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Solution Methods of Overhead Transmission Line Mechanics

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Summary: Paper is focused to mechanics calculation of overhead transmission lines. In first part of this paper is presented mathematical model for complex solution of overhead line mechanics in a plan. Non-level overhead line can be considered. The second part of paper deals with a solution method of the mechanics of overhead line conductors in three-dimensional space in non- level span. This method enables to define three-dimensional location, vector of mechanics stress of suspended line in optional point of line under combined load caused by the ice and the wind of optional direction.

Keywords:
overhead transmission lines mechanics, mechanical tension, catenaries curve

1. INTRODUCTION

Calculation of transmission lines mechanics is necessary not only for design of overhead transmission lines but for practical application on existing lines, too. From this point of view is needed to use general methods that calculate also non - symmetrical states of mechanical load. Extended application of computers allows using accurate methods of solution without same simplifications.

Mathematical method of solution of overhead power lines mechanics consider with plane location of the all anchor distance of the line. Direction of this plane is given by direction of resultant mechanics load caused by weight of conductors, insulators and ice coating. This mathematical model insufficient provides activity of the wind in arbitrary direction. It is presented approach of calculate of overhead line conductor under combined wind and ice load, therefore is created the base of solution of overhead line conductors mechanics in three-dimensional space in non level span.

2. MATHEMATICAL MODEL OF SOLUTION CONDUCTOR MECHANICS

If overhead line is described as unit, not only mechanical properties of wires or insulators should be considered. Mechanical properties of towers and its foundations must be respected too.

The method solves real state mechanics of overhead transmission lines. The proposed mathematical model is coming out from till ignored assumptions. Mainly, it is elasticity of tower frame and instability of tower foundation. From mathematical point of view, the calculating of mechanics with rigid tower assumption has many advantages, but it does not reflect real state of suspension tower under extreme longitudinal loads.

In additional, next factors will be considered in presented mathematical model:
— spans with different suspension point level
— non-fixed suspension insulator strings

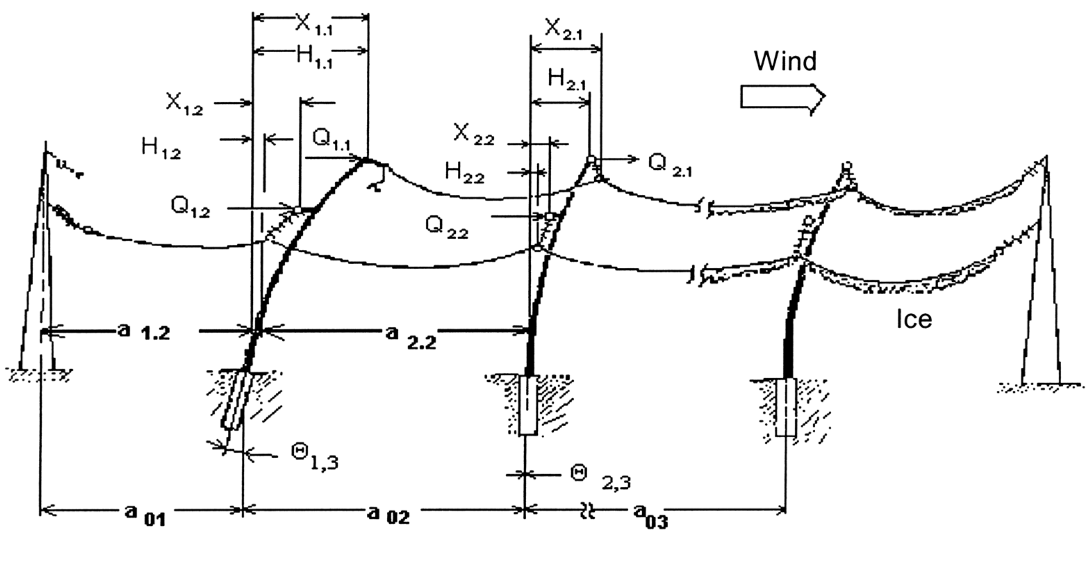

Fig. 1. Static loads of cables and towers

— non-elastic catenaries form of suspension cable

For static loads of cables and towers in the transmission line plane it is possible to calculate such climatic effects as ice, wind, temperature variation etc. (see -Figure1).

2.1. Conditions of calculating

1. The transmission line system is essentially straight containing no large angles.
2. The insulators may be the suspension type (single-string or V-string) or dead-end type
3. The suspension towers are linearly elastis undergo longitudinal strain and displacements. The dead-end towers are rigid structures, to give a boundary conditions of solution.
4. The equilibrium state of transmission line system is given by suspension towers equilibrium thus; equilibrium equations at each of attachment points have a zero value (or equation value is under required accuracy of solution).

2.2. Equilibrium equations

With these assumptions, a new solution method was developed based on a direct stiffness formulation for line system. This method involves the determination of stiffness relationships for every element (the relationships that express the forces in the elements in terms of displacements). The equilibrium equations at each of the suspension tower attachment points are written in terms of displacements by use of stiffness relationships. Finally, the equilibrium equations are solved for the displacements. The equilibrium equation for i-th suspension tower has a next shape

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_7 \\ 0 \end{Bmatrix} + \begin{Bmatrix} R_{i,1} \\ R_{i,2} \\ \vdots \\ R_{i,7} \\ 0 \end{Bmatrix} - \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{18} \\ & k_{22} & \cdots & k_{28} \\ & & \ddots & \vdots \\ & & & \\ SYM & & & k_{88} \end{bmatrix} \cdot \begin{Bmatrix} X_{i,1} \\ X_{i,2} \\ \vdots \\ X_{i,7} \\ \Theta_{i,8} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{Bmatrix} \quad (1)$$

where:

Q — is vector of longitudinal wind forces,

R_i — is vector of mechanical tension differences in wires of adjacent spans,

k — is stiffness matrix of the suspension support, where the element $k_{i,j}$ represents force reaction in point i when point j is replaced about 1 m in line direction; element $k_{8,8}$ represents moment properties of tower base

X_i — is vector of suspension tower attachment point displacements

Wind forces, in equilibrium equation (1), are acting in attachment points. Therefore is necessary, that total wind pressure is re-calculated to the equivalent partial forces acting only in attachment points of suspension tower.

To determine of each element of vector **R** is needed to define horizontal parts of mechanical tension or force $F_{Hi,j}$ in each wire of anchor line section. These forces we can obtain as solution of the system of the next non-linear equations:

$$f_{i,j} = \frac{2.F_{H1i,j}}{q_{i,j}} \cdot arcsin\,h \cdot \quad (2)$$

$$\cdot \left(\frac{q_{i,j}}{2.F_{H1i,j}} \cdot \sqrt{l_{0i,j}^2 . \left(1 + \alpha_{i,j}.\Delta\vartheta + \frac{F_{H1i,j} - F_{H0i,j}}{E_{i,j}.S_{i,j}} \right)^2 - h_{1i,j}^2} \right) -$$

$$-a_{i,j} + H_{i,j} - H_{i,j+1}$$

where:

$i = 1 \ldots N$; N — number of wires in the span

$j = 1 \ldots M$; M — is number of spans

b — insulator length [m]

E — modulus of elasticity [MPa]

$F_{H1(0)}$ — horizontal part of wire force in state '1' or '0' [N]

F_{V1} — vertical part of wire force in state '1' [N]

h_1 — level difference of attachment points [m]

q — weight of wire [N.m^{-1}]

S — cross section of wire [m^2]

α — coefficient of linear thermal expansivity [K^{-1}]

$\Delta\vartheta$ — difference of temperatures in state '1' and '0' [K]

$l_{0i,j}$ — length of wire in state '0' [m]

System of equations is possible to solve by the Newton method of non-linear equations solution.

Displacements of nodal points $H_{i,j}$ (see — Figure 2) are given by the equilibrium condition of suspended insulator:

$$H_{i,j} - X_{i,j} = \frac{R_{i,j}.b_{i,j}}{\sqrt{R_{i,j}^2 + F_{V1i,j}^2}} \quad (3)$$

When all values of vectors **R** and **Q** for any suspension towers are known, system of equilibrium equations (1) can be created. Number of equations is equal number of suspension towers. Results of solution are nodal point

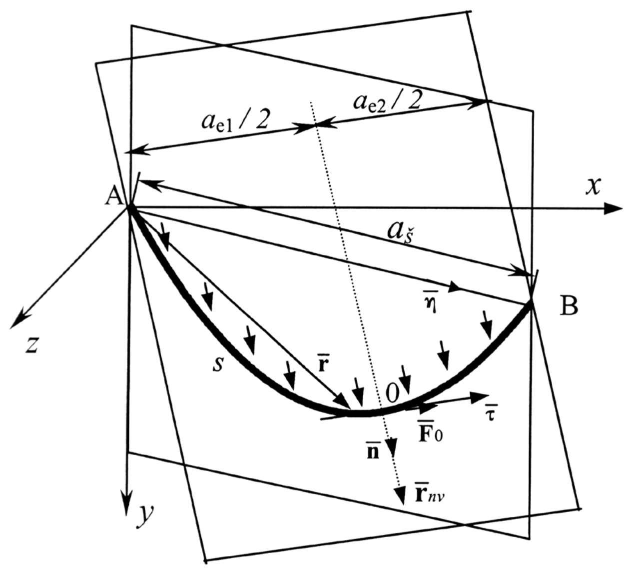

Fig. 2. Deflected conductor in vector so-ordinate system

Equeation 4. (see text)

$$\frac{\partial \bar{\mathbf{F}}(s)}{\partial s}\cdot\Delta s+\left(\bar{\mathbf{q}}_1+\bar{\mathbf{q}}_2\right)\cdot\Delta s+w_0\cdot\left(\frac{v}{29,6}\right)^2\cdot\alpha\cdot C\cdot d\cdot\left|\bar{\boldsymbol{\eta}}\times\bar{\boldsymbol{\rho}}\right|\cdot\bar{\boldsymbol{\rho}}\cdot\Delta s=0$$

$$\underbrace{\Delta\bar{\mathbf{F}}}\ +\ \underbrace{\underbrace{\Delta\bar{\mathbf{g}}}\ +\ \underbrace{\Delta\bar{\mathbf{r}}_v}}_{\Delta\bar{\mathbf{r}}_{nv}}\ =0$$

displacements of any suspension towers, insulators displacements and mechanical tensions in any wires of anchor line section.

3. VECTOR METHOD OF SOLUTION CONDUCTOR MECHANICS

The most universal method solving of mechanics of overhead lines under optional overloading is vector method. Supposition of this method is continually distributed mechanics load along conductor length. It is the shape of the suspended conductor know as catenaries curve. The position of conductor is given by the direction of resultant load and flow line of suspension points.

Deflected conductor in vector co-ordinate system (Equation 4) where:

$\Delta\bar{\mathbf{F}}$ — resultant force functioning to the conductor element with length Δs,

$\bar{\mathbf{F}}(s)$ — vector of tension force in conductor in position s. This vector is given by equation:

$$\bar{\mathbf{F}}(s)=F(s)\cdot\frac{\partial\bar{\mathbf{r}}}{\partial s}\qquad(5)$$

q_1 — unit weight of conductor [N.m^{-1}],

q_2 — ice load [N.m^{-1}],

$\bar{\boldsymbol{\rho}}$ — unit vector of wind direction,

$\bar{\boldsymbol{\eta}}$ — direction of the span,

$\Delta\bar{\mathbf{g}}$ — vector of weight of conductor element with unit weight of conductor q_1 [N.m^{-1}],

$\Delta\bar{\mathbf{r}}_v$ — vector of static component of wind loading functioning to the conductor element with length Δs in direction of vector $\bar{\boldsymbol{\rho}}$ at speeds of v [m.s^{-1}],

$\Delta\bar{\mathbf{r}}_{nv}$ — vector of resultant functioning to the conductor element with length Δs

ω_0 — standard pressure of wind [Nm^{-2}]

α — coefficient of non-uniformity of wind pressure [-]

C — shaping coefficient of conductor [-]

d — diameter of conductor [m]

Determination of positional vector $\bar{\mathbf{r}}$ and tension force $\bar{\mathbf{F}}$ dependency of conductor length s is given by solution of system of equation (4) and (5), i.e.:

$$\bar{\mathbf{r}}=\left[\frac{a_{e1}}{2}+\frac{F_0}{r_{nv}}\cdot arcsinh\left(\frac{r_{nv}}{F_0}\cdot s-sinh\frac{r_{nv}\cdot a_{e1}}{2\cdot F_0}\right)\right]\cdot\bar{\boldsymbol{\tau}}+$$
$$+\frac{F_0}{r_{nv}}\left(cosh\frac{r_{nv}\cdot a_{e1}}{2\cdot F_0}-\sqrt{1+\left(\frac{r_{nv}}{F_0}\cdot s-sinh\frac{r_{nv}\cdot a_{e1}}{2\cdot F_0}\right)^2}\right)\cdot\bar{\mathbf{n}}\qquad(6)$$

$$\bar{\mathbf{F}}=F_0\cdot\bar{\boldsymbol{\tau}}-r_{nv}\cdot\left(s-\frac{F_0}{r_{nv}}\cdot sinh\frac{r_{nv}\cdot a_{e1}}{2\cdot F_0}\right)\cdot\bar{\mathbf{n}}\qquad(7)$$

Equation of state of conductor in vector form

Equation of state of conductor in vector form determines conductor's mechanics stress σ_H dependency of ambient temperature ϑ and of overloading z caused by ice and wind load. It's assigned by form (8), where index "0" indicates conductor in initial state (temperature –5°C and ice load in middle ice locality), index "1" indicates conductor in sought state.

$$\sigma_{H1}^3+\sigma_{H1}^2\cdot sin\left(\bar{\boldsymbol{\eta}}\cdot\bar{\mathbf{n}}_1\right)\cdot\qquad(8)$$
$$\cdot\left[A\cdot\left(\frac{a_{\check{s}}\cdot z_0}{\sigma_{H0}}\right)^2\cdot sin^4\left(\bar{\boldsymbol{\eta}}\cdot\bar{\mathbf{n}}_0\right)+B\cdot\left(\vartheta_1-\vartheta_0\right)-\frac{\sigma_{H0}}{sin\left(\bar{\boldsymbol{\eta}}\cdot\bar{\mathbf{n}}_0\right)}\right]=$$
$$=A\cdot\left(a_{\check{s}}\cdot z_1\right)^2\cdot sin^5\left(\bar{\boldsymbol{\eta}}\cdot\bar{\mathbf{n}}_1\right)^2$$

A, B — constants respected the mechanics elasticity and thermal extensibility of line conductor.

Presented equation of state regards conductor's ice loading, wind loading in optional direction (vector $\bar{\mathbf{n}}$) and computes with optional level difference of hanging points of conductor (vector $\bar{\boldsymbol{\eta}}$).

4. CONCLUSION

Mathematical methods used up to now were designed on simplified assumptions to make easer of calculating process. The on–line applications calculating accuracy must be very high, therefore inaccurate methods using is not possible. Designed mathematical model is more accurate and it allows calculating such parameters as mechanical tension in wires, insulators deflection, elastic towers deflection in nodal points for non-symmetrical longitudinal loads (ice accretion, wind, wire breakdown etc.). When solution of foundation stability is known, it can be included to the calculating.

Vector method deals with effect of combined wind and ice load of line conductor. Approach of calculation three— dimensional location and force tension of suspended line, which is deflected by the wind in optional direction, was determined by vector method. Versatility of this method is modified by equation of state of conductor in vector form. Mathematical model shown in this paper is basis of complex solution of overhead line mechanics in non-level span in three- dimensional space.

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