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Viscoelastic Model of Waterhammer in Single Pipeline – Problems and Questions

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Abstract

In the paper, viscoelastic model of waterhammer in a single polymer pipeline is analysed. The theoretical background of viscoelastic behaviour of the structure is shown and the mathematical model of waterhammer in a polymer pipeline is presented. The main problems connected with applying the model are discussed. The main emphasis is on the question of parameter estimation. Important aspects of wave speed calibration are presented. Estimation of a second group of parameters – retardation time and creep compliance values – was analysed. Problems and questions connected with the number of parameters, methods of estimation, potential non-uniqueness of the solution and accuracy of obtained calculations were discussed.

Key words: waterhammer, viscoelasticity, wave speed, relaxation time, creep compliance

Notations

a – wave speed,
A – cross section area,
c_1 – constant dependent on the pipe fastening,
D – internal pipe diameter,
e – thickness of the pipe walls,
E – Young's modulus,
E_0 – instantaneous tensile modulus,
E_i – tensile modulus of i-th Kelvin-Voigt element,
E^* – complex tensile modulus,
g – acceleration due to gravity,
H – water head,
J – tensile compliance of the spring; creep function,
K – liquid bulk modulus of elasticity,
N – number of elements in viscoelastic model of the structure,
p – pressure,

Q – discharge,
r – internal radius of the pipe,
t – time,
T – oscillation period,
v – average in cross-section velocity of the stream,
x – distance,
ε – strain; unitary elongation of the internal circumference of the pipe,
η – viscosity coefficient,
θ, ψ – parameters in four-point scheme,
λ – linear friction factor,
ρ – density of liquid,
σ – stress,
τ – time of retardation,
ω – circular frequency.

1. Introduction. Viscoelastic Behaviour of the Structure

The behaviour of polymers, understood as a strain reaction for applied stress, is the consequence of their characteristic structure. In this specific construction each of the molecules has the form of flexible thread and may ceaselessly change the shape of its contour, curling and twisting with change of energy (Ferry 1965). Such complex structure determines viscoelastic effects in which each macroscopic strain is accompanied with complicated strains in molecular structure. In consequence, the reaction of polymers to the applied load is totally different from the behaviour of elastic materials, e.g. steel.

The description of viscoelastic behaviour of the body is usually developed on the basis of one of two approaches (Aklonis et al 1972). The first is connected with so-called "mechanical analogues". The group of models basing on this kind of approach refer to the combinations of mechanical elements, usually springs and dashpots, which more or less faithfully reproduce the viscoelastic response of real systems. The behaviour of the body is then predicted on the basis of mechanical parameters of these elements.

The second group of models is developed on the basis of molecular theories and the motion of the representative form of molecules in viscous medium is deducted (Aklonis et al 1972). In this case, the behaviour of the structure is predicted on the basis of molecular parameters. As the authors prove, the two approaches, at least in the case when polymer structure is analysed, are equivalent.

The model of waterhammer in the pipeline of the walls made of material of viscoelastic type is developed according to the approach of the first type. The response of the polymer pipeline walls during waterhammer refers to the behaviour of a combination of springs and dashpots and the analysis is based on the mechanical properties of these elements, described by mechanical parameters and basic equations governing stress-strain relations. The springs in the model are treated as pure Hookean elements responsible for the elastic behaviour of the structure, and are described by the equation:

$$\sigma = E\varepsilon, \tag{1}$$

where σ represents stress, ε – strain and E is Young's modulus.

The linear viscous response of the structure, characteristic for liquids, is represented by the behaviour of dashpots (pistons in cylinders), each one described by Newton's law:

$$\sigma = \eta \frac{\partial \varepsilon}{\partial t}, \tag{2}$$

where t is time and η a viscosity coefficient.

On the basis of these elements several viscoelastic models may be derived, according to the assumed combination of elements. The most basic models include Maxwells model and the Voigt model.

In Maxwells model the structure is represented by a set of a spring and dashpot in series (Fig. 1a, Reiner 1958, Ferry 1965, Aklonis et al 1972). The instantaneous tensile modulus E characterizes elastic behaviour of the spring, while η – the viscosity of a liquid in the dashpot is responsible for viscous response of the element. The relation between the parameters may be stipulated as (Aklonis et al 1972):

$$\eta = \tau E, \tag{3}$$

where τ – the proportionality constant is known as relaxation time of the element.

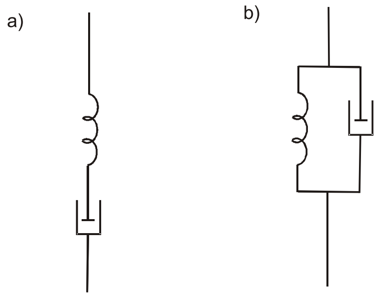

Fig. 1. Basic models of viscoelastic behaviour of the structure: a) Maxwell model, b) Voigt model

The equation of the motion of the Maxwell element is (Reiner 1958, Ferry 1965, Aklonis et al 1972):

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}. \tag{4}$$

According to the type of experiment, the solution of Eq. 4 leads to different results describing the behaviour of the body in different situations. The elementary tests are carried out with the assumption that the stress varies sinusoidally with angular frequency f ($\omega = 2\pi f = 2\pi/T$, where T is oscillation period), and can be decomposed on two components – *real*, which is in phase with strain, and *imaginary*, which is $\pi/2$ out of phase with strain. The typical test results are shown in Tab. 1.

Table 1. Response of Maxwell and Voigt models to elemental tests (on the basis of Aklonis et al 1972)

Experiment	Maxwell element	Voigt element
Creep test	$J(t) = J + t/\eta$	$J(t) = J(1 - e^{t/\tau})$
Stress relaxation	$E(t) = E\ e^{-t/\tau}$	$E(t) = E$
Sinusoidal dynamic experiments	$J^* = J' - iJ''$ $J' = J$ $J'' = \dfrac{1}{\omega\eta}$ $E^* = E' + iE''$ $E' = \dfrac{E\omega^2\tau^2}{1+\omega^2\tau^2}$ $E'' = \dfrac{E\omega\tau}{1+\omega^2\tau^2}$	$J^* = J' - iJ''$ $J' = \dfrac{J}{1+\omega^2\tau^2}$ $J'' = \dfrac{J\omega\tau}{1+\omega^2\tau^2}$ $E^* = E' + iE''$ $E' = E$ $E'' = \omega\eta$
where: $J = 1/\ E$ – tensile compliance of the spring (Aklonis et al 1972), creep function (Covas et al 2004) J^* – complex tensile creep compliance; E^* – complex tensile modulus J', E' – real components of J^* and E^* J'', E'' – imaginary components of J^* and E^*		

Contrary to the Maxwell element, the Voigt model is based on parallel structure of spring and dashpot (Fig. 1b). The stress then is the sum of stresses of two individual elements and the fundamental equation is:

$$\sigma(t) = \varepsilon(t)E + \eta\frac{d\varepsilon(t)}{dt}. \tag{5}$$

The solution of Eq. 5 for elemental tests is shown in Tab. 1.

The basic models mentioned above are not usually sufficient to describe viscoelastic behaviour of the body efficiently. Thus more generalized models are often used. If a few Maxwell elements are put in parallel structure, the Maxwell-Weichert model is achieved (Fig. 2a). If several Voigt elements are connected in series, the Kelvin-Voigt model is obtained (Fig. 2b). The response of the models to elemental tests are shown in Tab. 2.

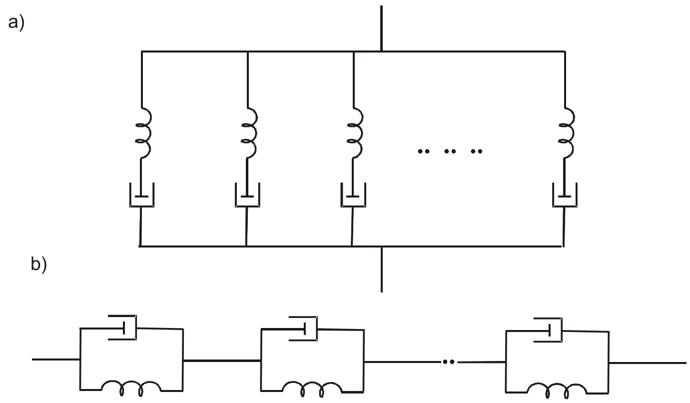

Fig. 2. Generalized models of viscoelastic behaviour of the structure: a) Maxwell-Weichert model, b) Kelvin-Voigt model

Table 2. Response of Maxwell-Weichert and Kelvin-Voigt models to elemental tests (on the basis of Aklonis et al 1972)

Experiment	**Maxwell-Weichert model**	**Kelvin-Voigt model**
Creep test		$J(t) = \sum_{i=1}^{N} J_i(1 - e^{-t/\tau_i})$
Stress relaxation	$E(t) = \sum_{i=1}^{N} E_i e^{-t/\tau_i}$	
Sinusoidal dynamic experiments	$E^* = E' + iE''$ $E' = \sum_{i=1}^{N} \frac{E_i\omega^2\tau_i^2}{1+\omega^2\tau_i^2}$ $E'' = \sum_{i=1}^{N} \frac{E_i\omega\tau_i}{1+\omega^2\tau_i^2}$	$J^* = J' - iJ''$ $J' = \sum_{i=1}^{N} \frac{J_i}{1+\omega^2\tau_i^2}$ $J'' = \sum_{i=1}^{N} \frac{J_i\omega\tau_i}{1+\omega^2\tau_i^2}$
where: J_i, E_i – creep or tensile compliance of i-th element, E, J – see Tab. 1, N – number of elements		

2. Viscoelastic Model of Waterhammer

The traditional description (Chaudhry 1979, Streeter and Wylie 1979, and others) of the waterhammer run in a pipeline can be presented as a set of two equations:

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + R_0 Q|Q| = 0, \quad \text{where} \quad R_0 = \frac{8}{g\pi^2}\frac{\lambda}{D^5} \tag{6a}$$

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} = 0, \tag{6b}$$

where Q is the rate of discharge, H – water head, g – acceleration due to gravity, a – wave celerity, A – cross-section area, D – internal pipe diameter and λ is the linear friction factor. However, thanks to the appreciable improvement of measuring techniques and rapid progress in mathematical modelling, comparison of observed characteristics of waterhammer run (mainly pressure changes during the phenomenon) with results of calculation on the basis of Eq. (6a, b) was possible. Such comparison proved significant differences in both types of characteristics – measured and calculated, which in consequence led to the conclusion that traditional description of waterhammer is not complete and adequate. As a result, the set of equations (6a, b) was often replaced by more complicated description, in which the main emphasis was on the modification of friction term in momentum equation. One can find many different approaches of such modification, from the simplest idea of multiplying it by some constant (even up to 10 and more), to more complicated ones. The results obtained by calculations became closer to observations, however the problem was still not solved to a satisfactory degree.

The difference in calculations and observations is particularly clear for pipes made of polymers. The reason for this fact is the viscoelastic behaviour of this material as the reaction on stress. The equations (6a, b) describing the elastic model may be applied for steel pipes and preliminary calculations for plastic pipes. If more accurate calculations are needed, it is necessary to develop the form of mathematical description taking into account viscoelastic character of pipe wall deformations (Brunone et al 2000, Covas et al 2004).

The features of polymers used for pipes and the behaviour of this material in stress-strain aspect proved that it may be assumed that the polymeric pipe exhibits a linear viscoelastic behaviour. The characteristic feature is that the material does not respond instantaneously to the applied load, but a kind of lag behind the applied stress is observed Thus the viscoelastic behaviour is characterized by an instantaneous elastic strain followed by a gradual retarded strain for an applied load (Covas et al 2005). This time-dependent strain behaviour resulting from constant loading is defined as creep and depends on molecular structure of the body, stress-time history and temperature. In consequence, the phenomenon of waterhammer in polymeric pipeline may be described with the use of Kelvin-Voigt model of several number of elements (*N* elements in general approach, Covas et al 2005, Ghilardi and Paoletti 1986, Pezzinga and Scandura 1995), slightly modified in comparison to its 'classical' version presented in Fig. 2b. This modified form of Kelvin-Voigt model is shown in Fig. 3 and is considered to be appropriate to model the specific kind of strain development that polymers reveal.

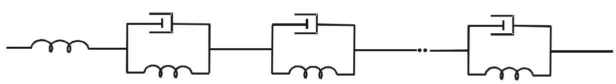

Fig. 3. Modified Kelvin-Voigt model of viscoelastic behaviour of the structure

The strain of the polymer pipe during waterhammer is understood as unitary elongation of the internal circumference of the pipe, which can be defined as (Ghilardi and Paoletti 1986):

$$d\varepsilon = \frac{dr}{r}, \tag{7}$$

where r is the internal radius of the pipe.

The total strain ε of the material described by the model shown in Fig. 3 can be expressed then as a sum of instantaneous and retarded components, ε_0 and ε_r respectively:

$$\varepsilon = \varepsilon_0 + \varepsilon_r, \tag{8}$$

where ε_r represents the total retarded strain resulting from behaviour of each element i:

$$\varepsilon_r = \sum_{i=1}^{N} \varepsilon_i. \tag{9}$$

The instantaneous strain component in Eq. (8) is calculated as in the elastic model, where E_0 is Young's modulus, thus:

$$\varepsilon_0 = \frac{\sigma}{E_0}, \tag{10}$$

while retarded components ε_i in Eq. (9) represent the viscous behaviour of each Kelvin-Voigt element ($i = 1 \ldots N$). Thus, on the basis of Eq. (5), for each element i one can write

$$\sigma = E_i \varepsilon_i + \eta_i \frac{d\varepsilon_i}{dt} \tag{11}$$

and – in consequence:

$$\frac{d\varepsilon_i}{dt} = \frac{1}{\tau_i}\left(\frac{\sigma}{E_i} - \varepsilon_i\right), \tag{12}$$

where E_i and τ_i are the tensile modulus and retardation time for i-th Kelvin-Voigt element.

Now, assuming that the material of the pipe is homogenetic and isotropic, for which the Poisson modulus is constant and all the shear stresses and inertia effects may be neglected, the stress for the pipe subjected to the internal pressure p can be expressed as (e.g. Wylie and Streeter 1983):

$$\sigma = \frac{pDc_1}{2e}, \tag{13}$$

where D is the internal diameter of pipe, e is its wall thickness and c_1 is a constant dependent on the pipe fastening. Introducing Eq. (13) to (12), one can write:

$$\frac{d\varepsilon_i}{dt} = \frac{1}{\tau_i}\left(\frac{pD\,c_1}{2eE_i} - \varepsilon_i\right). \tag{14}$$

The Eq. (14) put into (9) defines the retarded component of the strain, which introduced to Eq. (8) both with instantaneous component (10) describe the total strain of viscoelastic material.

The last step of creating the viscoelastic model of waterhammer is introducing the viscoelastic behaviour into unsteady flow equations. The analysis of the influence of viscoelasticity is easier if the following form of the continuity equation is used (Ghilardi and Paoletti 1986):

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho v A) = 0, \tag{15}$$

which can be expanded to:

$$\rho\frac{\partial A}{\partial t} + A\frac{\partial \rho}{\partial t} + \rho A\frac{\partial v}{\partial x} + \rho v\frac{\partial A}{\partial x} + vA\frac{\partial \rho}{\partial x} = 0, \tag{16}$$

where ρ is water density and v is average in the cross-section velocity of the stream.

Taking into account the definition of ε expressed by (7), one can write:

$$\frac{\partial A}{\partial t} = \frac{\partial A}{\partial \varepsilon}\,\frac{\partial \varepsilon}{\partial t} = 2A\,\frac{\partial \varepsilon}{\partial t}, \tag{17a}$$

$$\frac{\partial A}{\partial x} = \frac{\partial A}{\partial \varepsilon}\,\frac{\partial \varepsilon}{\partial x} = 2A\,\frac{\partial \varepsilon}{\partial x}. \tag{17b}$$

What is more, the relation between density ρ and pressure p can be expressed by state equation for water (with the assumption of neglecting the entropy variation):

$$\frac{\partial \rho}{\partial p} = \frac{\rho}{K}, \tag{18}$$

where K is the liquid bulk modulus of elasticity.

Introducing Eqs. (17a, b) and (18) to continuity equation (16), after transformation, leads to:

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + 2K\left(\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial x}\right) + K\frac{\partial v}{\partial x} = 0. \tag{19}$$

In consequence, after introducing linear viscous behaviour described by the Kelvin-Voigt model into Eq. (19), one obtains:

$$\frac{\partial p}{\partial t} + v\frac{\partial p}{\partial x} + \rho a^2\frac{\partial v}{\partial x} + 2\rho a^2\frac{d\varepsilon_r}{dt} = 0, \tag{20}$$

where wave speed a can be defined as:

$$a = \sqrt{\frac{\frac{K}{\rho}}{1 + c_1\frac{KD}{E_0}e}}. \tag{21}$$

If the terms $v(\partial p/\partial x)$ and $v(\partial\varepsilon_r/\partial x)$ in Eq. (20) are neglected and if the dependent variables Q and H are taken into account (instead of v and p), the final set of equations for waterhammer in a viscoelastic pipe can be written as:

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + R_0 Q|Q| = 0, \text{ where } R_0 = \frac{8}{g\pi^2}\frac{\lambda}{D^5}, \tag{22a}$$

$$\frac{\partial H}{\partial t} + \frac{a^2}{gA}\frac{\partial Q}{\partial x} + \frac{2a^2}{g}\sum_{i=1}^{N}\frac{\partial\varepsilon_i}{\partial t} = 0, \tag{22b}$$

where:

$$\frac{\partial\varepsilon_i}{\partial t} = \frac{1}{\tau_i}\left(\frac{pD\,c_1}{2eE_i} - \varepsilon_i\right). \tag{22c}$$

The set of equations (22a–c) is the equivalent of (6a, b) for viscoelastic material of the pipe.

In conclusion it can be said that viscous behaviour influences only the continuity equation, in which the additional term connected with viscoelastic behaviour of the material appears.

The consequence of viscoelastic behaviour of pipeline material is the increase of wave dispersion which results in change of wave frequency (and so – also oscillation period) and additional dumping of the surge (visibly stronger than in the case of elastic material, e.g. steel), resulting in faster attenuation of the oscillations and shorter time of phenomenon run. As the maximal pressure increases and the time of phenomenon duration are much lower than in case of elastic material – from the practical point of view polymer pipes are much safer for the pipeline systems than elastic ones.

Applying viscoelastic model for the solution of waterhammer problem in polymeric pipelines improves the result, making it more consistent with measured pressure characteristics. The example of application of the model is presented in Fig. 4.

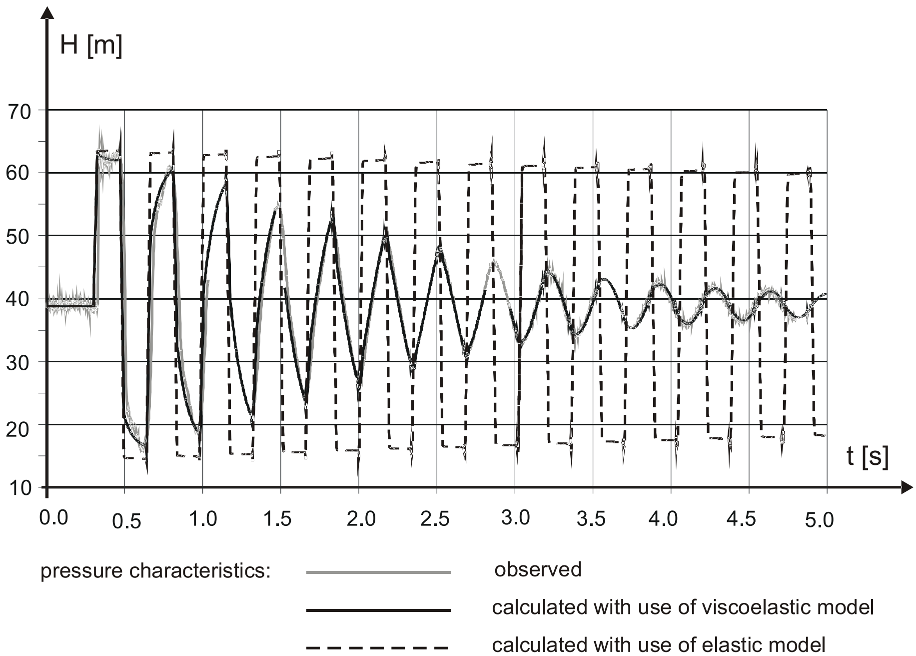

Fig. 4. Example of application of elastic and viscoelastic model of waterhammer (description in text)

The waterhammer in an MDPE pipe (SDR 10.7 PN = 10) of length L equal to 36 m, extrinsic diameter D equal to 50 mm and the wall thickness e equal to 4.6 mm was analysed. The measurements were carried out in the Laboratory of the Institute of Water Supply and Water Engineering of Warsaw University of Technology by Kodura (Kodura and Weinerowska 2005, 2007). The scheme of experimental model can be found in papers of Mitosek and Chorzelski (2003), Kodura and Weinerowska (2005) and others. The pipe was fed from a large pressure reservoir in which a constant value of pressure during the experiment was enforced. The test involved unsteady water flow resulting from the sudden closure of a ball valve mounted at the end of a pipe. The water hammer pressure characteristics were measured by tensiometer indicator located 0.39 m from the valve and recorded in the computer's memory. Time of valve closing was measured by precise electronic stop-watch connected to the valve.

For the presented example, initial water discharge in steady flow was equal to 0.744 dm^3/s, thus the steady flow velocity was equal to 0.57 m/s. The value of water head at the valve cross-section during steady flow was equal to 38.8 m. Sudden valve closure (closure time 0.024 s) caused waterhammer in the pipe. The measured pressure characteristic is presented in Fig. 4. The example was calculated using the 'traditional' model of waterhammer (described by Eq. 6a–b) and also viscoelastic model (Eq. 22a–c). The well-known Preissman scheme (with parameters

$\theta = \psi = 0.5$) was used to solve the equations in both cases. The pressure wave speed was determined on the basis of such measurements as $a = 423$ m/s. The presented solutions were obtained for space step $\Delta x = 1$ m and time step enabling to obtain Courant number equal to unity ($\Delta t = 0.00236$ s). Linear friction coefficient was calculated for assumed pipe roughness 0.004 mm from simplified Colebrook-White formula. In viscoelastic approach the easiest model of one Kelvin-Voigt element was used and the best solution obtained for the values of $\tau = 0.0541$ s and $J = 1/E = 0.9E - 10$ Pa^{-1} was shown in Fig. 4.

In the figure, the results of calculations and measurements are compared. As it can be seen, viscoelastic model of waterhammer enables the obtaining of a satisfactory solution, while the classical approach (elastic model) is not sufficiently accurate in the case of polymer pipe. Even if different attempts with increasing the friction factor are applied, it is not possible to obtain a satisfactory solution for an elastic model, as not only amplitudes of oscillations but also frequency and period of oscillations are incorrectly modelled. Thus one can notice the significant increase of accuracy of the solution if viscoelastic model is applied.

However, despite such "good" examples as the one presented above, there are still several problems that have not been sufficiently recognized so far. Many authors present different examples of successful implementation of this method, however – the way of obtaining such result is not "general" but individually found for each case separately. This means that the solution may not be of a universal character and thus there is still a problem with finding some global approach to this question. The problems that very strongly determine such a situation may be divided into some groups, from which the most important seem to be those connected with model parameters estimation and the ones connected with numerical aspects.

The second group of problems – numerical aspects – boils down mainly to the question of a choice of numerical scheme applied to solve the set of equations (22a–c) and choice of numerical parameters such as space and time step in calculations (in consequence also the value of Courant number) and numerical coefficients if they appear in a chosen numerical scheme. This question has been widely analysed in literature (e.g. Szymkiewicz 2006). It is known that numerical effects may significantly change the solution, not only as to the numerical quantities but – what is more important – the character of the phenomenon. The waterhammer is a distinct example of a problem in which it is particularly easy to lose the physical reality of the process and interpret numerical effects as physical ones. The amplitude and frequency of pressure oscillations are the essential elements of the phenomenon and the values obtained in the solution may be the resultant effect of both – physical character of waterhammer and numerical dispersion and dissipation. Thus it is very important to realize the influence of the numerical parameter values and to set these values clearly when the solution example is presented. Some authors pay great attention to this question and apply numerical schemes of a high accuracy and present the values of numerical parameters (coefficients, time and space steps etc.)

to enable the solution to be interpreted correctly (e.g. Covas et al 2005). However, there are still examples presented in different papers in which the authors do not show numerical details of the solution, making it impossible to repeat and interpret in the right way.

Even if problems of the kind given above are successfully solved – the chosen scheme is accurate and the parameters are chosen properly, there is still one very important question, which is in the presented problem of an essential matter. It is connected with model parameters, their estimation and sensitivity of the model to the values of these parameters. Some of these questions will be analysed in a more detailed way below.

3. Viscoelastic Model Parameters

General Remarks

One of the important group of problems connected with model parameters concerns the friction term. It is a question vivid not only in the case of polymeric pipes but for all types of pipelines. Thus it is not connected strictly with elastic or viscoelastic behaviour of the material and may be treated as a separate problem. The term connected with friction appears in momentum equation, no matter whether viscous features of the material are taken into account or not. The way of representing friction in this equation is important and influences the result very strongly. The approaches applied in this question are of various types – from the easiest, where the friction factor λ is calculated like a steady flow or – its modified version – the value of λ is multiplied (even 10 times or more) to take into account higher energy losses in unsteady conditions – to much more sophisticated approaches in which friction depends also on time and space derivatives of velocity and other factors. However, no matter which concept is considered – the problem of friction in waterhammer is a separate question. As it does not deplete the list of difficulties during waterhammer problems solution, and – even if solved in a very accurate way – does not guarantee the proper solution – it will not be considered in the paper.

The essential question of the viscoelastic model of waterhammer is connected with main parameters that must be estimated – wave speed a, numbers of elements in Kelvin-Voigt model, values of retardation time τ_ι and values of creep compliance J_i (where $J_i = 1/E_i$ and E_i is modulus of elasticity for a spring of i-th element) for each element i (i=1…N). As will be discussed further, most of these parameters are very difficult or even impossible to estimate theoretically. There are problems of different nature, one of the most important is connected with the fact, that – as the springs and dashpots are only hypothetical, conceptual elements used to model the phenomenon – most parameters are of a 'mathematical' character, with no strict attitude to the physical side of waterhammer. It makes identification much more difficult. If also taken into account that in some examples the number of elements in the Kelvin-Voigt model reaches even the value of 5 – the total number of parameters goes up to 11, which may make the solution not unique and difficult to interpret. Obtaining the solution is usually not so difficult as regards calculation, as the possibility exists to estimate parameters on the basis of optimization procedures, but the interpretation and obtaining of consistency of the model with physical features of the phenomenon is questionable. There may be a problem with the influence of the method of optimization and choice of objective function on solution and usually such parameters have only mathematical meaning, with no physical relation. What is more – such approach is possible only if there are measurements for waterhammer in the considered pipeline, which affords the possibility of applying optimization techniques. If the pipeline is being designed and there are no possibilities of verification – there are no clear rules as to how to estimate parameters to be able to carry out rough calculations of waterhammer modelling.

Wave Celerity

One of the most important questions is proper wave celerity estimation. In many examples authors take into account the value of wave speed calculated on the basis of the formula (21), (Chaudhry 1979, Wylie and Streeter 1993). Such approach brings many problems of different natures.

The first aspect concerning this question is proper E_0 choice. It is known that the value of Young modulus depends on the material of the pipeline. However, the value of E_0 is not strictly fixed for each material A big variety of polymers used for pipe walls is observed and for each type of polymer there are several modifications (Saechtling 2000). For example, from a big family of polyethylene pipes one can distinguish HDPE, MDPE, LDPE, HPPE and for each type the producers give the range of the values that Young modulus can take. If the values are taken from handbooks of polymers the range of values can also differ slightly. What is more, the ranges for different types of polymers can partly interpenetrate, which means that the limits of values for different materials are not strict. E.g. the modulus of elasticity for MDPE takes the values 0.6–0.8 GPa, while for HDPE one can find the values of Young's modulus of a range 0.7–1.0 GPa (Covas et al 2004) or even 0.6–1.4 GPa (Saechtling 2000). In consequence, the values of a calculated on the basis of Eq. (21) may differ significantly. For example, for the pipeline of inner diameter D equal to 50.6 mm and wall thickness e equal to 3 mm, for $K = 2.19$ GPa and $c_1 = 1$, the wave speed calculated according to Eq. (21) may vary from 197.8 m/s (for $E_0 = 0.6$ GPa) to 298 m/s (for $E_0 = 1.4$ GPa) which makes the difference of about 100 m/s depending only on the assumed value of E_0. What is more, the viscoelastic features of polymers that result in rheological behaviour have considerable influence on mechanical performance of the material (Ferry 1965, Aklonis et al 1972, Covas et al 2004), which results in that short-term modulus of elasticity being higher than long-term modulus of elasticity (Janson 1995, Covas et al 2004). The factors given above cause that the value of a – if one tries to estimate it theoretically – may be significantly different. That consequently makes a significant difference in the oscillation period and values of pressure amplitudes. If the velocity of steady flow in the pipeline was 1 m/s, the theoretical pressure increase may differ from 19.7 m to 29 m for the extreme values of E_0 given above, which is a difference of 0.1 MPa.

What makes this question even more complicated, is that the essential aspect – as observed – was that the real value of wave celerity in waterhammer differs from that obtained in Eq. (21). Experiments proved that the observed values of a are much higher that those calculated for Young modulus for elastic behaviour. And also, as was observed, the maximal pressure amplitude calculated on the basis of the formula:

$$\Delta p = \rho a \Delta v, \tag{23}$$

(where ρ is water density and Δv is the difference between the values of velocity in steady state v_0 and final velocity v_1 near the valve after the rapid change; usually v_1 equal to zero) as the values of a presented above is also lower than observed, and the difference may be up to about 10–25%. The phenomenon is recognized as *line packing effect* (Wylie and Streeter 1993, Covas et al 2004). Thus the two values of empirical wave speed and theoretical wave speed (for elastic model, calculated from Eq. (21)) must be distinguished. For the purpose of this paper this second wave speed (theoretical) will further be denoted as a_0.

What is more, the variation of wave speed value during the phenomenon run is observed (Covas et al 2004). This value changes both in time (decreases during the phenomenon) and in space (differs along the pipeline). These values can change e.g. from 423 m/s at the beginning of the phenomenon to about 380 m/s in further phase (Covas et al 2004), which is about 40 m/s (about 10%). Thus for the purpose of the viscoelastic model of waterhammer expressed by Eq. (22a–c) only some kind of average value may be assumed. This average value may be estimated on the basis of measurements, by observing the oscillation period, which seems to be the best way to fit this value to the analysed phenomenon run. However, it cannot usually be done in sufficiently accurate way, and even so – it is valid only for the considered case of waterhammer in a particular pipeline. Thus there are also attempts to express the viscoelastic model of the phenomenon in a different way.

Instead of influencing the additional term in mass equation (22b) which is a consequence of approach shown in Eqs. (7–14), there is an alternative way of taking into account rheological behaviour of the material. In this approach the variable during phenomenon wave speed a is considered and introduced to continuity equation (Covas et al 2005, Meißner and Franke 1977, Mitosek and Chorzelski 2003 and others). This is a concept of frequency-dependent wave speed and also time-dependent behaviour of the pipe-wall, that can be described in terms of angular frequency ($\omega = \pi/T$). This approach was analysed among others by Meißner and Franke (1977), Rieutford (1982), Franke and Seyler (1983) and Mitosek and

Chorzelski (2003). By considering creep compliance function and introducing it to momentum and continuity equations, Meißner and Franke (1977) derived a complex formula, enabling to calculate the velocity for pressure wave propagation. As they presented, the value of wave celerity a depended not only on fluid density, internal diameter of the pipe, liquid bulk modulus of elasticity and pipe wall thickness, but also the frequency of pressure oscillations ω, the storage and loss compliances of the wall material J' and J'' (defined as in Tab. 1 and Tab. 2) and so-called linearized friction factor. If the influence of the friction on wave speed is neglected, the formula for thin-walled viscoelastic pipes can be written as follows (Meißner and Franke 1977, Mitosek and Chorzelski 2003):

$$a = \sqrt{\frac{\frac{2}{\rho}}{\sqrt{\left(\frac{1}{K} + J'\frac{D}{e}\right)^2 + \left(J''\frac{D}{e}\right)^2} + \frac{1}{K} + J'\frac{D}{e}}}. \tag{24}$$

This alternative approach also affords some problems, one of which is proper J' and J'' estimation. Some authors present the equations on the basis of which one can determine the values of the storage and loss compliances J' and J''. Schwarzl (1970) and Mitosek and Chorzelski (2003) present the formulas in which J' and J'' are the functions of creep compliance $J(t)$ and circular frequency ω. However, to calculate J' and J'', the explicit formula of $J(t)$ must be known. This, as is known, is not an easy problem, as the form of this function depends on specific features of the material and should be determined for each material separately, experimentally. Although some authors present some formulas for $J(t)$, they are valid only for the analysed case of a pipe of particular diameter, wall thickness and – what makes things even worse – for the particular temperature of the stream. In such situation proper determination of wave speed a in this way is very difficult, and in most practical cases almost impossible, as the formulas for $J(t)$, which are of an empirical character, for most of the pipes are not known and difficult to determine.

The second way of taking into account frequency-variable wave speed a is the approach presented by Lisheng and Wylie (1990). They proposed to apply the known formula for pressure wave velocity (21), but with use of complex Young modulus E^* (defined as in Tab. 1 and Tab. 2) instead of Young modulus E_0. As $E^*(\omega)$ is a complex function of wave frequency, in consequence, wave speed a is also frequency-variable. As it is easy to guess, the problem is how to determine $E^*(\omega)$. The best way is to get such data from material tests in rheo-vibration apparatus. However, it is not easy to carry out such tests and for many practical cases they are inaccessible. What is more, some authors prove (Lisheng and Wylie 1990) that the value of E^* so obtained can be even 40% smaller than the true physical value of the dynamic modulus. Thus one can see that this kind of approach also leads to many problems of practical use of the mentioned formulas.

However, the approaches presented above show explicitly that pressure wave speed a is frequency depended. In a consequence – it depends on period of oscillations T. Taking into account that:

$$a = \frac{2L}{T}, \tag{25}$$

it can be proved that a also depends on the length of the pipe. Mitosek and Chorzelski (2003) presented a study in which they examined the relationship between observed a (for pipes of the same material, diameter and wall thickness) and the length of the pipe which varied in different experiments. The diagrams showing the $a(\omega)$, $a(L)$ and $E^*(\omega)$ dependences for the analysed cases were presented and the empirical formulas for $a(L)$ functions for the analysed materials were determined. Such formulas may be helpful in the determining of a. The problem in application of this method is that the formulas are valid only for the pipes examined experimentally thus for the pipes of the same material, diameter and wall thickness as those used in tests. As we also know, the value of pressure wave speed is influenced by many different factors and thus the empirical formulas do not have a 'general' character.

To sum up the previous considerations, there is no firm formula enabling a estimation if there are no measurements. One way may be increasing the value of a_0 by about 10–25% (Covas et al 2004) or even more as the author's experiments proved, however the question is which value of E_0 to calculate a_0 should be chosen, and second – how much it should be increased (which value of the range presented above to chose). The answers to these questions may influence the solution significantly. The second way of estimating a may be comparing to previously analysed examples for which the observations were accessible and trying to find some rule for each type of material separately. However it seems that there is no objective method to estimate a value in a proper way.

Thus one can notice that the basic problem of waterhammer – wave celerity – is not a trivial one, and seemingly unimportant factors influence the solution to a significant degree. Even if the problem of waterhammer is considered only from the practical point of view and the proper simulation of the whole phenomenon run is not particularly important, proper estimation of the highest amplitude is of significant meaning.

Kelvin-Voigt Element Parameters

Although, as we have already presented, there are attempts to apply frequency--dependent wave speed in waterhammer model, the approach leading to modified waterhammer equations (22a–c) is more popular. In such an approach, next, an even more difficult problem, is the estimation of J_i and τ_i for each element in the Kelvin-Voigt model. As already mentioned, the springs and dashpots are only conceptual elements of the model, with no physical equivalent and thus the estimation of J_i and τ_i is hindered as the parameters have no physical interpretation. The values of the parameters are strongly related to the assumed number of elements in Kelvin-Voigt model. The higher number of *a priori* assumed elements, the better potential coincidence between calculations and observations. However, the only way of estimating such high numbers of parameters is optimization and the results of such estimation are valid for the particular, analysed case only. For a different pipe of the same material or for a different discharge rate or number of elements – the set of parameters has different values. Thus it is very difficult to find any global, general relation or formula or at least procedure as to how to deal with such cases or how to predict the pressure characteristics during waterhammer for a pipeline for which there are no measurements (e.g. during the stage of planning and designing). As far as is known – there are no objective procedures for finding the "optimal" number of elements N in Kelvin-Voigt model nor any relations enabling estimation of retardation time for each dashpot and creep compliance for each spring in the model. The usual way is to assume the number of elements – usually from 1 to 5. Covas et al (2004) state that increasing the number of parameters above 5 does not result in solution improvement. If the number of elements is low – 1 or 2, retardation time and the related values of creep compliance are usually estimated on the basis of trial and error method or with the use of optimization procedures (e.g. Pezzinga and Scandura 1995). If the number of elements is greater (e.g. 4 or 5) the values of τ_i are assumed and the values of creep compliance are estimated on the basis of optimization (Covas et al 2004). As a result the least square error of calculated values of water head related to observed is very low (even down to 0.05 m^2), but the chosen values of parameters (retardation time and creep compliances) are valid only for this particular case. Thus it would be important to find some more global relation between the parameters and some kind of more general procedure enabling predicting waterhammer runs also for pipelines for which the observations are not possible. That is particularly important, as it should be considered that in practice there are many much more complicated cases. The optimization procedures with such big numbers of parameters may be possible for a single pipeline, but if the network of pipelines is considered – there are much more factors influencing the solution and it would be important to be able to predict at least approximate values of parameters on the basis of more a physically-related method than optimization procedure.

On the other hand, finding any global approach, based more on theoretical considerations or examination of different experiments, and trying to find any regularity occurs to be difficult. Covas et al (2004) present a wide range of experiments leading to creep characterization and estimation of values of creep compliance J for pipes made of PE. In tests presented the creep compliance function was estimated by running creep tests in longitudinal samples of pipe. In addition, dynamic mechanical tests were carried out.

The creep compliance values for each temperature change in time (increase). The obtained values varied from about 0.08 E-8 Pa^{-1} to 0.25 E-8 Pa^{-1}. The dynamic tests let real and imaginary components of complex modulus of E^* be examined (see Tab. 1 and Tab. 2). In conclusion authors state that in none of the tests was it possible to obtain a good definition of creep function for a very short time (2–3 s), nor exact value of the elastic creep component J_0. What is more, the authors proved that the values obtained in tests do not correspond exactly with the exact creep function of PE integrated in pipe systems, and the creep compliance for existing pipe systems should be estimated on the basis of calibration procedures.

The numerical tests carried out by the same authors (Covas et al 2005) for different cases, different numbers of Kelvin-Voigt elements and for assumed times of retardation led to optimal creep compliance values varying from about 9E-11 to 2E-10 Pa^{-1}, which is considerably different from the values obtained in creep tests. Thus the optimization procedures seem to be the best way of determining model parameters, however, one should remember that each set of obtained values is valid only for the analysed case and also for assumed times of retardation.

Optimization procedure cannot be applied if measurements are not accessible. Thus numerous cases (e.g. designing of pipelines, existing pipeline systems where it is impossible to carry out the measurements etc.) cannot be solved using such a means of calibration. Evaluation of retardation time and creep compliance in such case still seems to be an open question.

4. Final Remarks and Conclusions

The considerations presented constitute the kind of discussion on the question of viscoelastic model application. The main aspects of theoretical background of the viscoelastic behaviour of the structure were presented and main problems connected with viscoelastic model application were discussed. The general conclusions may be summarized as below:

- For polymer pipelines it is necessary to apply viscoelastic models of waterhammer as the traditional approach valid for elastic materials (e.g. steel), leads to considerable discrepancy between calculated and observed values of pressure. Some authors present calculations for polymeric pipelines without considering the viscoelastic model of waterhammer, however the viscoelastic effects of material behaviour are then 'hidden' in values of other parameters (e.g. numerical) or other terms (e.g. friction term in momentum equation), giving a similar effect of additional damping of oscillations. However, such approach seems not to be suitable as the physical side of the phenomenon is not modelled in a proper way which makes interpretation of the solution difficult and may lead to wrong conclusions.

- Application of a viscoelastic model is connected with many problems of different natures, of which very important is parameter estimation.
- The choice of a the method of parameter estimation and accuracy of obtained results depend, among other things, on the assumed approach to considering viscoelasticity of the material. If the approach of frequency-dependent pressure wave speed is taken into account, most of the parameters should be estimated on the basis of material tests, which makes this approach difficult in practical cases.
- If the Kelvin-Voigt model of viscoelastic behaviour is taken into account (instead of frequency-dependent pressure wave speed introduction to the 'traditional' waterhammer model equations), the important question of parameter estimation is the number of parameters. The number of viscoelastic model parameters depends on the assumed number of Kelvin-Voigt elements. For each element two parameters – retardation time and creep compliance must be found. In addition, the waterhammer wave speed must be estimated. The bigger number of parameters gives the potential possibility of obtaining higher accordance between observations and calculations. However, there is a limit of Kelvin-Voigt element number (Covas et al 2005 give the value of 4 or 5) above which no improvement in measurement-calculations coincidence is observed. What is more, the greater number of parameters causes the higher possibility of non-unique solution obtaining and bigger problems with interpretation of the influence of the parameters in the result.
- Wave speed may be estimated in different ways. The most proper seems to be the estimation on the basis of observations. However, such an approach has also disadvantages. First, it requires the results of measurements, thus it is valid for existing systems for which experiments may be carried out. Second – waterhammer wave speed changes both in space and time during the phenomenon run and, moreover, it depends on many factors, e.g. frequency of oscillations and temperature of the liquid. Thus the value that is introduced to the model described by Eqs. (22a–c) may be only a kind of average wave speed.
- The other way of wave speed estimation takes into account the theoretical value of a_0 calculated according to Eq. (21). This approach also gives rise to two main problems. First, there is the problem with proper estimation of modulus of elasticity which can vary for considered material in a relatively wide range, leading to vivid results in obtained waterhammer pressure characteristics. Second – it was proved that real values of a differ from the theoretical value of a_0 significantly (10–20%) or even more. Thus in consequence such approach may lead to results far from those observed.
- There is a serious problem to estimate a when there is no possibility of carrying out the experiments. In such a situation the approach presented above seems

to be very helpful, however, it may give only a rough estimation. The second solution may be comparing the analysed case to previously examined, and trying to conclude on the basis of some kind of analogy. However, this may also be difficult.

- Parameters connected with Kelvin-Voigt element – retardation time and creep compliance are very difficult to estimate, as they are of a mathematical character only, with no attitude to technical parameters of the system. If the number of elements in a Kelvin-Voigt model is small (1 or 2), usually all parameters are estimated on the basis of optimization procedures. If the number is higher, usually retardation times are assumed and responding creep compliance values are optimized. As proved (Covas et al 2004), the values obtained due to optimization procedures differ from those obtained from creep tests. It suggests that the only way of estimating these parameters is optimization, which is possible only when measurements are accessible. Thus the problem arises, what to do if the pipeline is analysed on design stage or for any other reason experiments are not possible. It seems there has been no general way to estimate creep compliance in such situations, so far. One should also remember that even if measurements are accessible and optimization procedure can be carried out – the parameter values obtained in such a way are valid for this particular case only and will be different not only if a different pipeline is considered, but also for the same pipeline if a different number of Kelvin-Voigt elements is taken into account or if different values of retardation times are assumed. Thus, such values do not have any global character.

- For many of the reasons mentioned above, especially for cases where measurements are not accessible, it seems to be very important to try to find a more effective approach of relatively global character, more vivid regularity in parameter estimation, to be able to better model those cases for which an experiment is not possible. It seems to be particularly important when one takes into consideration that the problems presented above concerned only the easiest example of simple pipeline of constant diameter. In real systems there is often a need for carrying out calculations for more complicated systems, e.g. pipeline networks. Thus it seems reasonable that in such cases one will tend to decrease the number of parameters for each pipeline, to enable calibration of the whole system. Thus some kind of regularity may be essential for such complicated cases.

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