

# Multi-threshold signature

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**Abstract**—The work presents a new signature scheme, called the multi-threshold signature, which generalizes the concept of multisignature and threshold signature. This scheme protects the anonymity of signers in a way the group signature does – in exceptional circumstances the identities of signers may be revealed. Due to the new party – completer, in our scheme the threshold size may vary together with the message to be signed. The presented scheme is based on the RSA signature standard, however other signature standards might be applied to it as well.

**Keywords**—public key cryptography, threshold signature, multisignature, secret sharing.

## 1. Introduction

Threshold and multiparty cryptography represent a wide and important area of the modern cryptography. The large part of it deals with the signature schemes such as threshold signatures and multisignatures.

Threshold signatures (c.f. [3, 9]) allow any group of  $l$  users to create a signature provided  $l \geq t$  (where  $t$  is a threshold level, fixed in advance). The multisignature allow any group of members to sign the given message. The identities of signers are recognized in the verification phase and then the decision if the signature is accepted is made (see [1, 4–6]). The verification of the signature applies the public keys of corresponding signers.

This paper is motivated by the following problem: given the group  $G$  of cardinality  $l$  and the pair  $(m, t)$  we are interested in the cryptographic scheme that allows any subgroup of at least  $t$  members to sign the message  $m$ . In distinction to the common  $(t, l)$ - threshold type scheme, here the value of  $t$  is not fixed in advance, but may vary together with the message  $m$  to be signed. Thus it might be very useful in applications, where the number of members required to agree upon the given document depends for instance on the document's "priority".

Another motivation is to propose the flexible signature scheme, which according to the requirement is anonymous or admits the signer's identification. This flexibility was not the subject of the previous papers, which generally speaking treat both solutions in separate schemes (c.f. [2] for example). From the practical point of view this ability seems to be significant in applications, and the proposed scheme provides the essential computational savings by joining both options within one cryptographic scheme. Therefore as an input for the signing algorithm is the triple  $(m, t, b)$ , where  $b \in \{0, 1\}$  points out if the signature should be anonymous or with the signer's identification. The resulting signature is to be verified by any user apply-

ing the public key related to  $G$ . Similarly as in the conventional threshold signature scheme we require, that any subgroup of cardinality less than  $t$  is not able to generate the valid (i.e., accepted in the verification phase) signature, attached to the pair  $(m, t)$  (in fact it is not able to obtain any nontrivial information about the group  $G$  secret value related to its public key).

In the conventional threshold signature the group public key corresponds to the given value of the threshold size  $t$ . The idea of our solution relies on the enlarging somewhat the original group  $G$ , so that the public key corresponds to the bigger threshold size  $t'$ . Then the additional shares (handled by the additional (trusted) party  $C$ ) will ensure the valid threshold size  $t \leq t'$  of the original signer's group. One could extend the above idea considering not necessarily trusted completer (e.g.,  $C$  being another group of signers). Such a development in direction of dynamic groups was considered in [10]. The presented scheme is based on the RSA [7] cryptosystem, and the Shamir secret sharing protocol [8]. The paper contains the detailed description of the corresponding multi-threshold signature scheme and the proof of its correctness.

## 2. General system model

### 2.1. Participants

We assume there are three parties involved in the protocol:

1. Group  $G = \{P_1, P_2, \dots, P_K\}$ .
2. The trusted dealer  $D$  responsible for the generation of private and public key of  $G$  and the corresponding shares for the group members  $P_1, P_2, \dots, P_K$  and completer  $C$ .
3. The trusted completer  $C$  responsible for flexibility of the threshold level.

We assume that the dealer  $D$  is connected with the members  $P_i$  and the completer  $C$  by the secure channels. Communication between  $C$  and members of  $G$  goes through group message board ( $GMB$ ) where all the partial signatures are published (only  $C$  and  $G$  have an access to it).

### 2.2. Notation

Throughout the paper  $N$  is a positive integer such that:  $N = pq$ , where  $p = 2p' + 1$ ,  $q = 2q' + 1$  and  $p, q, p', q'$  are prime numbers satisfying  $\min(p'q') > 2K$ , where  $K = |G|$ .

By  $\lambda$  we denote the Carmichael lambda function defined as

$\lambda(\prod_p(p^{\alpha_p})) = \text{lcm}_p \lambda(p^{\alpha_p})$ , where:

$$\begin{aligned} \lambda(2) &= 1 & \lambda(2^\alpha) &= 2^{\alpha-2} \text{ for } \alpha \geq 3, \\ \lambda(4) &= 2 & \lambda(p^\alpha) &= p^{\alpha-1}(p-1) \text{ if } p \text{ is an odd prime number.} \end{aligned}$$

Conventionally the elements  $e$  and  $d$  are mutually inverse elements in  $Z_{\lambda(N)}^*$ , i.e.,  $ed \equiv 1 \pmod{\lambda(N)}$ .

We assume that every member  $P_i \in G$  is equipped with the RSA keys  $(N_i, e_i, d_i)$ , respectively, needed for the authentication process within corresponding parties or members. To assure the uniqueness of  $m'$  at the end of the verification process we assume that  $N_1 < N_2 < \dots < N_K$ :

$$A_i = \prod_{\substack{j=1 \\ j \neq i}}^{2K} (x_i - x_j) \pmod{\lambda(N)} \text{ for } i = 1, 2, \dots, 2K$$

( $x_i$  are numbers assigned to  $P_i$ ).

Moreover we let  $A'_i = A_i / 2^{\bar{\omega}_i}$ , where  $\bar{\omega}_i$  is the highest power of 2 dividing  $A_i$  and  $\bar{\omega} = \max_{i \in I} \bar{\omega}_i$ .

Throughout the paper  $h$  will denote a given secure hash function.

### 3. Initialization phase

In the initialization phase the dealer  $D$  performs the following steps:

1. Generates the key pair  $(d, e)$  and a random polynomial:  

$$f(x) = d + c_1x + c_2x^2 + \dots + c_Kx^K \in Z_{\lambda(N)}^*[x].$$
2. Computes the shares  $s_i = f(x_i)(A'_i)^{-1} \pmod{\lambda(N)}$  and sends them to  $P_i$  ( $i \leq K$ ) and to  $C$  (for  $K+1 \leq i \leq 2K$ ).

*Remark 1.* Since  $\min(p', q') > 2K$  the odd numbers  $A'_i$  are invertible  $\pmod{\lambda(N)}$ .

3. Selects  $g \in Z_N^*$  of order equal to  $\lambda(N)$  and sends  $g^{-1} \pmod{N}$  and  $z_i = (g^{s_i})^{-1} \pmod{N}$  to the completer  $C$ .
4. He publishes the group public key  $gpk = (N, e, \bar{\omega})$ .

### 4. The anonymous signing phase

Assume that the tuple  $(m, t, b)$  ( $m$  is the message,  $t \in \{1, 2, \dots, K\}$  is the threshold level and  $b \in \{0, 1\}$  points out the signature type (anonymous or with signers identification)), is given to  $G$  and  $C$  in order to be signed by a given subgroup of  $G$ . Then the following steps are performed:

1. **Completer's computation.** The completer  $C$  computes  $m^* = h(m, t, b)$  and applies the partial signa-

ture generation algorithm to compute and publish in the *GMB* the following partial signatures:  $\sigma_{i_{t+1}}(m^*)$ ,  $\sigma_{i_{t+2}}(m^*), \dots, \sigma_{i_{K+1}}(m^*)$  (where  $\sigma_i(m^*) = (m^*)^{s_i} \pmod{N}$ ) together with the sequence  $i_{t+1}, i_{t+2}, \dots, i_{K+1}$  of terms contained in the interval  $(K, 2K]$ .

2. **Group signing.** The group members who decide to sign  $m$ , compute  $m^* = h(m, t, b)$  and publish their partial signatures  $\sigma_{i_1}(m^*), \sigma_{i_2}(m^*), \dots, \sigma_{i_t}(m^*)$  ( $1 \leq i_1 < i_2 < \dots < i_t \leq K$ ) in the *GMB*.
3. **Partial signature verification.** The completer selects a random  $r \in Z_{\lambda(N)}$  computes  $v^* = (\frac{m^*}{g})^r \pmod{N}$  and sends it to the members participating in the signature generation. Next he computes  $v_j = (z_j \sigma_j(m^*))^r \pmod{N}$ . Each member compute  $v_j^* = (v^*)^{s_j} \pmod{N}$  and sends it to the completer. Completer accepts the partial signature  $\sigma_j$  if and only if  $v_j = v_j^*$ .
4. **Generation of the full signature.** With the aid of the share combining algorithm the  $K+1$  valid signatures  $\sigma_{i_1}(m^*), \sigma_{i_2}(m^*), \dots, \sigma_{i_{K+1}}(m^*)$  allow any delegated signer to compute the anonymous signature  $((m, t, 0), \sigma_B(m^*))$ , where:

$$B = \{i_1, i_2, \dots, i_t, i_{t+1}, \dots, i_{K+1}\},$$

$$\sigma_B(m^*) = \prod_{j \in B} \sigma_{i_j}^{a_j}(m^*) \pmod{n} \text{ and}$$

$$a_i = 2^{\bar{\omega} - \bar{\omega}_i} \prod_{\substack{j \in B \\ j \neq i}} (0 - x_j) \prod_{j \notin B} (x_i - x_j)$$

is the appropriate Lagrange coefficient for the group  $B$ .

When the signature  $\sigma_B(m^*)$  (verified by any signer using *gpk*) occurs in the *GMB*, the anonymous signing is finished and  $\sigma_B(m^*)$  is published.

### 5. The authorization phase

In the following part the members  $P_i \in B$  authorize subsequently their signature using the private keys  $d_i$ . We remark that the description of  $B$  contains the subscripts of the corresponding signers. They perform the following steps:

1.  $P_1$  computes the message  $m' = h(m^*, \sigma, B)$ , signs it using his private key  $d_1$  and sends the obtained ciphertext  $\delta_1 = (m')^{d_1} \pmod{N_1}$  to the second member  $P_2$ .
2.  $P_2$  verifies if  $(\delta_1)^{e_1} \equiv m' \pmod{N_1}$  if so, he computes  $\delta_2 = (\delta_1)^{d_2} \pmod{N_2}$  and sends it to  $P_3$  (otherwise he publishes in *GMB* information about this disagreement and stops the protocol).
3. Similarly  $P_3$  verifies the obtained ciphertext  $\delta_2$  using the public keys  $(e_2, N_2)$  and  $(e_1, N_1)$ , respectively, computes  $\delta_3 = (\delta_2)^{d_3} \pmod{N_3}$ , sends it to  $P_4$  and so on.

4. The last member  $P_t \in B$  verifies  $\delta_{t-1}$  using the public keys:

$(e_{t-1}, N_{t-1}), (e_{t-2}, N_{t-2}), \dots, (e_1, N_1)$  and, if the verification is correct, he computes  $\delta_t = (\delta_{t-1})^{d_t} \bmod N_t$  and publishes it in  $GMB$ .

5. The full signature of the message  $m$  is the 5 – tuple:  $((m, t, \sigma), B, \delta_t)$ , where  $P_i \in B$  are ordered as above.

According to the requirements, the chosen member of the group  $G$  publishes the anonymous signature  $((m, t, 0), \sigma)$  or the full signature  $((m, t, 1), \sigma, B, \delta_t)$ .

The anonymous signature does not imply any information about the identities of the members of  $B$ . It proves only that at least  $t$  members of group  $G$  have signed the document  $m$ .

## 6. The verification phase

After receiving the anonymous signature  $(m, t, \sigma)$  the verifier uses the group public key  $(e, N, \varpi)$  to compute  $m^* = h(m, t, b) \bmod N$  and then accepts it provided  $\sigma^e \equiv (m^*)^{2^\varpi} \bmod N$ . To verify the full signature  $(m, t, \sigma, B, \delta_t)$  one first computes  $m' = h(m^*, \sigma, B)$  and then accepts the full signature provided  $(\dots((\delta_t^{e_t} \bmod N_t)^{e_{t-1}} \bmod N_{t-1}) \dots)^{e_1} \equiv m' \bmod N_1$ .

*Theorem 1.* The correctly created signature will be accepted in the verification phase.

*Proof.* First let us consider the anonymous signature  $(m, t, \sigma)$ . It is sufficient to prove that  $\sigma^e \equiv (m^*)^{2^\varpi} \bmod N$ .

By definition we have  $\sigma = \prod_{i \in B} \sigma_i^{a_i} = (m^*)^{\sum_{i \in B} s_i a_i}$ , where:

$$s_i = f(x_i)(A'_i)^{-1} \bmod \lambda(N), \quad (1)$$

$$a_i = 2^{\varpi - \varpi_i} \prod_{j \in B, j \neq i} (0 - x_j) \prod_{j \notin B} (x_i - x_j). \quad (2)$$

It remains therefore to show that  $\sum_{i \in B} s_i a_i = 2^\varpi d = 2^\varpi f(0) = F(0) \bmod \lambda(N)$ .

In this connection we apply the Lagrange interpolation formula for

$F(x) = 2^\varpi f(x) \in Z_{\lambda(N)}^*[x]$  whose graph passes by the points  $(x_{i_1}, F(x_{i_1}))$ ,

$(x_{i_2}, F(x_{i_2})), \dots, (x_{i_{K+1}}, F(x_{i_{K+1}}))$ , where  $B = \{i_1, i_2, \dots, i_t, i_{t+1}, \dots, i_{K+1}\}$ .

We have  $F(x) = \sum_{i \in B} f(x_i)(2^\varpi \Lambda_i(x)) \bmod \lambda(N)$ , where:

$$\begin{aligned} 2^\varpi \Lambda_i(x) &= 2^{\varpi - \varpi_i} \prod_{j \in B, j \neq i} \left( 2^{\varpi_i} \frac{x - x_j}{x_i - x_j} \right) = \\ &= 2^{\varpi - \varpi_i} \prod_{j \in B, j \neq i} (x - x_j) \prod_{j \notin B} (x_i - x_j) \prod_{\substack{j=1 \\ j \neq i}}^{2t} \frac{2^{\varpi_i}}{(x_i - x_j)} \bmod \lambda(N). \end{aligned}$$

Hence in view of Eqs. (1) and (2) and definition of  $A'_i$  we obtain:

$$\begin{aligned} F(0) &= \sum_{i \in B} f(x_i)(A'_i)^{-1} \cdot 2^{\varpi - \varpi_i} \prod_{j \in B, j \neq i} (0 - x_j) \prod_{j \notin B} (x_i - x_j) = \\ &= \sum_{i \in B} s_i a_i \bmod \lambda(N), \text{ as claimed.} \end{aligned}$$

To verify the full signature  $(m, t, \sigma, B, \delta_t)$  we use the bijectivity of transformation  $x \mapsto x^{d_i} \bmod N_i$  ( $1 \leq i \leq t$ ) and the inequalities:

$N_1 < N_2 < \dots < N_K$  (that assure the uniqueness of  $m'$  at the end of the verification process).

Taking  $\delta_t$  to the power  $e_t$  we obtain the unique  $\delta_{t-1} \bmod N_{t-1}$  then (using  $e_{t-1}$ ) the unique  $\delta_{t-2} \bmod N_{t-2}$  and finally the unique

$$((m')^{d_1})^{e_1} = m' \bmod N_1 \text{ as required.} \quad \blacksquare$$

## 7. Conclusions

Two basic benefits of the presented scheme are the scalability (in threshold size) and generality – it might be useful for the applications typical for the threshold-type signatures or multisignatures.

The final output is the pair: anonymous  $G$ -signature and the full signature (containing the signers' identifications).

The completer can be regarded as a well protected machine which for the input value  $(m, t, b)$  outputs the corresponding partial signatures.

As proved in [10] the multi-threshold device with  $C$  regarded as another group of signers could be developed in the direction of dynamic groups signatures schemes.

## Appendix – an example

1. System parameters:

$$\begin{aligned} p &= 23 & q &= 47 & N &= 1081 & \lambda(N) &= 506 \\ p' &= 11 & q' &= 23 & t &= 3 & \min(p', q') &> 6 = 2t \\ e &= 13 & d &= 39 & m^* &= 7 & G &= \{P_1, P_2, P_3\} \end{aligned}$$

2. Dealer generates random polynomial:

$f(x) = 3x^3 + 5x^2 + 7x + 39$  and sets  $x_i = i$  which implies:

$$A_i = \prod_{\substack{j=1 \\ j \neq i}}^6 (i - j) \bmod 506$$

3. We have:

$$\begin{aligned} A_1 &= 386 & A'_1 &= 193 & \varpi_1 &= 1 & (A'_1)^{-1} &= 409 & f(1) &= 54 \\ A_2 &= 24 & A'_2 &= 3 & \varpi_2 &= 3 & (A'_2)^{-1} &= 169 & f(2) &= 97 \\ A_3 &= 494 & A'_3 &= 247 & \varpi_3 &= 1 & (A'_3)^{-1} &= 295 & f(3) &= 186 \\ A_4 &= 12 & A'_4 &= 3 & \varpi_4 &= 2 & (A'_4)^{-1} &= 169 & f(4) &= 339 \\ A_5 &= 482 & A'_5 &= 241 & \varpi_5 &= 1 & (A'_5)^{-1} &= 21 & f(5) &= 68 \\ A_6 &= 120 & A'_6 &= 15 & \varpi_6 &= 3 & (A'_6)^{-1} &= 135 & f(6) &= 403 \\ \varpi &= \max_i \varpi_i = 3 \end{aligned}$$

4. Dealer, using the table above, computes:

$$s_i = f(i) * (A_i')^{-1} \pmod{\lambda(N)}$$

$$s_1 = (54 * 409) \pmod{506} = 328$$

$$s_2 = (97 * 169) \pmod{506} = 201$$

$$s_3 = (186 * 295) \pmod{506} = 222$$

$$s_4 = (339 * 169) \pmod{506} = 113$$

$$s_5 = (68 * 21) \pmod{506} = 416$$

$$s_6 = (403 * 135) \pmod{506} = 263$$

5. Dealer selects  $g = 3$  and sends to the completer the following values:

$$g^{-1} \pmod{N} = 3^{-1} \pmod{1081} = 721 \text{ and}$$

$$z_1 = (g^{s_1})^{-1} \pmod{N} = 331$$

$$z_2 = (g^{s_2})^{-1} \pmod{N} = 259$$

$$z_3 = (g^{s_3})^{-1} \pmod{N} = 639$$

$$z_4 = (g^{s_4})^{-1} \pmod{N} = 949$$

$$z_5 = (g^{s_5})^{-1} \pmod{N} = 538$$

$$z_6 = (g^{s_6})^{-1} \pmod{N} = 647$$

6. We assume that  $m^* = h(m, 2, 0) = 7$  and  $B = \{2, 3, 4, 6\}$ .

7.  $P_2$  and  $P_3$  generate and send to the completer their partial signatures:

$$\sigma_2 = (m^*)^{s_2} \pmod{N} = 7^{201} \pmod{1081} = 711$$

$$\sigma_3 = (m^*)^{s_3} \pmod{N} = 7^{222} \pmod{1081} = 3$$

8. Completer verifies partial signatures created by  $P_2$  and  $P_3$ .

He selects  $r = 5$  and sends  $v^*$  to  $P_2$  and  $P_3$ , where

$$v^* = (m^* g^{-1})^r \pmod{N} = (7 \cdot 721)^5 \pmod{1081} = 732.$$

Next the completer computes:

$$v_2 = (z_2 \cdot \sigma_2(m^*))^r \pmod{N} = (259 \cdot 711)^5 \pmod{1081} = 948$$

$$v_3 = (z_3 \cdot \sigma_3(m^*))^r \pmod{N} = (639 \cdot 3)^5 \pmod{1081} = 16$$

9. Members  $P_2$  and  $P_3$  compute and send to the completer:

$$v_2^* = (v_2)^{s_2} = 732^{201} \pmod{1081} = 948$$

$$v_3^* = (v_3)^{s_3} = 732^{222} \pmod{1081} = 16$$

10. Completer accepts  $\sigma_2$ ,  $\sigma_3$  and creates two missing partial signatures:

$$\sigma_4 = (m^*)^{s_4} \pmod{N} = 7^{113} \pmod{1081} = 964 \text{ and}$$

$$\sigma_6 = (m^*)^{s_6} \pmod{N} = 7^{263} \pmod{1081} = 79$$

11.  $P_2$  (as a delegated user) computes the interpolation coefficients:

$$a_2 = 2^1(0-3)(0-4)(0-6)(2-1)(2-5) \pmod{506} = 216$$

$$a_3 = 2^2(0-2)(0-4)(0-6)(3-1)(3-5) \pmod{506} = 262$$

$$a_4 = 2^1(0-2)(0-3)(0-6)(4-1)(4-5) \pmod{506} = 216$$

$$a_6 = 2^0(0-2)(0-3)(0-4)(6-1)(6-5) \pmod{506} = 79$$

and finally he computes the anonymous signature:

$$\sigma = \prod_{i \in B} \sigma_i^{a_i} =$$

$$(711^{216} * 3^{262} * 964^{216} * 79^{386}) \pmod{1081} = 354$$

12. To verify the signature  $((m, 1, 0), 354)$  we use the public key  $(1081, 13, 3)$  and compute:

$$\sigma^e \pmod{N} = 354^{13} \pmod{1081} = 909$$

$(m^*)^{2^{\sigma}} \pmod{N} = 7^8 \pmod{1081} = 909$  and accept the signature.

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