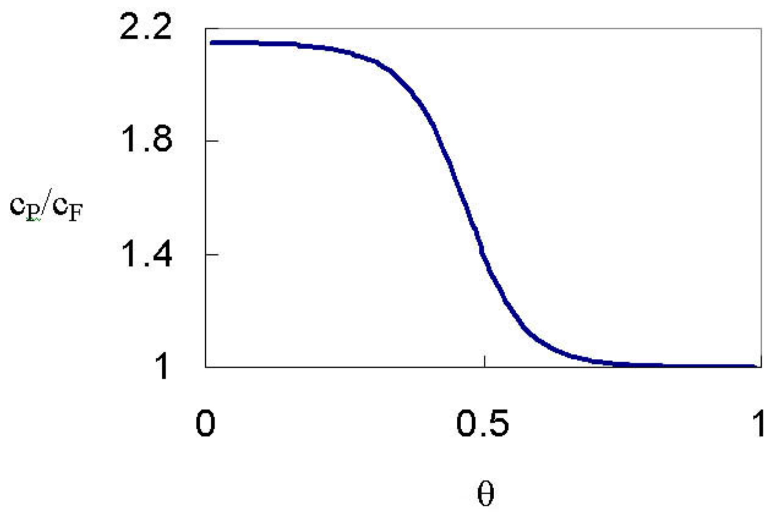

Fig. 4. Relative concentration of a key-component in a product flow versus a cut number (q=1.1).

To understand the influence of the cut number value on the total flow, a series of the cascades providing identical enrichment on the key-component were calculated. The result for q = 1.1 is given in Fig. 5. As in the case above, the dependences for q=1.3 and q=1.5 are similar to the indicated one differing in the flow axis scale.

As the next step of the research, the optimization was made for the separation cascade under investigation with the use of two efficiency criteria: thermodynamic power and separative work. Besides, as it is known from the theory of the ideal cascade, the optimal cut number for it can be calculated by the well-known formula ($\theta_{id} = 1 / (q^{1/2} + 1)$).

In Tab. 1, the optimum cut numbers θ for various overall separation factors q and the corresponding values of the total flows are listed (θ_{td} corresponds to the maximum of the thermodynamic power which is necessary for isotope separation [4] and θ_{sw} is related to the maximum of separative work [8]).

Tab. 1. Total flows in a cascade and optimal cut numbers for various efficiency criteria

		θ_{td}	θ_{id}	θ_{sw}
q = 1.1	θ	0.484	0.488	0.492
	$(\sum L/P)$	7398	7119	7419
q = 1.3	θ	0.456	0.467	0.478
	$(\sum L/P)$	924	888	924
q = 1.5	θ	0.432	0.450	0.466
	$(\sum L/P)$	478	457	477

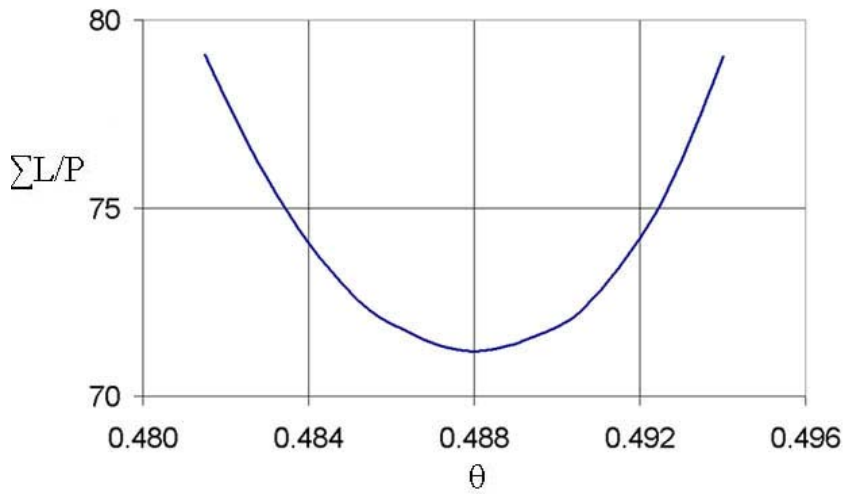

Fig. 5. Total flow in a cascade versus a cut number (q=1.1).

One can see that the minimum of the total flow corresponds to the number of θ_{id}. The analysis of the results obtained (see Fig. 5 and Table 1) allows to draw the same conclusion. As for the total flows corresponding to the maximum of thermodynamic power and the maximum of separative work of the cascade, they practically are undistinguishable. In addition, the difference in values of the optimum total flow defined by these criteria in comparison with $(\sum L/P)_{id}$ is equal to 3.9%; 4.1% and 4.4% for q = 1.1; 1.3 and 1.5, respectively. Obviously, in the case of numbers of separation elements big enough, the total flow in a multistage installation with the optimum cut numbers will be much lower in contrast to the non-optimum one. Finally, it will mean much lower capital expenditure for the construction of a separation installation.

CONCLUSIONS

1. An analytical solution for the component concentration distributions of a binary isotope mixture over a cascade with constant cut numbers is obtained.

2. It paved the way to analyze major mass transfer regularities in such a cascade using various efficiency criteria: the maximum value of thermodynamic power necessary for an isotope separation and the maximum value of separative work of the cascade. Besides, the cut number at the stages of the separation cascade was used as an optimization parameter. The optimization calculations made on the basis of the two efficiency criteria above gave similar results.

3. It is shown that the minimum of the total flow in the separation cascade with constant cut numbers at the stages corresponds to the value predicted by the theory of the ideal cascade. It gives a possibility to apply this theory to arbitrary enrichment coefficients at the cascade stages. The optimum total flows in the cascade obtained by means of two other efficiency criteria exceeds the value calculated with the help of the ideal cascade theory for approximately 4 - 4.5%.

LIST OF SYMBOLS

c	concentration entering a separation stage
c'	concentration in an up-flow leaving a stage
c''	concentration in a down-flow leaving a stage
c_F	concentration in a feed flow
c_P	concentration in a product flow
c_W	concentration in a waste flow
F	feed flow
f	feed flow point
G	flow of c component in feed flow
G'	flow of c component in up flow
G''	flow of c component in down flow
$L(s)$	current flow in separation cascade
$\sum L/P$	$\dfrac{\sum_{s=1}^{N} L(s)}{P}$
N	total number of stages in multistage installation

P	product fraction flow
q	overall separation factor
s	number of a current stage
W	waste fraction flow
θ	cut number
ρ	relative cut number, $(1-\theta)/\theta$
ω	root of the characteristic equation

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