

Practical Application of 1-D Sediment Transport Model

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(Received April 13, 2007; revised March 06, 2008)

Abstract

The aim of the paper is to present a numerical simulation of sediment transport in a mountain river. Two one-dimensional sediment transport models: RubarBE and Metoda are applied to predict variation of longitudinal bed profile along the river reach and changes in the cross-sectional geometry due to erosion or deposition of sediment. In the current paper, simulation of the sediment transport is applied for the experimental reach of the River Raba – a mountain tributary of the Vistula River. The reach is located from 81.829 km to 77.751 km upstream from the Dobczyce Dam. This part of the river was completely changed due to the project concerning the extension of the Kraków–Zakopane road, which is located next to the Raba River. The results of simulation of riverbed evolution, before and after river training carried out by both models, are analysed and discussed.

Key words: mountain rivers, riverbed erosion, 1-D model

Notations

- A – cross-sectional flow area (m^2),
- A_s – bed-material area (m^2),
- B_{akt} – the active width (m),
- C – coefficient of local losses,
- C_s – sediment transport capacity (m^3/s),
- D_{50} – median diameter of sediment (m),
- G – sediment discharge (N/s),
- g – acceleration due to gravity (m/s^2),
- h – water depth (m),
- J – friction slope,
- K – Manning-Strickler coefficient ($m^{1/3}/s$),
- k – ratio between the velocity of the main flow and the axis velocity of the lateral flow,
- La – active width (m),

- Q – water discharge (m^3/s),
- q – lateral water flow per unit of length (m^2/s),
- Q_s – sediment discharge (m^3/s),
- q_s – lateral sediment flow per unit of length (m^2/s),
- R – hydraulic radius (m),
- S_0 – slope of riverbed (–),
- S_f – slope of the energy line (–),
- t – time (s),
- x – streamwise coordinate (m),
- z – water surface elevation (m),
- β – the coefficient of quantity of movement,
- γ – specific weight of water (kg/m^3),
- ε – established acceptable closeness,
- ρ – density of water (kg/m^3),
- ρ_s – density of sediment (kg/m^3),
- Δx – distance between cross-sections (m).

1. Introduction

Sediment transport is one of the main process which occurs with the high intensity, especially in mountainous rivers. This natural process is still acting and may generate huge consequences on human activities or the environment.

The aim of current paper is simulation and analysis concerning the evolution of the erosions and depositions that occur along the experimental reach of the mountainous Raba River. The analysis of the riverbed morphology on this reach should be known, principally after river training.

The Raba River, in southern Poland, is a mountain tributary of the Vistula River. In this region, the topography of drainage catchments is highly varied. In the Carpathian rivers a high variety of water stages is observed: a rapid-growing flow appears, especially in spring and early summer while, in the dry season or during the long-lasting snow-cover on rivers, a low-flow period can be observed. In the Raba River, during the flood, discharge can reach the value $Q = 800 \text{ m}^3/\text{s}$ in the upper course, and about $Q = 1500 \text{ m}^3/\text{s}$ in the lower course. The Raba River is characterized by erosion and deposition process which occurs with varied intensity along the river course. The highest erosion process is observed particularly downstream from the Dobczyce Dam (km 60.000) (Lenar-Matyas and Łapuszek 2000).

In the current paper, we present simulation of the sediment transport and riverbed evolution for the reach which is located (Fig. 1) from 81.829 km to 77.751 km. The experimental reach is sited about 20 km to the south of Kraków (Cracow) and presently close to the important road Kraków–Zakopane. The river in the presented reach (near Stróża Town) was repeatedly straightened and narrowed during



Fig. 1. The experimental Raba River reach: 81.829–77.751 km

the 20th century. The hydraulic engineering activity was the most important factor of high erosion of riverbed, which appeared there especially in the 50th and 60th of the previous century.

In the year 2000 an idea of extension of the Kraków–Zakopane road located next to the river Raba was set up. According to this plan, the new project of partially replacing the main channel along the road was also applied. The engineering activity on the experimental reach started in spring 2003. Works are still going on throughout the whole reach, but the main channel is now formed completely. Therefore morphology of the evolution of the Raba River could be examined.

This paper is limited to computing and analysing the riverbed changes before and after river training. In order to predict variation of longitudinal bed profile along the river reach and changes in the cross-sectional geometry due to erosion or deposition of sediment, two one-dimensional sediment transport and riverbed evolution models Rubarbe and da are used. This paper recalls also the problems that may appear and the main processes and parameters concerned.

2. Formulation of the Problem

Most of the sediment transport occurs during high flows. The flood events should be modelled in a detailed way to assess the main morphological changes. This fact implies that unsteady flow modelling should be possible. The unsteadiness may be linked with the characteristics of the flow linked with the river basin but also with the evolution of the riverbed during the flood itself. Coupling of sediment transport modelling and flow modelling are thus necessary although for engineering studies, the approximation of steady flow can often be used if the flood lasts for a long time, and the bottom evolution is very slow. This point is also due to the difficulties of calibration of a sediment transport model that generally depends on the measurements performed before and after the flood event only.

The processes inside the river itself could then be followed with both small space and time steps. However, except for solving local questions (for instance around a structure), the main questions remain at the scale of the whole reach of a river (corresponding to the scale of the river basin) for which the river flow can be described by a 1-D model. It implies that, in most cases, a 1-D sediment transport model is the more relevant; it takes into account a smaller set of parameters, which also means a simpler way for model calibration (Paquier 2003).

The models used for the computation have two components: one to simulate the flow and a component to characterise the changes in river morphology due to erosion or deposition of sediment. The models rely on:

- de Saint Venant equations for water (Paquier 2003):

$$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial x} = q, \quad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\beta \frac{Q^2}{S} \right) + gS \frac{\partial z}{\partial x} = -g \frac{Q^2}{K^2 S R^{4/3}} + kq \frac{Q}{S}; \quad (2)$$

- equation for conservation of sediment mass (Paquier 2003):

$$(1 - p) \frac{\partial S_s}{\partial t} + \frac{\partial Q_s}{\partial x} = q_s \text{ and} \quad (3)$$

- sediment transport capacity relation (Meyer-Peter and Müller 1948):

$$C_s = \frac{8L_a \sqrt{g}}{(\rho_s - \rho) \sqrt{\rho}} (\rho J R - 0.047 D_{50} (\rho_s - \rho))^{3/2}. \quad (4)$$

Data requirements for our models are modest, involving only a few parameters. Thus, the models are relatively easy to calibrate and implement.

3. The RubarBE Software and METODA Model Description

The Hydrology and Hydraulics Research Unit of Cemagref has developed a 1-D model RubarBE, for predicting variation of longitudinal riverbed profile along rivers and changes in the cross-sectional geometry. METODA 1-D model is developed in the Institute of Water Engineering and Water Management of the Cracow University of Technology.

In both models, sediments are represent only by a mean diameter D_{50} . This parameter does not clearly fully describe the processes that occur in many channels such as armouring. Therefore, RubarBE represents sediments by a mean diameter D_{50} and a complementary parameter, the standard deviation σ . The standard deviation is assessed as the square root of the ratio between D_{84} and D_{16} .

In RubarBE model the space lag effects are taken into account by introducing the following equation:

$$\frac{\partial Q_s}{\partial x} = \frac{C_s - Q_s}{D_{char}}, \quad (5)$$

in which D_{char} is a distance that characterizes the ability of sediment transport to reach the value of the sediment transport capacity. For bed load transport in rivers, this value is generally very short (a few meters), which means that it is shorter than space step and thus can be neglected. In METODA model it is not taken into account at all.

The method for solving the set of equations in RubarBE model is based on several steps. First, de Saint Venant equations are solved by a Godunov type second order finite difference scheme that makes possible the calculation of flow variables even if critical flow appears (Paquier 1995). Then, the sediment transport capacity is calculated. Solving the spatial lag equation inside a cell is carried out according to the scheme shown on Figure 2, which calculates the sediment discharge downstream Q^{ds} from the upstream sediment discharge Q^{us} while distinguishing the sediments that are only transferred Q^{tra} to the ones that interfere with the sediments previously present in the cell (deposited Q^{dep} and eroded Q^{ero}).

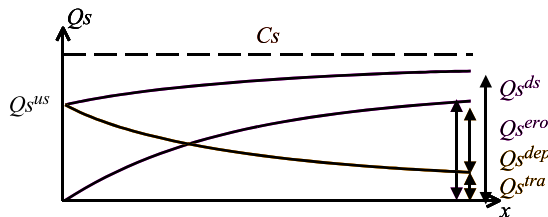


Fig. 2. Scheme for spatial lag calculation (Balayn 2001)

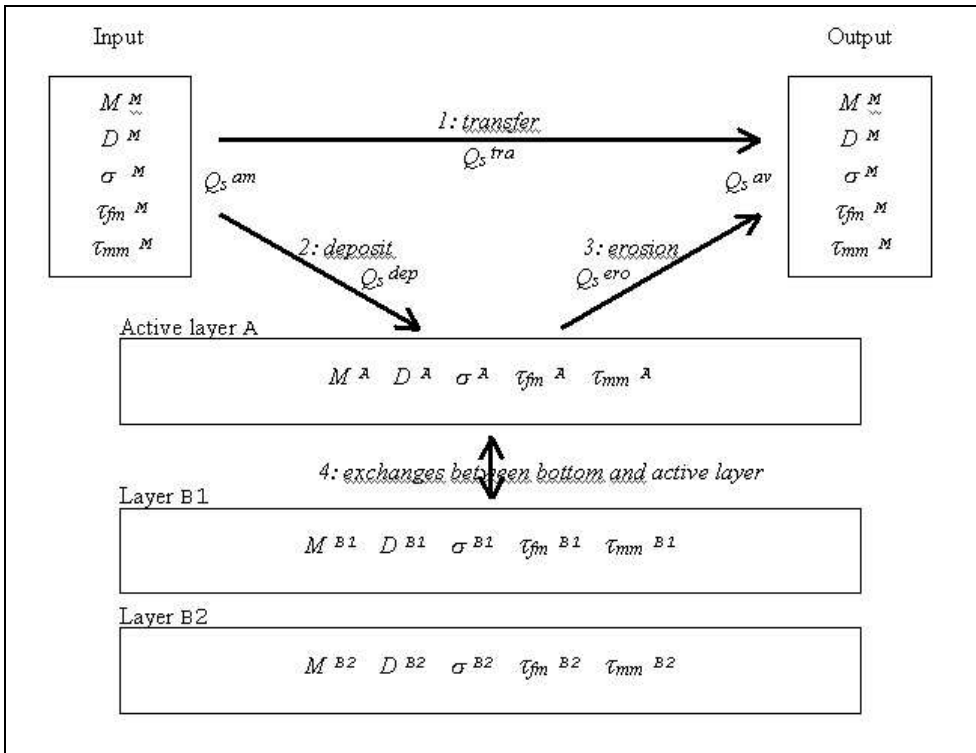


Fig. 3. Scheme of distribution of sediments inside one cell (Balayn 2001)

Then the sediment continuity (equation 3) is applied to every cell on the basis of Figure 3. This leads to a change of A_s that should be translated in a change of the shape of the cross-section. This change will then change the water elevation as the hypothesis of no change in the water depth and velocity is applied.

In Figure 3, the active layer that corresponds to the sediments that move during the time step has its thickness fixed from the sediment transport capacity, the velocity of the flow and the space step. The deposits and erosions occur when this active layer is respectively too thick or not thick enough.

Thus, inside one cell, a sedimentary compartment corresponds to a set of sediments that have a coherent behaviour and three compartments are defined:

- A compartment M^M of movable sediments: the contents of the water column where particles are moving at one time. We distinguish a compartment M_{am} of input sediments and a compartment M_{av} of output sediments.
- A compartment A of the active layer: a layer near the bed where sediment particles slide or roll at one moment during the time step.
- A compartment B of one or several substrate layers: it reflects historical deposition of sediments on the riverbed or undisturbed subsurface. Some of the layers can be created or can disappear.

It was mentioned, that the simplest one-dimensional models represent sediments only by a mean diameter D_{50} . This simplification clearly does not fully describe the processes that occur in many channels, as armouring or presence of mixed sediments of various sizes. Therefore, in RubarBE model a complementary parameter was added, the standard deviation σ , that appears convenient to describe grain size distribution in a river for which sediments are homogeneous (Shih and Komar 1990). Extra parameters of one compartment are the shear stress τ_{mm} for beginning of the movement and τ_{fm} shear stress at the end of the movement. Generally, these two last parameters are set equal and determined from D_{50} . During the phases 1 to 4 of Figure 3, sediments are mixed or shared into two fractions of different characteristics. The relations used are specific averages (equations (6), (7) and (8) for respectively the mass, the diameter and the standard deviation).

$$M = M_1 + M_2, \quad (6)$$

$$D = D_1^{(M_1/M_1+M_2)} \times D_2^{(M_2/M_1+M_2)}, \quad (7)$$

$$\sigma = \sigma_1^{(M_1/M_1+M_2)} \times \sigma_2^{(M_2/M_1+M_2)}, \quad (8)$$

in which indexes 1 and 2 refer to the quantities of sediments that are added and no index to the mixture, M is the mass, D the mean diameter and σ the standard deviation. These relations are the only ones for which addition of once a double mass and addition of twice a unit mass are strictly equivalent.

For sharing, the diameter D_2 of the coarser sediment can be calculated from a relation (9) introducing an additional parameter C , the expression of which is still to be determined although various relations were tested (Palussi re 2002) and concluded to stable and realistic results only for C less than 0.1. The standard deviation σ_2 can be calculated from Eq. (10) in order to keep D_{84} .

$$D_2 = D \times (1 + C(\sigma - 1)), \quad (9)$$

$$\sigma_2 = \sigma \frac{D}{D_2}. \quad (10)$$

For the change of the shape of the cross-section, various alternatives were tested:

- in the case of erosion: the entire movable bed under water lowers uniformly, possibly considering areas without any possible erosion;
- in the case of deposition: either, the volume of deposited sediment is spread across the channel width, starting from the bottom, or the deposits are identical for all the points below water;

- more complicated types of evolution have also been developed by using a calculation of local shear stress called the Merged Perpendicular method (Khadashenas and Paquier 1999); some of these methods make possible erosion and deposition at the same time in one cross-section (Paquier 2005).

METODA model is based also on the system of de Saint Venant equations for water (1), (2), equation for conservation of sediment mass (3) and sediment transport capacity relation (4), as in RubarBE model.

In the METODA model equations (1) and (2) are simplified by the assumption, that study reach of the river is in dynamic equilibrium at time T .

The dynamic equation for flow in METODA model is written for:

- steady flow:

$$S_0 - S_f = 0, \tag{11}$$

- nonuniform flow:

$$\frac{\partial}{\partial x} \left(\beta \frac{Q^2}{A} \right) + gA \frac{\partial z}{\partial x} + gAS_f = 0, \tag{12}$$

where the equation (12) is developed as:

$$z_{i+1} + \frac{\alpha_{i+1}}{2g} \left(\frac{Q}{A_{i+1}} \right)^2 = z_i + \frac{\alpha_i}{2g} \left(\frac{Q}{A_i} \right)^2 + h_e, \tag{13}$$

$$h_e = \Delta x S_f + C \left| \frac{\alpha_i v_i^2}{2g} - \frac{\alpha_{i+1} v_{i+1}^2}{2g} \right|. \tag{14}$$

Equation for conservation of sediment mass is solved using the explicit finite difference scheme (Fig. 4).

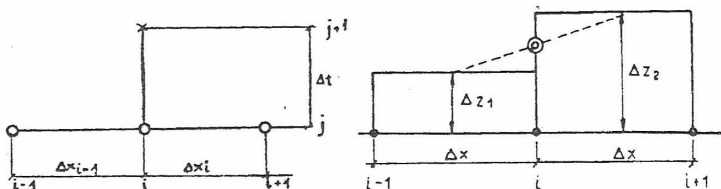


Fig. 4. The numerical scheme for the sediment continuity equation

The computation of riverbed deformation is based on the assumption, that changes of river bed between the studied cross-sections are linear. The value of the increment in the time step t_{j+1} is written by the formula:

$$\Delta z_i = \frac{-\Delta T}{\gamma} \left(\frac{q_{s_{i+1}}^j - q_{s_{i-1}}^j}{\Delta x_i} + \frac{q_{s_{i+1}}^j - q_{s_i}^j}{\Delta x_{i+1}} \right). \quad (15)$$

The ordinate of increment in time t_{j+1} is expressed as:

$$\Delta z_i = z_i^{j+1} - z_i^j. \quad (16)$$

The distance between the studied cross-sections can be varied, therefore, the ordinate of increment in time t , is described by the equation:

$$\Delta z_i = \frac{\Delta z_{i-1,i} \times \Delta x_{i-1,i} + \Delta z_{i,i+1} \times \Delta x_{i,i+1}}{\Delta x_{i-1,i} + \Delta x_{i,i+1}}. \quad (17)$$

The above formula is the basis for ordinate of increment in time $j + 1$ determination. The ordinate in each band of the cross-section is calculated as:

$$z_{k,i}^j = z_{k,i}^{j-1} - \frac{\Delta t}{\gamma \Delta x_i} (q_{s_{k,i}}^j - q_{s_{k-1,i}}^j). \quad (18)$$

In the study case of the natural cross-section, the established quantity of q_s in each band of $k - 1$ -cross-section is summed up, and afterwards the value is distributed uniformly on each band of k -cross-section, by the rule:

- sediments incoming, are distributed uniformly in each band, where sediments move,
- established ordinates of increment are averaged in each node between adjacent bands.

The ordinate in each band is established by the formula (Piwowarczyk-Ogórek 2003):

$$z_{k,i}^j = z_{k,i}^{j-1} - \frac{\Delta t}{2\gamma \Delta x_i} (q_{s_{k,i}}^j - q_{s_{k-1,i}}^j). \quad (19)$$

In METODA model, the system of equations of water and sediment is solved separately for each time step by the finite difference method, with the computation grid as of the Fig. 5.

The maximum value of time step is written by the formula (Ratomski 1983):

$$\max |t| = \frac{\gamma}{2} \varepsilon \Delta x \times \min \left(\frac{(B_{akt_i} + B_{akt_{i+1}})(h_i + h_{i+1})}{G_{i+1} - G_i} \right), \quad (20)$$

with the assumption, that the value of increment in i -cross-section cannot exceed the established acceptable value ε . If ΔT is exceeded, then the time interval is divided into n time intervals Δt .

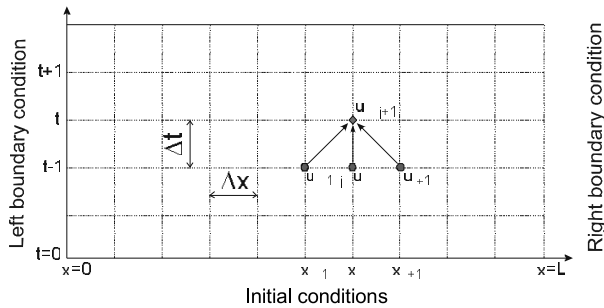


Fig. 5. Computation grid

The initial and boundary conditions are as follows:

- the initial condition for equations of water;
- for time $t = 0$: $Q_1 = Q(t) = \text{const}$; $z = f(x, t)$; $q_r = f(x)$;
- the initial condition for equation of sediments is the initial geometry of cross-section in time $t = 0$, with the assumption of steady flow throughout the whole reach in time $t = 0$.

The boundary conditions are established with the assumption, that the above river reach is in the hydrodynamic equilibrium.

- Left boundary condition:

The sediment discharge corresponds to the hydrograph of water flow for $x = 0$, $q_r = f(Q, t)$. Left terminal interval is prolonged by the value Δx_1 , behind the experimental river reach.

For $x = 0$: $z = z(t)$

$Q = Q(t)$ – flow hydrograph,

$q_r = q_r(Q, t)$ – sediment transport quantity incoming to the experimental reach,

$\Delta z(Q, t)$ – increment established in the right terminal cross-section,

- right boundary condition can be established for two cases:

1. on the end of the experimental reach hydraulic structure is located,
2. no structure, and the uniform flow, with interval, prolongation behind the experimental river reach.

For $x = L$: $z = z(Q, t)$ – from the right boundary condition,
as: $z_L = (Q, t)$,

$q_r = f(x, t)$ – established increment,

$\Delta z(Q, q_r, t)$ – established increment in the left terminal cross-section.

4. The Study Cases and Results of Computation

In order to study the changes of riverbed erosion in the experimental reach and to compare both models also (Piwowarczyk-Ogórek 2003), we carried out the simulation for the following cases:

- discharge $Q = 250 \text{ m}^3/\text{s}$ with the duration $t = 12$ hours,
- flood discharge hydrograph (Fig. 6) chosen from data set of 30 years (1971–1999),
- long-lasting flow sequence (years: 1961–1980).

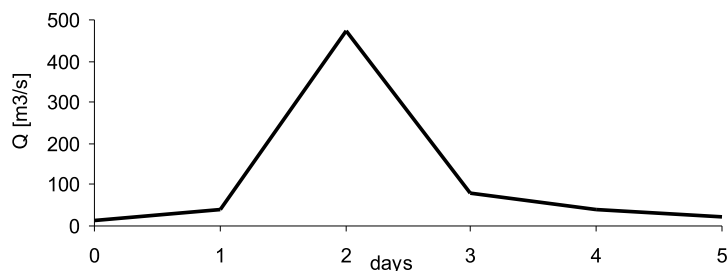


Fig. 6. Flood discharge hydrograph

The period of 20-years daily flows contains a great number of low flows which lasted for long time. Therefore, the period of the only high flows which could play part in the process of channel forming were selected by the rule as follows:

- for RubarBE model the sequence of high flows was established manually, judging by the well-known value of channel-forming flow for the studied reach of Raba River,
- in METODA the selection of channel-forming flows with the upper ones is established by the model.

44 prismatic cross-sectional channel geometry for the experimental river course before the project execution, were taken into account by the RubarBE and METODA models. The initial conditions for sediment transport is $Q_s = 0 \text{ kg/s}$, and the sediment boundary conditions are $Q_s = 0 \text{ kg/s}$. The downstream conditions are given by the rating curve for the hydraulic structure (km 77.751) which is located there.

The second part of our computation simulated sediment transport in the experimental reach after the river training. In that case, simulation was done also for 44 prismatic new cross-sectional channel geometry. In order to have a full simulation of the riverbed evolution, a computation was carried out for the more complete flow sequence (20 years).

An extensive comparison between results of calculations from RubarBE and from METODA with as similar as possible conditions was carried out, and the results of a few cases are shown here below.

Table 1. Verification of the computation results for $Q = 250 \text{ m}^3/\text{s}$ (Łopuszek et al 2003)

Km	Computed riverbed level		The real evolution of riverbed
	Δz [m] METODA	Δz [m] RubarBE	
81.501	0.099	0.51	Deposition: The reach is located just downstream from the bridge. The reach is straight and a slight deposition is observed.
81.421	0.26	0.3	
81.288	-0.156	-0.131	Erosion: The area of cross-section is characterized by strong erosion of the left bank.
81.144	-0.138	-0.283	
81.080	0.201	0.186	Deposition: The reach is rather straight with small layers of bed load observed.
81.861	0.011	0.036	
80.735	-0.0236	-0.0586	Strong process of erosion: The reach meanders and the strong eroded banks at the riverbed can be observed.
80.595	0.061	0.101	High deposition: This reach is straight, with the extensive layers of bed load located in the channel. It points, being the reach characterized by process of deposition.
80.517	0.095	0.0	
79.977	-0.122	-0.122	Erosion: The reach is located about 2300 m downstream of the bridge. It is characterized by high process of riverbed erosion.
79.893	-0.062	-0.057	
79.723	-0.0139	-0.0139	
79.091	0.034	0.039	Deposition: This reach is straight and regulated. The extensive layers of bed load are located in many parts of the channel. Which means, that this reach is characterized by process of deposition.
78.981	0.156	0.431	
78.873	-0.2886	-0.6836	Erosion: There observed soft meanders with the remarkable process of erosion on the left bank.
78.798	-0.1402	-0.0902	
78.309	0.023	0.023	High deposition: This reach is characterised by the bed load deposition. The extensive layers of bed load located in many parts of the channel point to the process of deposition is being high.
78.229	0.075	0.11	
78.170	0.172	0.332	
78.89	0.176	0.211	

The first comparison is performed for the computation carried out for discharge: $Q = 250 \text{ m}^3/\text{s}$, with the total simulation time of 12 hours and for 44 cross-sections before project execution (Fig. 7) (El Kadi et al 2003).

Computational results obtained by both models were verified by field observations carried out in 2001, before the project execution. The experimental reach is 4 km long with 44 cross-sections taken into computation. However, for simplicity, Table 1 contains verification performed only for the selected 20 cross-sections in

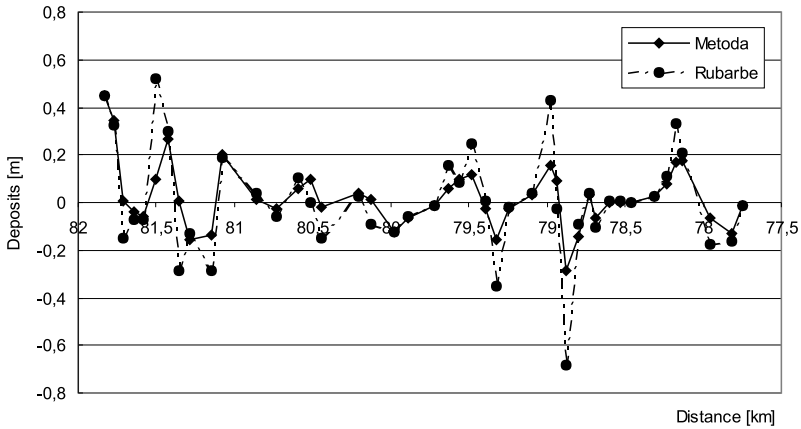


Fig. 7. Bottom level change for 44 cross-sections before project execution (for $Q = 250 \text{ m}^3/\text{s}$)

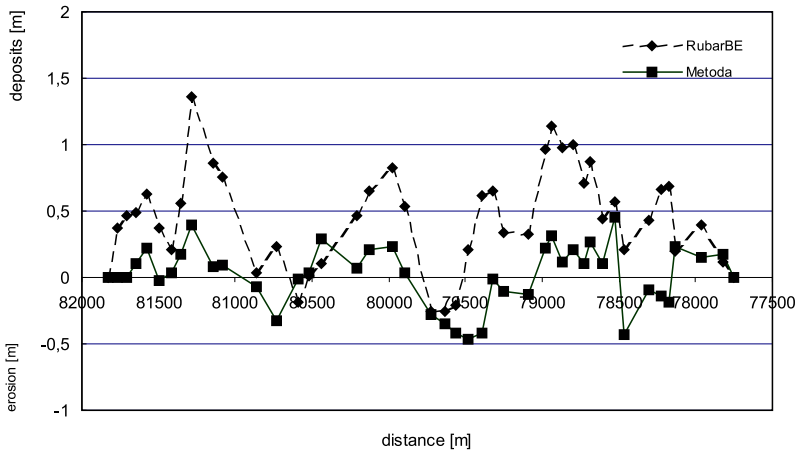


Fig. 8. Bottom level change for 44 cross-sections and long-lasting flow sequence (years: 1961–1980)

which clear conclusions have been established in the field. The agreement between results is such that erosion and deposition are located in the same cross-sections with the same order of magnitude (Łapuszek et al 2003).

The next comparison performed, is the result of the full flow sequence of 20 years (Fig. 8).

The very important part of our computation was analysing the impact of project execution on the future riverbed evolution. Sediment transport movement computations were carried out for the new cross-sections obtained after river training. Computations were made for the flood hydrograph (Fig. 6) and for discharge 250

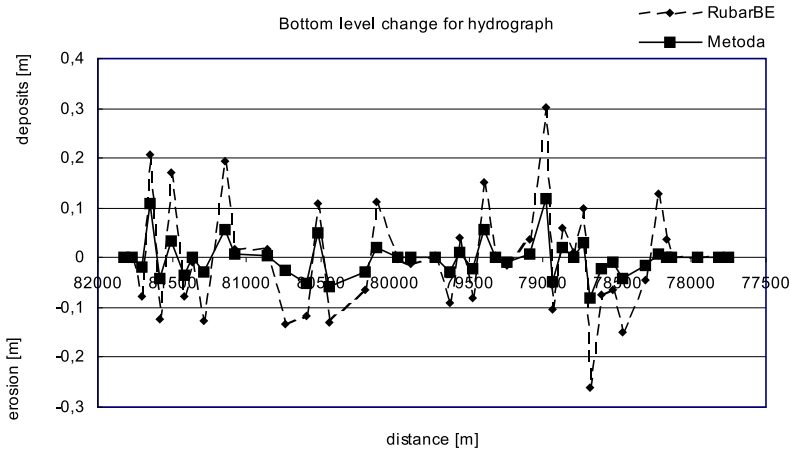


Fig. 9. Bottom level change for 44 cross-sections after river training and for the hydrograph

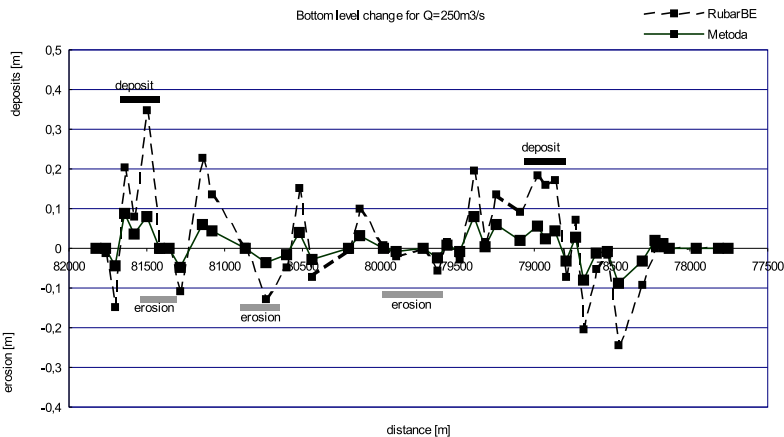


Fig. 10. Bottom level change for 44 cross-sections after river training and for $Q = 250 \text{ m}^3/\text{s}$

m^3/s . It was important to locate erosion and deposition areas particularly for a new channel's conditions (Figs. 9, 10).

The preliminary verification of obtained computation was carried out. And on Fig. 10 are pointed out the areas of slight erosion and deposition, which were noticed during field observation. However, the field observation and measurements must be continued in the future. Now, it is only possible to observe some slight tendency of riverbed evolution, because the river training activity has just been finished, and no flood appeared in our experimental reach till now.

5. Analysis of the Results and Conclusions

For a mountainous river such as the Raba, it is important to localise erosion or deposition areas, particularly after the river channel modification. METODA and RubarBE, two 1-D sediment transport models solving similar equations provide similar trends on the presented test cases of a 4 km Raba reach. For simple cases, the results are quite similar, but if the case is complicated to approach reality in a better way, the differences increase although the tendencies remain rather the same. These trends of erosion and deposition correspond to the field observations although more accurate comparison would have required both complete topographies on two dates and discharges during the period between these dates. Finally, the parallel use of the two models tends to validate the quality of the numerical schemes and of the results at least in term of trends.

For future study, it is important to make the new field measurements throughout the whole modified by the project, experimental reach just after the flood and also after a longer period of time.

The result of the full flow sequence of 20 years shows that tendency of erosion and sedimentation is rather different in both models. The reason of these differences might be affected by the unsteadiness of flow, because floods appeared frequently during a period of 20 years. Another reason could be the different method of the channel-forming and upper flows selection in both models. Moreover, the problem which could appear during very long period of computation, were digitization and the succession of different flow values in the models. Finally, it could lead to the different balance of sediment observed in both models. Therefore this part of computation should be repeated in the future.

References

- Balayn P. (2001) *Contribution of numerical modelling of the riverbed evolution caused by floods*, PhD Thesis, Lyon, Université Claude Bernard Lyon (in French).
- El Kadi K., Lenar-Matyas A., Paquier A., Piwowarczyk-Ogórek J. (2003) One-dimensional Sediment Transport Models: Application to Raba River, *3rd IAHR Symposium on River, Coastal and Estuarine Morphodynamics, RCEM*, Barcelona, 1–5 September 2003, 200–210.
- Khodashenas S. R., Paquier A. (1999) A geometrical method for computing the distribution of boundary shear stress across irregular straight open channels, *Journal of Hydraulic Research*, **37** (3), 381–388.
- Lenar Matyas A., Łapuszek M. (2000) The riverbed stabilization in the case of cutting off the sediment transport by the Dobczyce Dam, *10th International Conference on Transport and Sedimentation of Solid Particles*, Wrocław, Poland, 239–250.
- Łapuszek M., El Kadi K., Paquier A. (2003) One dimensional sediment transport model and its application to the mountainous river (Raba), *Selected Problems of Water Engineering, Politechnika Krakowska – Cemagref – results of cooperation*, 9–11 October 2003, Seminary, Cemagref Editions 2004, BP 44, 92163 Antony, France, 86–97.
- Meyer-Peter E., Müller R. (1948) Formulas for bed-load transport, *Report on second meeting of IARH, IAHR*, Stockholm, 39–64.

- Palussière S., (2002) *Transport de sédiments en rivière, Validation 1D et 2D, Fluid Mechanics*, Université Claude Bernard Lyon, Lyon.
- Paquier A. (1995) *Modelling and simulation of the wave propagation developed by breaking of the dam*, PhD Thesis, Saint-Etienne: Université Jean Monnet (in French).
- Paquier A. (2003) What are the problems to be solved by a 1-D river sediment transport model? Example of RubarBE software, *Selected Problems of Water Engineering, Politechnika Krakowska – Cemagref – results of cooperation*, 9–11 October 2003, Seminary, Cemagref Editions 2004, BP 44, 92163 Antony, France, 75–85.
- Piwowarczyk-Ogórek J. (2003) *Closed to the nature river cross-section design in the case of the sediment transport equilibrium*, PhD thesis, Cracow University of Technology (in Polish).
- Ratomski J. (1983) The hydraulic method of predicting the bedload sedimentation in the reservoirs, Cracow University of Technology, *Zeszyty Naukowe*, No. 4, Kraków (in Polish).
- Shih S. M., Komar P. D. (1990) Hydraulic controls of grain-size distributions of bedload gravels in Oak Creek, Oregon, USA, *Sedimentology*, **37**, 367–376.