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TRAJECTORY PLANNING OF END-EFFECTOR WITH INTERMEDIATE POINT

PLANOWANIE TRAJEKTORII RUCHU CHWYTAKA Z PUNKTEM POŚREDNIM*

The article presents the Polynomial Cross Method (PCM) for trajectory planning of an end-effector with an intermediate point. The PCM is applicable for designing robot end-effector motion, whose path is composed of two rectilinear segments. Acceleration profile on both segments was described by the 7th-degree polynomial. The study depicts an algorithm for the method and the research results presented as the runs of resultant velocity, acceleration and linear jerk of the stationary coordinate system.

Keywords: *trajectory planning, polynomial acceleration profile, jerk.*

W pracy zaprezentowano metodę PCM (Polynomial Cross Method) do planowania trajektorii ruchu chwytaka z punktem pośrednim. PCM ma zastosowanie do planowania ruchu chwytaka, którego tor składa się z dwóch odcinków prostoliniowych. Profil przyspieszenia na obu odcinkach opisany został wielomianem siódmego stopnia. W pracy przedstawiono algorytm metody oraz wyniki w postaci przebiegów prędkości, przyspieszenia i udaru liniowego.

Słowa kluczowe*: planowanie trajektorii, chwytak, wielomianowy profil przyspieszenia, udar.*

1. Introduction

Trajectory planning proves to be the first and critical phase in the operation of robotic workstations (such as supporting of machines, painting, welding, sealing, gluing, cutting, assembly, palletization and depalletization). This problem has been an active field of research and consequently vast literature addresses the issue. The authors have applied various techniques for trajectory generation. Some of them considered the minimization of adverse jerk that causes the practical limitation of trajectory mapping errors. The works of Visioli [10] and Dyllong and Visioli [3] highlight the unfavourable jerk effects at the initial and final point of the path for the cubic and third-order trigonometric splines. Interestingly, in some cases, jerk reduction was achieved by the fourth-order trigonometric spline introduction. One of the criteria for optimization of motion path design given by Choi et al. [2], was to keep the jerk within the specified limits. The obtained jerk profiles in the kinematic pairs are discontinuous and step shaped. At the initial and final point of the trajectory, the jerk is different from zero. Red [7] using the S-curves applied the constant (but different from zero) jerk values at the transition period between the constant phases of acceleration and deceleration. The analysis of the link acceleration profiles for Puma 560 manipulator presented by Rubio et al. [8] indicates that negative jerk effect in the kinematic pairs occurs at the initial and final point of the trajectory. That agrees with the observations made by Saramago and Ceccarelli [9] in their study on jerk runs in the kinematic joints. According to Huang et al. [5], the jerk profiles in the kinematic pairs at the both start and end points motion are close to zero. The method proposed by Olabi et al. [6], generates smooth jerk limited pattern constrained by the laws of tool motion and taking into account the joints kinematics constraints. Very interesting research results on the jerk runs in kinematic pairs were reported by Gasparetto and Zanotto [4]. They obtained not only continuous jerk for the applied fifth-order-B-splines, but importantly, its values at the start and end path point were equal zero. The higher degree polynomials to describe acceleration profile were applied by Boryga and Graboś [1]. The authors analyzed the runs of velocity, acceleration and jerk for polynomials of the 5th, 7th and 9th degree. On the basis of the simulation tests performed, they achieved the lowest values of the linear and angular jerks for the 7th-degree polynomials.

The authors proposed the Polynomial Cross Method (PCM) algorithm, which allows the design of trajectory comprising two rectilinear segments in the robot workspace. There were formulated the following assumptions concerning the manipulator end-effector motion:

- acceleration profile on both rectilinear segments depicted with 7th-degree polynomial,
- acceleration profile at the initial and end path points is tangent to the time axis that eliminates adverse jerk effect,
- change of run-up phase into brake one occurs at the intermediate point,
- linear acceleration value for any coordinate does not exceed the preset maximum value a_{max},
- end-effector motion proceeds so that resultant velocity does not change at the intermediate point (where rectilinear segments connect).

As a consequence of the presumed constant resultant velocity value at the intermediate point, resultant acceleration is equal to zero. It is noteworthy that at the intermediate point, a direction of the resultant velocity vector gets changed due to the preset path of the end-effector. Substantial advantage of the presented algorithm proves to be a fact that coefficients of the polynomials depicting the acceleration profile on any coordinate are established solely on coordinate increment and preset maximum acceleration. In general, the jerk elimination at the initial and final trajectory point influences the accuracy of trajectory mapping. That appears to be very helpful as far as technological processes such as pick and place, painting, assembly, welding, sealing, gluing, palletization and depalletization are concerned. Layout of the paper comprises the following sections: Section 2 depicting a trajectory planning technique with the 7th-degree polynomial application utilizing the root of an equation multiplicity; Section 3 presenting an algorithm, which was divided into initial computations, computations for a longer and shorter rectilinear segment and final computations; Section 4 demonstrates the example of the proposed algorithm practi-

(*) Tekst artykułu w polskiej wersji językowej dostępny w elektronicznym wydaniu kwartalnika na stronie ***www.ein.org.pl*** cal employment, while Section 5 summarizes the simulation results. The conclusions are presented in the last section of the paper.

2. Trajectory planning with polynomial use

The planning of robot end-effector trajectory can be accomplished by using higher-degree polynomials, that facilitate acceleration profile development. The study of Boryga and Graboś [1] showed that among the polynomials of 5th, 7th and 9th-degree describing the acceleration profile, the lowest values of linear and angular jerks were reported for the 7th-degree polynomial. Therefore in this paper, the acceleration profile on coordinate x_i is described with the 7th-degree polynomial in the form of

$$\ddot{x}_i(t) = -a_i \cdot (t)^2 \cdot (t-0.5t_e)^3 \cdot (t-t_e)^2 \tag{1}$$

where: a_i – coefficient of polynomial on coordinate x_i,
$i = 1, 2, 3$ – coordinate number,
t_e – time of motion end.

Acceleration profile described with the 7th-degree polynomial is presented in Fig. 1.

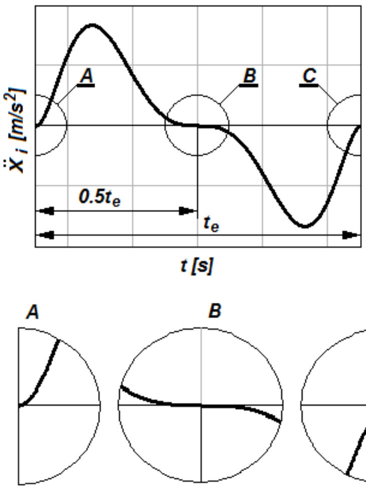

Fig. 1. Acceleration profile described by a 7th - degree polynomial

Acceleration profile is depicted by a continuous function for each coordinate of the Cartesian coordinate system – x_i. Change of the run-up phase into brake phase proceeds at t=0.5t_e and the acceleration profile for t=0, t=0.5t_e and t=t_e is tangent to the time axis. Thus, jerk effect is eliminated at these points. The polynomials describing the profiles of velocity and displacement determined through analytical integration of the dependence (1) go as follows:

$$\dot{x}_i(t) = -a_i \cdot \left(\frac{1}{8}t^8 - \frac{1}{2}t_e t^7 + \frac{19}{24}t_e^2 t^6 - \frac{5}{8}t_e^3 t^5 + \frac{1}{4}t_e^4 t^4 - \frac{1}{24}t_e^5 t^3 \right) \tag{2}$$

$$x_i(t) = -a_i \cdot \left(\frac{1}{72}t^9 - \frac{1}{16}t_e t^8 + \frac{19}{168}t_e^2 t^7 - \frac{5}{48}t_e^3 t^6 + \frac{1}{20}t_e^4 t^5 - \frac{1}{96}t_e^5 t^4 \right) \tag{3}$$

The obtained value $x_i(t)$ is a distance tracked by the robot end-effector on the coordinate i. In order to establish the end-effector coordinate at any moment of time, the following points should be taken into account initial coordinate of end-effector on coordinate i – denoted as x_{bi}, and direction of end-effector motion concordant or discordant with the axis versor orientation. The end-effector coordinate on the coordinate i is defined by the equation

$$x_i(t) = x_{bi} \pm a_i \cdot \left(\frac{1}{72}t^9 - \frac{1}{16}t_e t^8 + \frac{19}{168}t_e^2 t^7 - \frac{5}{48}t_e^3 t^6 + \frac{1}{20}t_e^4 t^5 - \frac{1}{96}t_e^5 t^4 \right) \tag{4}$$

If the end-effector motion is concordant with the versor of the axis i, the plus sign should appear in the equation and the minus one if it is discordant.

3. Planning trajectory with intermediate point

3.1. Polynomial Cross Method (PCM)

PCM is employed to generate end-effector trajectory whose path is composed of two connected rectilinear segments BM and ME (Fig. 2). Implementation of polynomial acceleration profile of the robot end-effector defined with the equation (1) for the preset segments BM and ME could cause that the end-effector velocity at the intermediate point was equal to zero. Thus, the problem of trajectory motion planning would be simplified to the motion with a stop at the intermediate point M.

For that matter, the ancillary points E' and B' are introduced. The E' point arises from the axial symmetry of point B reflected across the intermediate point M, whereas the point B' through the axial symmetry of the point E reflected across the intermediate point M. Description of acceleration defined by the equation (1) includes the segments BE' and $B'E$ (termed total segments in the algorithm). On both total segments, change of the run-up phase into brake one proceeds at the intermediate point M. Maximum acceleration of robot end-effector was limited to a_{max} value. It was assumed that at the transition from

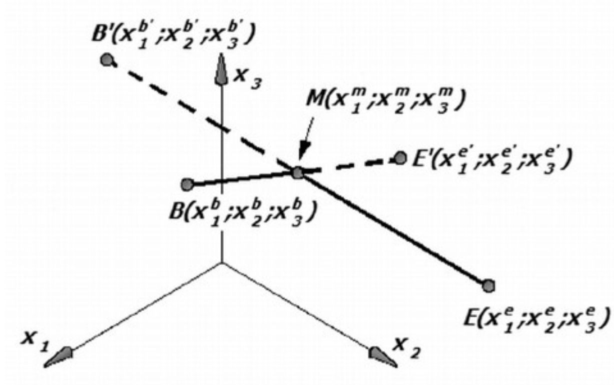

Fig. 2. Planned trajectory BME and ME'and B'M ancillary segments

the BM segment to the ME one (at the intermediate point M), the resultant velocity does not change, while resultant acceleration is equal to zero. Change of the direction and orientation of the velocity vector at the point M is imposed by the predetermined trajectory of end-effector motion. At the intermediate point M, there occured a rotation of the resultant velocity vector from the BM direction towards the ME direction. The problem can be solved through the introduction of the arc connecting the rectilinear segments or an alternative stop in the intermediate point. Coefficients of polynomials depicting acceleration profile on the total segments are determined separately for each coordinate x_i. The motion time is calculated using only the path increments and preset maximum acceleration – a_{max}. Velocity value at the intermediate point is established performing the substition of t=0.5t_e into the dependence describing velocity profile (2). The resultant velocity vector at the intermediate point displaces from one segment to the other and projects on the axes of the stationary coordinate system. That facilitates the determination of coefficients of a polynomial depicting aceleration profile on the other total segment. As the motion time on both total segments may vary, it is necessary to perform an appropriate translation in the time of acceleration, velocity, displacement and jerk profile.

3.2. PCM algorithm

3.2.1. Initial computations

Step 1. Assumption of coordinates of the initial, intermediate and final points are denoted by $B(x_1^b;x_2^b;x_3^b)$, $M(x_1^m;x_2^m;x_3^m)$ and $E(x_1^e;x_2^e;x_3^e)$. The points should belong to the workspace.

Step 2. Determination of the coordinates of ancillary points $B'(x_1^{b'};x_2^{b'};x_3^{b'})$ and $E'(x_1^{e'};x_2^{e'};x_3^{e'})$ are made on the grounds of dependence

$$x_i^{b'} = 2x_i^m - x_i^e \quad \text{for} \quad i = 1, 2, 3 \tag{5}$$

$$x_i^{e'} = 2x_i^m - x_i^b \quad \text{for} \quad i = 1, 2, 3 \tag{6}$$

The B' and E' ancillary points need not belong to the workspace. The ancillary distances $B'M$ and ME' are used only to construct an appropriate form of the acceleration profile.

Step 3. Determination of path increments on each coordinate of the total segments

$$\Delta x_i^{BE'} = \left| x_i^{e'} - x_i^b \right| \quad \text{for} \quad i = 1, 2, 3 \tag{7}$$

$$\Delta x_i^{B'E} = \left| x_i^e - x_i^{b'} \right| \quad \text{for} \quad i = 1, 2, 3 \tag{8}$$

Step 4. Scheduling of the coordinate increments starting from the highest, with denotation by subscript in the brackets, in the schedule sequence

$$\Delta x_{\{1\}}^{BE'} \ge \Delta x_{\{2\}}^{BE'} \ge \Delta x_{\{3\}}^{BE'} \tag{9}$$

$$\Delta x_{\{1\}}^{B'E} \ge \Delta x_{\{2\}}^{B'E} \ge \Delta x_{\{3\}}^{B'E} \tag{10}$$

Step 5. Determination of maximum coordinate increment out of BE' and $B'E$ segments and denoting it as $\Delta x_{\{1\}}^L$, that is,

$$\Delta x_{\{1\}}^L = \max\{\Delta x_{\{1\}}^{BE'}, \Delta x_{\{1\}}^{B'E}\} \tag{11}$$

In the $\Delta x_{\{1\}}^L$ denotation, a superscript describes the longer total segment. If $\Delta x_{\{1\}}^{BE'} = \Delta x_{\{1\}}^{B'E}$ increments are equal then $\Delta x_{\{1\}}^{BE'} = \Delta x_{\{1\}}^L$.

Step 6. Assumption of end-effector maximum acceleration a_{max} on the coordinate of the maximum path increment, $\Delta x_{\{1\}}^L$. Thus, the accelerations on the other coordinates will not exceed the preset acceleration a_{max} that results from lower or equal path increments on these coordinates.

3.2.2. Computations longer total segment (L)

Step 1. Determination of polynomial $a_{\{1\}}^L$ coefficient and the end time of motion – t_e^L on coordinate $x_{\{1\}}^L$ requires solution of the equation system

$$\begin{cases} \dfrac{1}{c_1} a_{\{1\}}^L (t_e^L)^9 = \Delta x_{\{1\}}^L \\ -a_{\{1\}}^L (c_2 t_e^L)^2 (c_2 t_e^L - 0.5 t_e^L)^3 (c_2 t_e^L - t_e^L)^2 = a_{\max} \end{cases} \tag{12}$$

Having solved the above equation system, the below was obtained:

$$a_{\{1\}}^L = \frac{c_1}{(\sqrt{c_1 c_3})^9} \cdot \frac{a_{\max}^5}{\sqrt{a_{\max}}\,(\Delta x_{\{1\}}^L)^3 \sqrt{\Delta x_{\{1\}}^L}} \tag{13}$$

$$t_e^L = \frac{\sqrt{c_1 c_3}\sqrt{a_{\max}\Delta x_{\{1\}}^L}}{a_{\max}} \tag{14}$$

where:

$$c_1 = 10080\,,\ c_2 = \frac{1}{2} - \frac{1}{14}\sqrt{21}\,,\ c_3 = -\frac{1}{8}c_2^2(2c_2 - 1)^3(c_2 - 1)^2$$

Step 2. Determination of polynomial coefficients for the other coordinates [1]

$$a_{\{i\}}^L = \frac{c_1}{(t_e^L)^9}\Delta x_{\{i\}}^L \quad \text{for} \quad i = 2, 3 \tag{15}$$

Step 3. Determination of components and end-effector resultant velocity at M point

$$\dot{x}_{\{i\}M}^L = \frac{(t_e^L)^8}{6144} a_{\{i\}}^L \quad \text{for} \quad i = 1, 2, 3 \tag{16}$$

$$\dot{x}_M^L = \frac{(t_e^L)^8}{6144}\sqrt{\sum_{i=1}^{3}(a_{\{i\}}^L)^2} \tag{17}$$

3.2.3. Computations shorter total segment (S)

Step 1. Determination of direction cosines between the speed vector in the M point and the axes of the stationary coordinate system

$$\cos(\alpha_i) = \frac{x_i^{e'} - x_i^b}{\sqrt{\sum_{i=1}^{3}(x_i^{e'} - x_i^b)^2}} \quad \text{if} \quad \Delta x_{\{1\}}^{B'E} > \Delta x_{\{1\}}^{BE'} \tag{18}$$

$$\cos(\alpha_i) = \frac{x_i^e - x_i^{b'}}{\sqrt{\sum_{i=1}^{3}(x_i^e - x_i^{b'})^2}} \quad \text{if} \quad \Delta x_{\{1\}}^{B'E} \le \Delta x_{\{1\}}^{BE'} \tag{19}$$

where: α_I– angle between the speed vector at M point and axis x_i of the stationary coordinate system.
Speed vector orientation comes from a motion direction on a segment.

Step 2. Determination of speed components in M point

$$\dot{x}_{iM}^S = \dot{x}_M \cdot \cos(\alpha_i) \quad \text{for } i = 1, 2, 3 \tag{20}$$

According to the assumption, the resultant speed value in the M point $\dot{x}_M^L = \dot{x}_M^S = \dot{x}_M$, does not change.

Step 3. Determination of motion time for shorter total segment

$$t_e^S = \frac{105}{64}\frac{\Delta x_i^S}{\dot{x}_{iM}^S} \quad \text{for } i = 1 \tag{21}$$

Formula (21) results from a system of equations formed from the dependences (15) and (16). The same motion time is obtained when appropriate coordinate increments and appropriate velocity components at point M are substituted simultaneously.

Step 4. Determination of polynomial coefficients on each coordinate

$$a_i^S = \frac{6144}{(t_e^S)^8}\dot{x}_{iM}^S \quad \text{for } i = 1, 2, 3 \tag{22}$$

3.2.4. Final computations

Step 1. Determination of motion start time on the $B'E$ segment

$$t_b = \pm\frac{t_e^L - t_e^S}{2} \tag{23}$$

The time is established so as to obtain the same velocity at exactly the same moment in the M point for both move segments. If $\Delta x_{\{1\}}^{B'E} > \Delta x_{\{1\}}^{BE'}$, dependence (23) should acquire the minus sign, the opposite case should acquire the plus sign.

Step 2. Time displacement of the polynomial depicting the acceleration profile on the coordinates of $B'E$ distance by t_b value

$$\ddot{x}_i(t) = -a_i^L \cdot (t - t_b)^2 \cdot (t - 0.5t_e^L - t_b)^3 \cdot (t - t_e^L - t_b)^2 \ \text{ if } \Delta x_{\{1\}}^{B'E} > \Delta x_{\{1\}}^{BE'} \tag{24}$$

$$\ddot{x}_i(t) = -a_i^S \cdot (t - t_b)^2 \cdot (t - 0.5t_e^S - t_b)^3 \cdot (t - t_e^S - t_b)^2 \ \text{ if } \Delta x_{\{1\}}^{B'E} \le \Delta x_{\{1\}}^{BE'} \tag{25}$$

Analogical time displacement should be done for polynomials, describing the level of velocity, displacement, and jerk.

Step 3. Determination of motion time on the segments along the BME path

$$t_e = \frac{t_e^L + t_e^S}{2} \tag{26}$$

Table 1. The point coordinates for the planned trajectory and ancillary points

Point denotation	Point coordinates [m]		
	x_1	x_2	x_3
B	*0.5*	*0.5*	*1*
M	*0.5*	*0.75*	*1.25*
E	*0.75*	*0.75*	*1.5*
B'	*0.25*	*0.75*	*1*
E'	*0.5*	*1*	*1.5*

4. Numerical example

The point coordinates for the planned end-effector trajectory B, M, E and ancillary points E' and B' are presented in Table 1. The path increments on each coordinate are: $\Delta x_1^{BE'} = 0$, $\Delta x_2^{BE'} = \Delta x_3^{BE'} = 0.5\,m$, $\Delta x_1^{B'E} = \Delta x_3^{B'E} = 0.5\,m$, $\Delta x_2^{B'E} = 0$. Since $\Delta x_{\{1\}}^{BE'} = \Delta x_{\{1\}}^{B'E}$, BE' will be the first segment to study.

The maximum acceleration set is $a_{max} = 2\ m/s^2$ on coordinate x_2. The polynomial coefficients depicting acceleration level on each coordinate as well as motion time go as following: $a_{\{1\}}^L = a_{\{2\}}^L = 354.616\,m/s^9$, $a_{\{3\}}^L = 0$, $t_e^L = 1.343\,s$. As for the BE' distance, path increments are recorded for the coordinates x_2 and x_3 so consequently, the established coefficients $a_{\{1\}}^L$ and $a_{\{2\}}^L$ refer to these coordinates. Resultant speed in M point is $\dot{x}_M^L = 0.864\,m/s$. The direction cosines of a speed vector in M point for the $B'E$ segment go as follows $\cos\alpha_1 = \sqrt{2}/2$, $\cos\alpha_2 = 0$, $\cos\alpha_3 = \sqrt{2}/2$. The velocity components in M point on the $B'E$ segment are $\dot{x}_{1M}^S = \dot{x}_{3M}^S = 0.611\,m/s$, $\dot{x}_{2M}^S = 0$, whereas 1, 2 and 3 indices refer to the axis of the stationary coordinate system. Polynomial coefficients describing acceleration profile on each coordinate of the $B'E$ segment are $a_1^S = a_3^S = 354.616\,m/s^9$, $a_2^S = 0$. Move time recorded on the $B'E$ segment was $t_e^S = 1.343\,s$. Time of motion along the BME path is $t_e = 1.343\,s$, while $t_b = 0$.

5. Simulation tests results

According to the simulation tests performed, the following courses of kinematic characteristics of end-effector were recorded (Fig. 3 – 5). In each figure presented, a continuous line indicates the runs of kinematic characteristics of motion for the designed trajectory BME, while a dashed line – the courses for ancillary segments (ME' and $B'M$). A planned motion path and ancillary distances are displayed in the stationary coordinate system $x_1x_2x_3$, whereas kinematic characteristics of motion at two planes perpendicular to the plane determined by the points of the generated BME trajectory.

Fig. 3 displays the runs of end-effector speeds on the BE' and $B'E$ segments. The maximum velocity $\dot{x}_M^L = \dot{x}_M^S = 0.864\,m/s$ is obtained at the point M. The speed value at transition from the BM segment to ME did not change, while a direction of the resultant velocity vec-

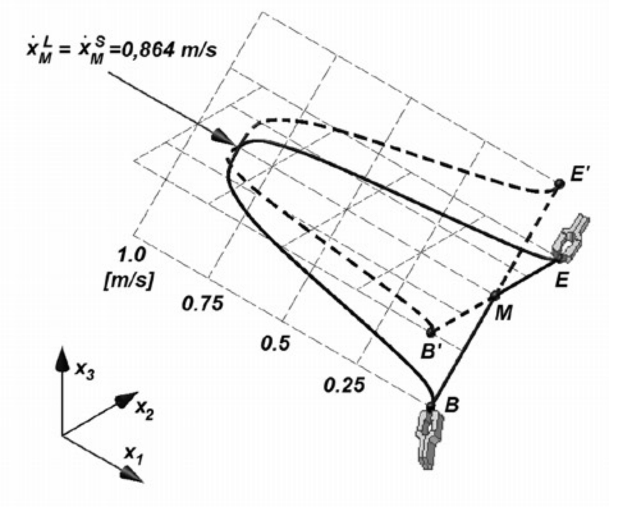

Fig. 3. Resultant velocity course along planned BME path and ancillary segments

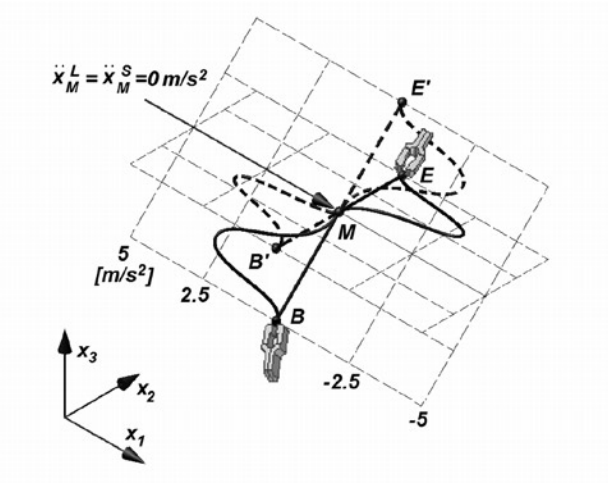

Fig. 4. Resultant acceleration course along planned BME trajectory and ancillary segments

tor changes from *BM* path to *ME*. The runs of end-effector resultant linear acceleration on the distances *BE'* and *B'E* are presented in Fig. 4. In the points *B*, *M*, *E* acceleration is equal to zero. The obtained absolute maximum acceleration at each coordinate does not surpass the set value a_{max} = 2 m/s^2 and the maximum resultant value reaches 2.83 m/s^2. A jerk value at points *B* and *E* is equal zero (Fig. 5). The maximum jerk value is 19.8 m/s^3.

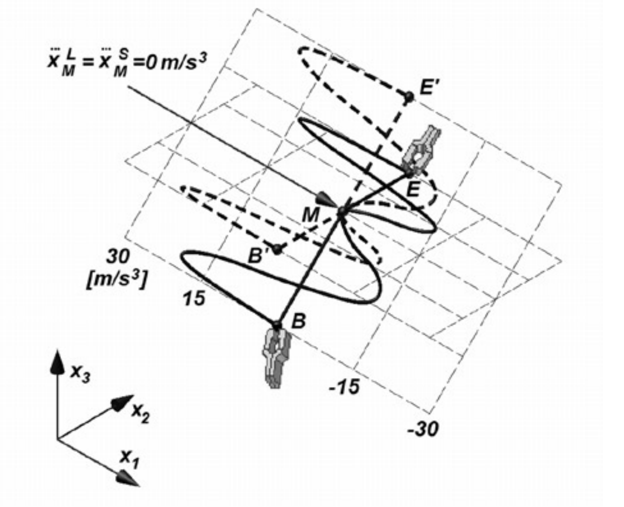

Fig. 5. Resultant jerk course along planned BME path and ancillary segments

6. Conclusions

On the basis of the simulation tests of the manipulator end-effector motion according to the PCM, the following conclusions were formulated:

a) The profiles of resultant velocity, acceleration and jerk obtained by the PCM application are continuous on the *BM* and *ME* segments. At the intermediate point *M*, resultant velocity value does not change, consistently with the underlying assumption. The velocity vector direction changes according to the motion direction (from *BM* towards *ME*).
b) Generation of trajectory according to the PCM may be utilized in some technological processes (pick and place, painting, assembly, welding, sealing, gluing, palletization and depalletization), where it is critical to eliminate jerk effect in the initial and final point of the trajectory.
c) If deformability of kinematic chain occurs, jerk elimination will result in vibration limitation that guarantees lower tracking errors.

Our further research will focus on the effect of trajectory of the robot manipulator end-effector (planned using PCM) on kinematics and manipulator dynamics.

References

1. Boryga M, Graboś A. Planning of manipulator motion trajectory with higher-degree polynomials use. Mechanism and Machine Theory 2009; 44: 1400–1419.
2. Choi YK, Park JH, Kim HS, Kim JH. Optimal trajectory planning and sliding mode control for robots using evolution strategy. Robotica 2000; 18: 423–428.
3. Dyllong E, Visioli A. Planning and real-time modifications of a trajectory using spline techniques. Robotica 2003; 1: 475–482.
4. Gasparetto A, Zanotto V. Optimal trajectory planning for industrial robots. Advances in Engineering Software 2010; 41: 548–556.
5. Huang P, Xu Y, Liang B. Global minimum-jerk trajectory planning of space manipulator. International Journal of Control, Automation, and Systems 2006; 4 no. 4: 405–413.
6. Olabi A, Béarée R, Gibaru O, Damak M. Feedrate planning for machining with industrial six-axis robots. Control Engineering Practice 2010; 18(5): 471–482.
7. Red E. A dynamic optimal trajectory generator for Cartesian Path following. Robotica 2000; 18: 451–458.
8. Rubio FJ, Valero FJ, Suñer JL, Mata V. Simultaneous algorithm to solve the trajectory planning problem. Mechanism and Machine Theory 2009; 44: 1910–1922.

9. Saramago SFP, Ceccarelli M. An optimum robot path planning with payload constraints. Robotica 2002; 20: 395–404.
10. Visioli A. Trajectory planning of robot manipulators by using algebraic and trigonometric splines. Robotica 2000; 18: 611–631.

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