ANNUAL OF NAVIGATION 11/2006

ALEKSANDER NOWAK, CEZARY SPECHT
Naval University of Gdynia

SNAPSHOT RAIM ALGORITHMS AVAILABILITY IN URBAN AREAS

ABSTRACT

This paper presents some theoretical considerations concerned usage of Snapshot RAIM algorithms in city navigation. Influence of urban areas on RAIM Availability and Approximate Radial-Error Protected (ARP) is taken into consideration. Some results of numerical experiments are presented, too.

INTRODUCTION

Navigation system integrity refers to the ability of the system to provide timely warnings to users when the system should not be used for navigation. The basic Global Positioning System (GPS) provides integrity information to the user via the navigation message, but this may not be timely enough for some applications. Therefore, additional means of providing integrity are necessary. Two different approaches can be considered. One of this is the receiver autonomous method, now referred to simply as RAIM (receiver autonomous integrity monitoring). A variety of RAIM schames have been proposed and all are based on some kind of self-consistency check among the available measurements. Of course, there must be some redundancy of information in order for RAIM to be effective. The other approach to providing an independent assurance of integrity is to have a network of ground monitoring stations whose primary purpose is to monitor the health of the GPS satellites [1]. Because of many obstructions which can occur in urban areas such as buildings and trees it is hardly possible to construct proper network of ground station, thus only the first mentioned approach can be implemented. Initially RAIM was considered as a part of air navigation. Therefore, all proposed algorithms assume that the process of navigation takes place in an open area (without obstructions) and only one satellite at time will be transmitting an unpredicted erroneous signal. Because more often and often GPS receivers are used as a part of land navigation systems it seems to be necessary to consider RAIM in aspect of city navigation. Some theoretical considerations and numeric simulations of RAIM availability in urban areas are presented in next sections.

THEORY OF RECEIVER AUTONOMOUS INTEGRITY MONITORING

A variety of RAIM schemes have been proposed in the literature. They all based on the same principle of carrying out a self-consistency check amongst redundant measurements using statistical decision theory. Two hypothesis-tests are considered:

1. Does a failure exist?
2. Which satellite is transmitting a faulty signal?

The first test is called Failure Detection (FD) the second Failure Identification (FI). For all these tests is assumed that only one satellite at a time will be transmitting an unpredicted erroneous signal. Three RAIM algorithm schemes which have been proposed for implementation are (known as Snapshot RAIM) [2]:

– Parity Method;
– Least-Squares-Residuals Method;
– Constant-Detection-Rate/Variable-Protection-Level Method.

Before these algorithms can be applied, a prediction is needed to determine whether the geometry of the available constellation of satellites will be sufficient to allow for RAIM – RAIM Availability (see section 1.3). With a minimum of five satellites it is possible to detect whether an error exists in one of the measurements – again assuming a certain geometrical quality of the constellation. At least six satellites are required to carry out Failure Detection and Identification (FDI).

Observation Equation and Measurement Model

The range measurement to an individual satellite can be described by following observation equation:

$$\rho = R + c \cdot \left(\delta t - \delta T\right) + d_{ion} + d_{trop} + \varepsilon_n, \tag{1}$$

where: ρ – measured pseudorange;
R – geometrical range;
c – speed of light;

δt – satellite clock error;
δT – receiver clock error;
d_{ion} – error connected with influence of ionosphere;
d_{trop} – error connected with influence of troposphere;
ε_n – measurement noise.

The observation equations for the individual satellites result the following linear equation system [2]:

$$\mathbf{y} = \mathbf{G}\mathbf{x} + \boldsymbol{\varepsilon}, \tag{2}$$

where: $\mathbf{y}$ – vector of linearised measurements compensated by a priori information;
$\mathbf{G}$ – geometrical matrix (direction cosine matrix);
$\mathbf{x}$ – innovation matrix;
$\boldsymbol{\varepsilon}$ – vector of Gaussian-distributed measurement errors.

This equation system is over-determined in case of more than four measurements and is usually solved by a least-square-adjustment. The least-square estimate of the position innovation vector $\left(\hat{\mathbf{x}}_{LS}\right)$ is given by:

$$\hat{\mathbf{x}}_{LS} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{y}. \tag{3}$$

The least-square solution can now be used to predict measurements in accordance with:

$$\hat{\mathbf{y}} = \mathbf{G}\hat{\mathbf{x}}_{LS}. \tag{4}$$

Now we can form vector of residuals $(\mathbf{w})$:

$$\mathbf{w} = \mathbf{y} - \hat{\mathbf{y}}. \tag{5}$$

The sum of the squares of the residuals plays the role of the basic observable in the snapshot RAIM methods. Parkinson and Axelrad called it (SSE) for sum of squared errors. We do also:

$$SEE = \mathbf{w}^T\mathbf{w}. \tag{6}$$

A Baseline Snapshot RAIM Scheme

To determine whether an error has occurred in one of the measurements, statistical hypotheses can be formulated about the assumption that a defined event – here: no failure – will occur [2]:

– Null-Hypothesis H_0 – assumption that no failure will occur;
– Alternate-Hypothesis H_1 – assumption that a failure will occur.

The decision process is implemented by a comparison of Decision Variable (D) with threshold (T):

$D < T$ – Null-Hypothesis H_0 accepted;
$D \geq T$ – Alternate Hypothesis H_1 accepted.

In snapshot RAIM method it is convenient to work with a test statistic that is derived form SSE. Very often as a Decision Variable (D) is used:

$$D = \sqrt{\frac{SSE}{n-4}}, \tag{7}$$

where: n – number of visible satellites.

With this definition, the radial position error and the test statistic have the same dimensions and their ratio is dimensionless.

If all elements of $(\boldsymbol{\varepsilon})$ have the same independent zero-mean Gaussian distribution, the statistical distribution of (D) is completely independent of the satellite geometry. But if any unexpected errors occur (D) will have non-central χ^2 distribution which is dependent of the satellite geometry. Thus, before snapshot RAIM algorithms can be applied, a prediction is needed to determine whether the geometry of the available constellation of satellites will be sufficient to allow for RAIM – RAIM Availability. This prediction is called Screening Out Poor Detection Geometries.

Screening Out Poor Detection Geometries

Various criteria have been used for evaluating the quality of satellite geometry for detection purposes. Of these, $\delta H_{\max}$ and ARP are probably the best [3]. The $\delta H_{\max}$ method proceeds as follows:

1. Compute the HDOPs associated with the *n* subset solutions. Call these $HDOP_i$, $i = 1, 2, \ldots, n$, where *n* – number of visible satellite.

2. Compute the HDOP associated with the full n-satellite least-squares solution. Call it $HDOP$.
3. Then:

$$\delta H_{\max} = Max_i \sqrt{HDOP_i^2 - HDOP^2} \tag{8}$$

The parameter $\delta H_{\max}$ then becomes an inverse measure of the quality of the satellite geometry for failure detection purposes (small values are best). A ceiling value for $\delta H_{\max}$ is then set accordance with the integrity specification and if calculated $\delta H_{\max}$ for geometry at hand exceeds the ceiling value, the geometry is declared inadmissible; otherwise, it is admissible.[1]

The ARP method derives from geometric considerations when we look at a plot of position radial error vs. test statistic (see fig. 1). A bias error on any particular satellite projects linearly into both the position-error and test-statistic domains. The slope, which relates the induced position error to the test statistic, can be readily calculated from the satellite geometry and it will be different for each satellite. For failure detection purposes the satellites whose bias error causes the largest slope is the one that is the most difficult to detect. It is the one that produces the largest position error (which we want to protect) for a given test statistic (which is what we can observe). We call the slope associated with the most-difficult-to-detect satellite $SLOPE_{\max}$ [1].

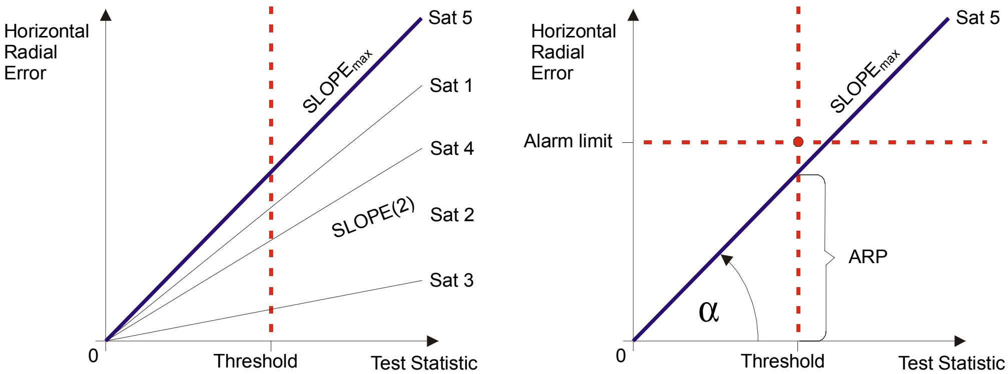

Fig. 1. Plot of position radial error vs. test statistic

The *ARP* value depends only on the satellite geometry and threshold setting and it is computed as follows [3]:

1. Define matrices **A** and **B** as

$$\mathbf{A} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}, \qquad \mathbf{B} = \mathbf{G}\left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}. \tag{9}$$

2. Compute a quantity called $SLOPE$ for each satellite in view:

$$SLOPE(i) = \sqrt{\frac{\left(\mathrm{A}_{1\mathrm{i}}^2 + \mathrm{A}_{2\mathrm{i}}^2\right)(n-4)}{1-\mathrm{B}_{\mathrm{ii}}}} \tag{10}$$

For $i = 1, 2, \ldots, n$

3. Define $SLOPE_{\max}$ as:

$$SLOPE_{\max} = Max_i\left[SLOPE(i)\right] \tag{11}$$

4. Then

$$ARP = SLOPE_{\max} \cdot threshold \tag{12}$$

The ARP value then becomes measure of the quality of the satellite geometry for failure detection purposes. As in case of parameter $\delta H_{\max}$ small values of ARP are best. A ceiling value for ARP (call it $ARP_{\max}$) is then set accordance with the integrity specification and if calculated ARP for geometry at hand exceeds the ceiling value, the geometry is declared inadmissible; otherwise, it is admissible.

INFLUENCE OF URBAN AREAS ON RAIM AVAILABILITY

ARP informs user about approximated value of radial position error which can be detected. Thus, we can define RAIM availability as follows:

– RAIM is available if calculated ARP at hand value is less or equal than $ARP_{\max}$;
– RAIM is unavailable if calculated ARP at hand value exceeds the $ARP_{\max}$.

As was shown in previous section, ARP value depends only on satellite geometry and threshold setting. R. G. Brown in [3] shown, that for test statistic defined in (7) both $\delta H_{\max}$ and ARP methods are equivalent. Because of many obstructions which can occur in urban areas such as buildings which block satellite signals, numbers of visible satellite decrease and satellite geometry can strongly degenerate. What is more, it could happen that less than five satellites will be visible and in consequence, RAIM procedure will be unavailable.

All in all, we can expect that ARP ($\delta H_{\max}$) values can dramatically increase in urban canyons and RAIM availability will be less than in open areas. To confirm this expectation the special computer application was done. Thanks it, it was possible to simulate influence of urban areas on RAIM availability.

REVIEW OF THE SIMULATION PROCEDURE

During the simulation we assumed that:

- to determine user's positions GPS system is used (nominal constellation defined in SPS-2001);
- GPS pseudorange measurements errors are independent zero-mean Gaussian random variables with the same standard deviation σ = 6m;
- to compute estimated position of the user all visible satellites are used;
- all computations are done in ECEF co-ordinates system;
- the user is located in urban area defined as it shown in fig. 2;
- user's true positions are random located on the surface of the Earth;
- the simulations are done for 24 hours with 1 second interval (1 Hz fixes frequency);
- the parameters $\delta H_{\max}$ and ARP are compute according to (1) and (5) for test statistic defined as in (7);
- the ceiling value for ARP is equal 100 m.

The simulation procedure was divided in two parts. In the first step the user was located at random in an open area and we simulated 86 400 fixes in each point. Then we placed the user at the same points as in previous step and we simulated 86 400 fixes again but an open area was replaced by model of urban area shown in fig. 2. The simulations results are presented in the next section.

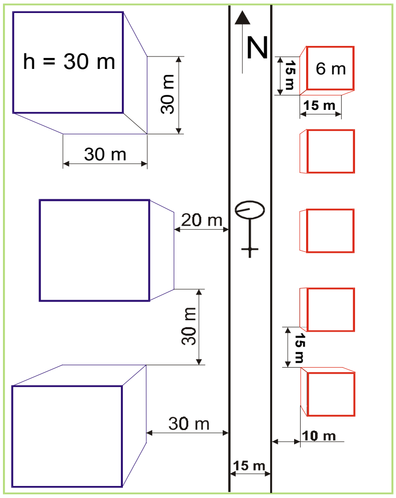

Fig. 2. Model of urban area used in the simulations

THE RESULTS OF SIMULATIONS

Due to restricted length of this paper only the results of simulations in one point are presented. The true position of the user in ECEF co-ordinate system was:

$$X = 6\,378\,137, \qquad y = 0, \qquad z = 0.$$

Fixes errors in plans XoY and YoZ are presented in fig. 3.

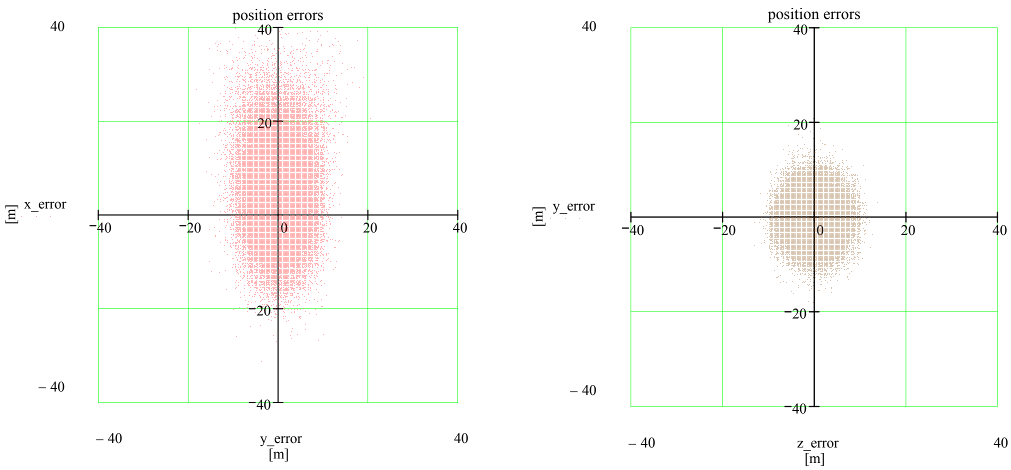

Fig. 3. Fixes errors in planes XoY and YoZ during the simulation

Values of root-mean-square errors (M rms) are contained in table below.

Table 1. Values of root-mean-squares errors during the simulation

Values	*Plane XoY*	*Plane XoZ*	*Plane YoZ*	*Space XYZ*
1M rms (p = 0.65)	10.15 m	9.93 m	4.67 m	10.57 m
2M rms (p = 0.95)	20.30 m	19.86 m	9.34 m	21.14 m
3M rms (p = 0.98)	30.45 m	29.79 m	14.01 m	31.71 m

It is easy to notice that values of M rms errors are similar to errors contained in SPS-2001. What is more, simulated nominal GPS constellation guaranteed that the number of visible satellites is more than 8 (see fig. 4) and in consequence, values of HDOP were less than 2 (see fig. 5) and parameters δH_{max} did not exceed 2.6 (see fig. 6).

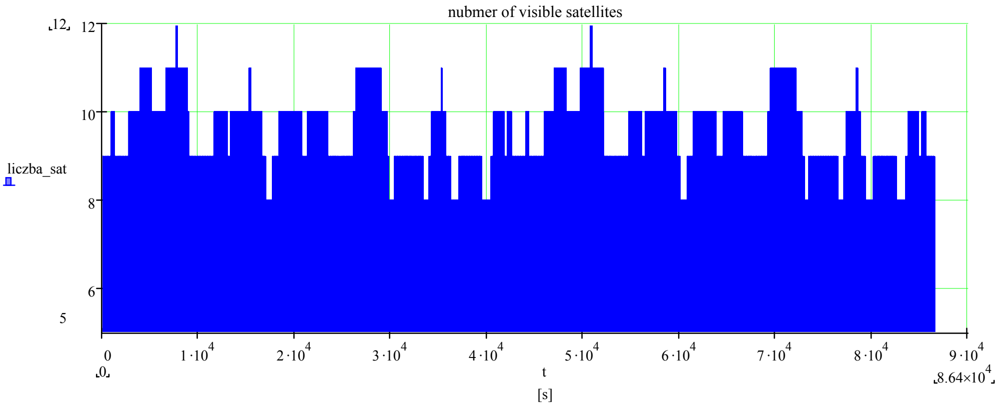

Fig. 4. Number of visible satellites during the simulation

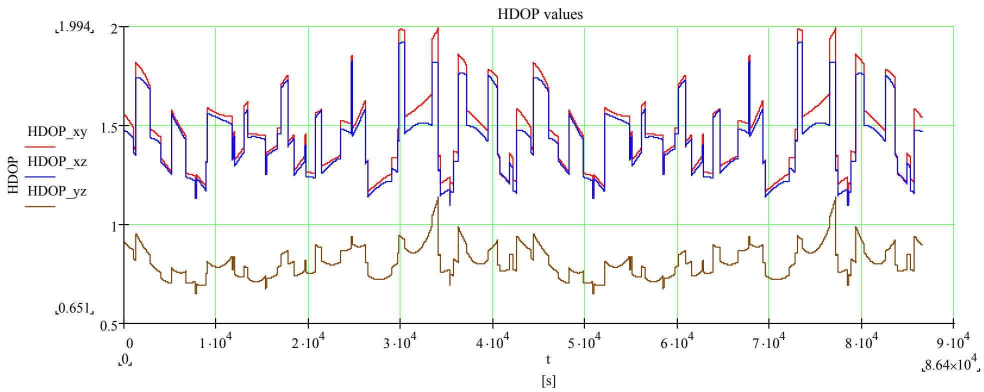

Fig. 5. HDOP values in planes XoY, XoZ and YoZ during the simulation

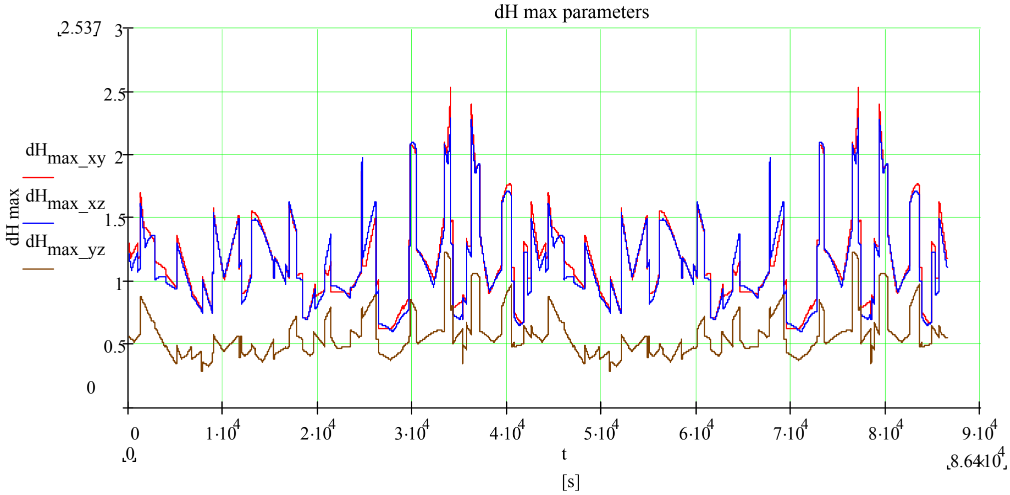

Fig. 6. Values of $\delta H_{\max}$ parameters in planes XoY, XoZ and YoZ during the simulation

In consequence, values of Approximate Radial-Error Protected (ARP) were less than 40 m in YoZ plane and less than 80 m in XoY and XoZ planes (see fig. 7).

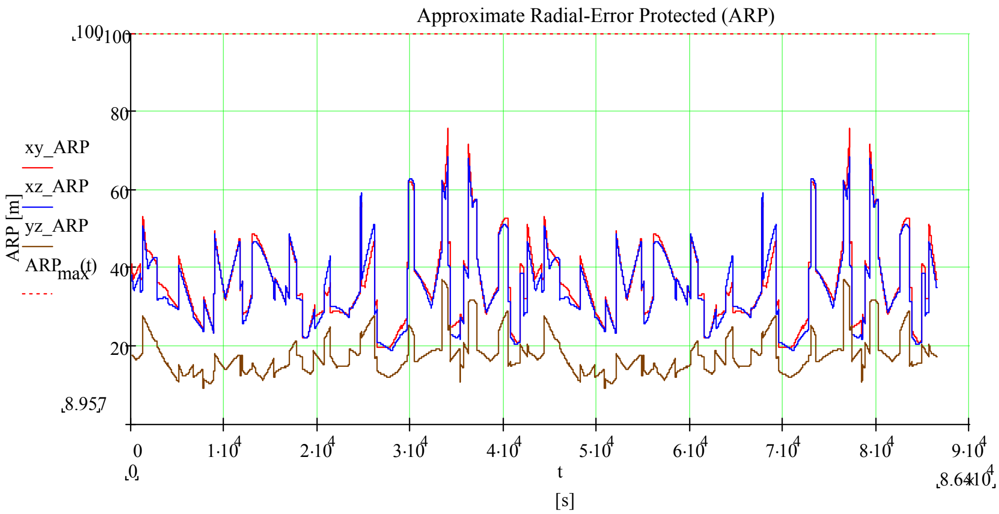

Fig. 7. Values of ARP parameters in planes XoY, XoZ and YoZ during the simulation. $ARP_{max}(t)$ value represents maximum acceptable (ceiling) value of ARP

On fig. 7 we can see that calculated *ARP* values during the simulation have never exceeded maximum acceptable value of *ARP* represented by red dot line. Thus, RAIM availabilities in all planes were equal 1.

The situation completely changed when an open area was replaced by the model of urban area (shown in fig. 2). Fixes errors increase more than two times (see fig. 8).

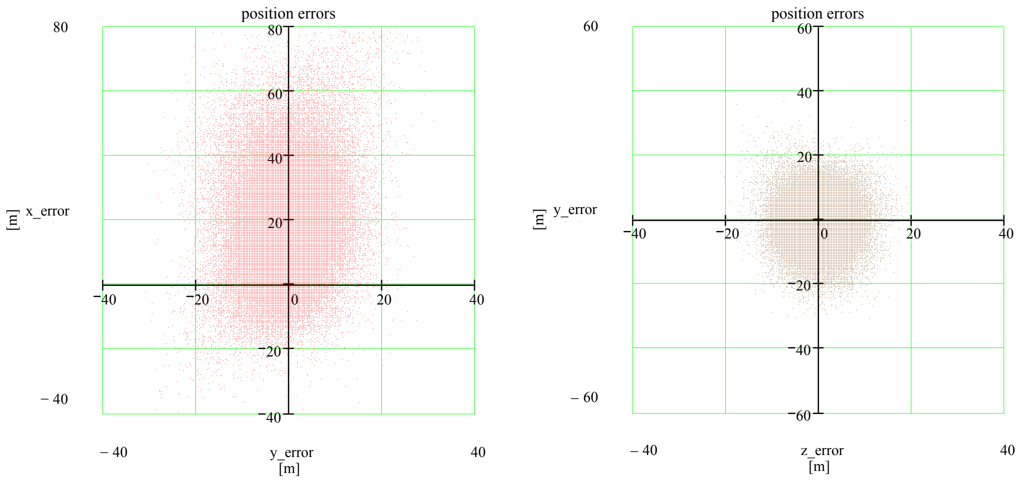

Fig. 8. Fixes errors in planes XoY and YoZ during the simulation

Values of root-mean-square errors (M rms) are contained in table below.

Table 2. Values of root-mean-squares errors during the simulation

Values	***Plane XoY***	***Plane XoZ***	***Plane YoZ***	***Space XYZ***
1M rms (p = 0.65)	25.34 m	24.89 m	7.67 m	25.71 m
2M rms (p = 0.95)	50.68 m	49.78 m	15.34 m	51.42 m
3M rms (p = 0.98)	76.02 m	74.67 m	23.01 m	77.13 m

The fixes errors increased because of less number of visible satellites. Introduced obstructions (buildings) caused that sometimes only 4 satellites were visible (see fig. 9).

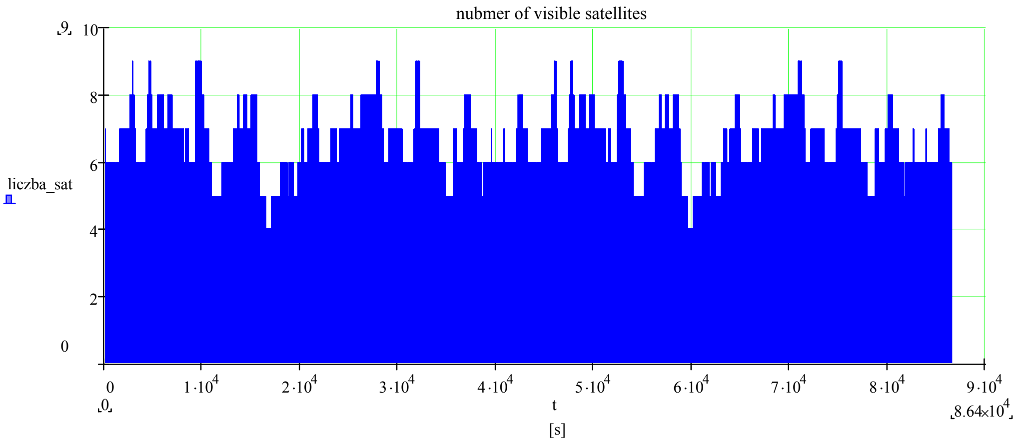

Fig. 9. Number of visible satellites during the simulation

Of course, less number of visible satellites means increasing HDOP values (see fig. 10).

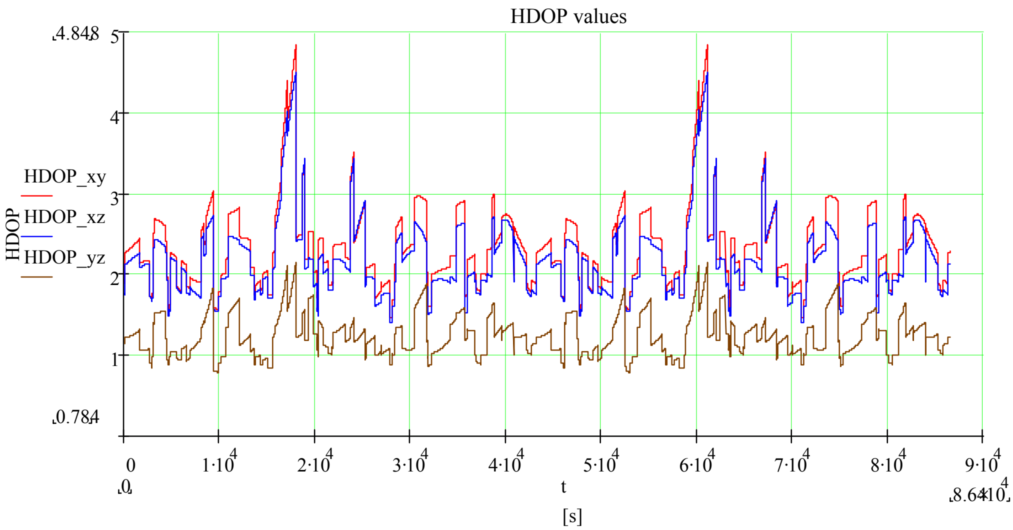

Fig. 10. HDOP values in planes XoY, XoZ and YoZ during the simulation

Less number of visible satellites also causes dramatically increasing of parameters δH_{max} (see fig. 11). Please notice that Y axis is restricted to 50.

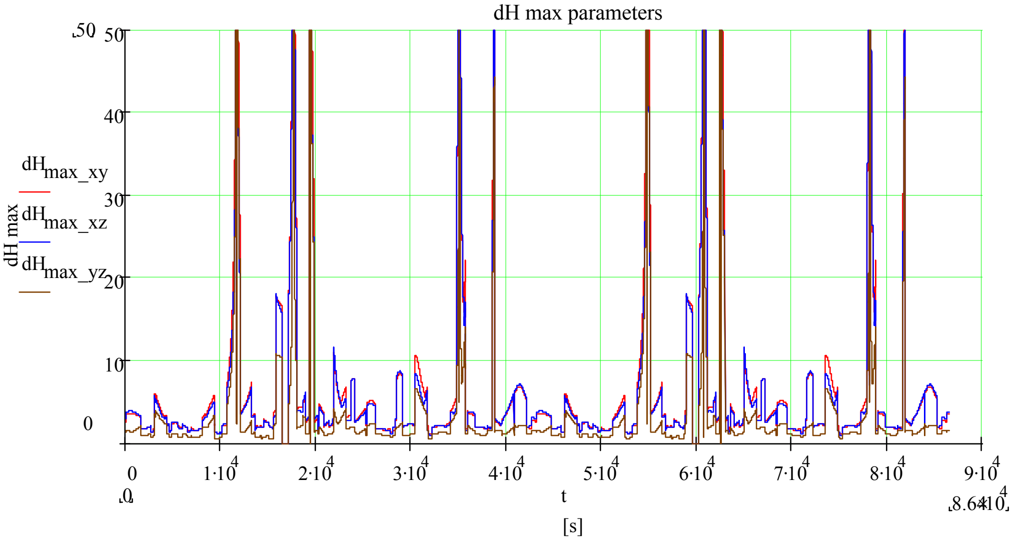

Fig. 11. Values of $\delta H_{\max}$ parameters in planes XoY, XoZ and YoZ during the simulation

In consequence, we have enormous values of Approximate Radial-Error Protected (ARP) (see fig. 12). In this case Y axis was restricted to 1000 m.

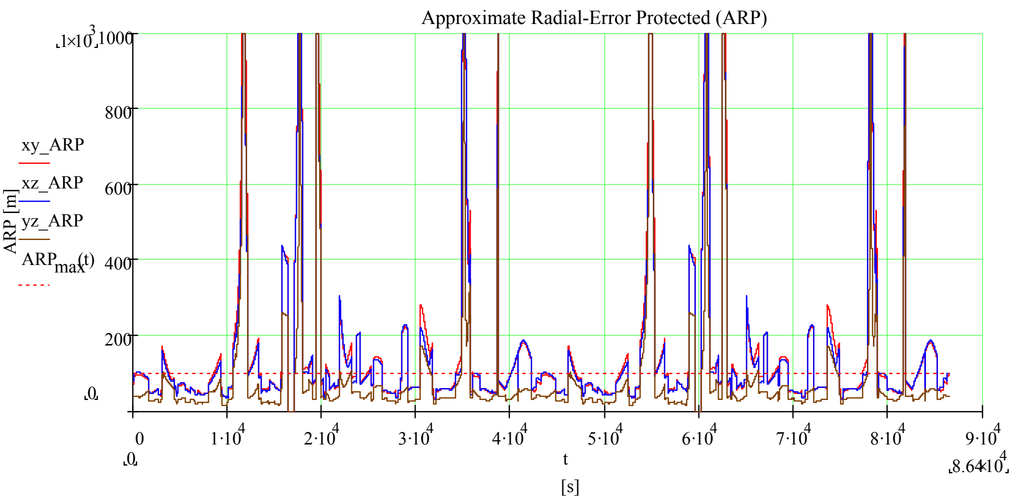

Fig. 12. Values of *ARP* parameters in planes XoY, XoZ and YoZ during the simulation. ARP_{max}(t) value represents maximum acceptable (ceiling) value of *ARP*

In contrast to the simulation which was done in an open area, during the simulation with urban area model, calculated *ARP* values exceeded maximum acceptable value of *ARP* represented by red dot line many times (see fig. 13). Thus, graphs of RAIM availabilities were as follows.

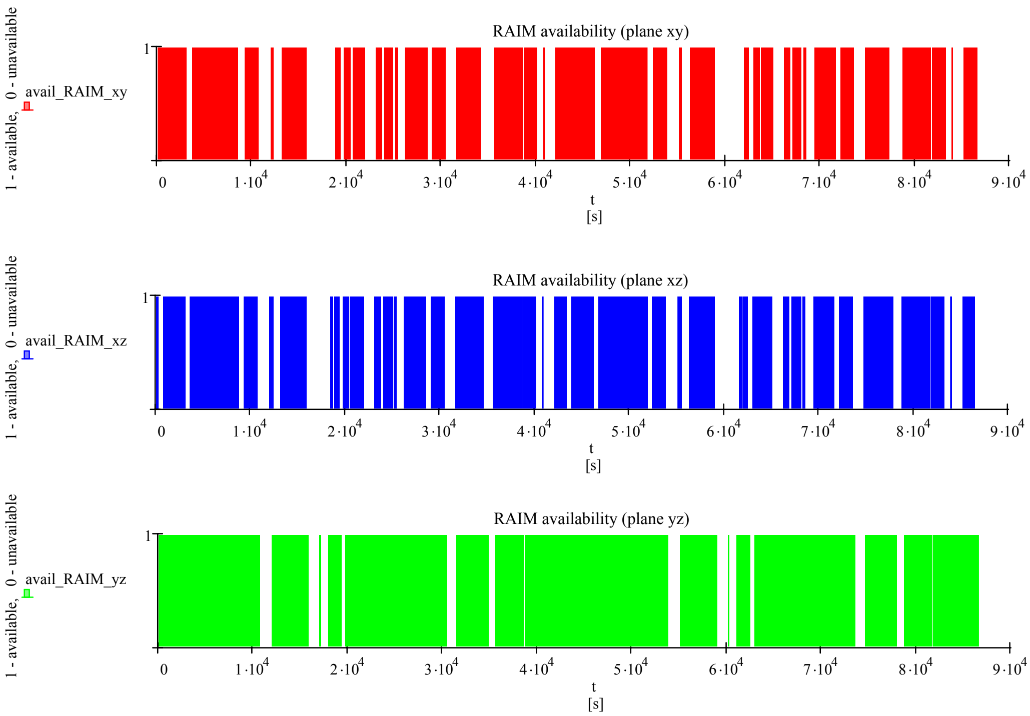

Fig. 13. RAIM availability in planes XoY, XoZ and YoZ during the simulation

Values of RAIM availability during the simulation are contended in table below.

Table 3. Values RAIM availability during the simulation

Values	*Plane XoY*	*Plane XoZ*	*Plane YoZ*
RAIM availability	0.61	0.62	0.85

CONCLUSIONS

The most important conclusions after the simulations are:

1. Present GPS constellation enables proper work of snapshot RAIM algorithms in an open area. RAIM availability is equal almost 1.
2. RAIM availability dramatically decreases in urban areas. It is connected with presence of buildings which block satellite signals. Because of that, number of visible satellites decrease and in consequence ARP values significantly incre-ase.
3. It is necessary to work out alternative methods of autonomous integrity monito-ring in urban areas.
4. Created computer application can be used to simulate of urban area influence on fixes accuracy and RAIM availability.

REFERENCES

[1] Kaplan E. D. (ed.), Understanding GPS: Principles and Applications, Artech House Inc.,1996.

[2] Eurocontrol Experimental Centre, Performance Evaluation of Satellite Naviga-tion and Safety Case Development, European Organisation for the Safety of Air Navigation, 2002.

[3] Brown R. G., A Baseline GPS RAIM Scheme and a Note on the Equivalence of Three RAIM Methods, NAVIGATION, Journal of The Institute of Naviga-tion, 1992, vol. 39, no. 3, pp. 301 – 316.

[4] Sairo H., Combined Performance of FDI and KDOP Analysis for User-Level Integrity Monitoring in Personal Satellite Navigation, 2003.

[5] Sturza M. A., Brown A. K., Comparison of Fixed and Variable Threshold RAIM Algorithms, Proceedings of the 3th International Technical Meeting of The Institute of Navigation Satellite Division, ION-90, 1990

[6] Tsakiri M., Kealy A., Stewart M., NAVIGATION, Journal of The Institute of Navigation, 1999, vol. 46, no. 3.

[7] Brown A. K., Sturza M. A., The Effect of Geometry on Integrity Monitoring Performance, Proceedings of the Institute of Navigation Annual Meeting, 1990.

[8] Brown R. G., Chin G. Y., Kreamer J. H., Update on GPS Integrity Requirements of RTCA MOPS, Proceedings of the 4th International Technical Meeting of the Satellite Division of the Institute of Navigation, ION GPS-91, 1991.

[9] Gajek L., Kałuszka M., Statistical reasoning, models and methods, WNT, Warszawa 1996 (in Polish).

Received October 2006
Reviewed November 2006