

# Decomposition of EMI Noise into Common and Differential Modes in PWM Inverter Drive System

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**Summary:** In this paper an analytical approach to decomposition of total phase EMI noises through algebraic calculations and the Fourier transform has been presented. Separating conducted EMI noise into different, common and differential mode (CM and DM) HF current components is important for a proper spectral analysis and application of EMI mitigation techniques. The results of calculations have been compared to EMI currents measured on the motor side of the PWM drive system. The proposed approach can be useful in comparative analysis of the influence of inverter control algorithms on spectra of CM currents in a given drive system.

**Keywords:** electromagnetic compatibility, common mode, differential mode, PWM inverter drive system

## 1. INTRODUCTION

Traditionally, EMI noises in equipment are classified in common (CM) and differential (DM) mode components according to their circulation paths. This separation is very important because usually the sources and equivalent circuits of these modes have a different nature and have to be distinguished carefully. Splitting conducted EMI noises into different modes, CM and DM, is important to the conducted EMI modeling and appropriate application of mitigation techniques. While CM/DM separation is well defined and understood for single-phase or DC systems there is no corresponding definition for three-phase converter systems, common for Adjustable Speed Drives (ASD). However, we can still define the CM noise for a three-phase system as “ground-included-loop” noise, and DM noise as “line-to-line” [8]. Such a “definition” in symmetrical, linear and time invariant three-phase systems allows decoupling of EMI noises into modes by means of orthogonal transformations. (Recently, a much more complicated noise couplings have been uncovered in motor drives with a diode front converter [5,7,8]). In this paper the method of EMI noise decomposition based on algebraic calculations and propagation characteristics

of CM and DM paths is presented. This decomposition allows for comparative analysis of the influence of control algorithms on EMI spectra in a PWM drive system especially in a CISPR A frequency band (9 kHz – 150 kHz).

## 2. EMI NOISE DECOMPOSITION

The origin of common mode conducted EMI (ElectroMagnetic Interference) in the PWM inverter drive system is unbalanced steep pulses of the output inverter voltages that can excite parasitic capacitive couplings in system components. The above causes a spreading of EMI currents over the drive system, including supply and grounding arrangement. Figure 1 schematically shows distributed parasitic couplings in a PWM inverter drive system, which determine main EMI currents paths.

Figure 2 shows the simplified HF model of a three phase system with pulse excitation in a phase b. This model allows prediction of the flows of EMI currents on the motor side of the drive where the highest level of EMI noises is expected.

Total EMI currents can be decomposed into common and differential mode currents, which are indicated as  $i_{DM}$  and

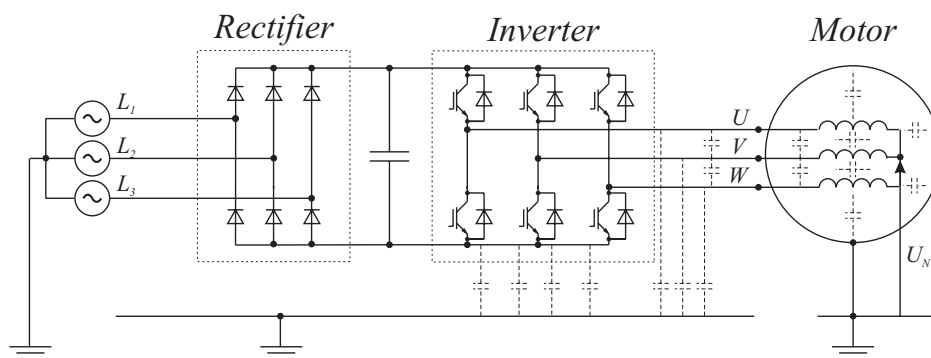


Fig.1. Distributed capacitive couplings in PWM inverter drive system

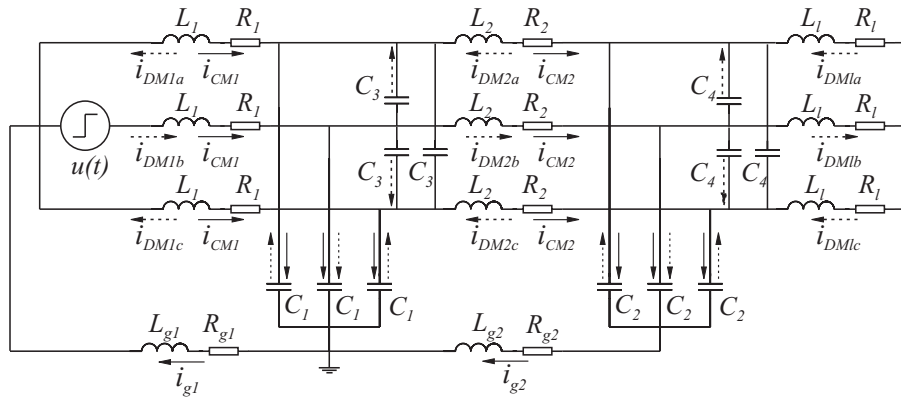


Fig. 2. Simplified HF model of three-phase system

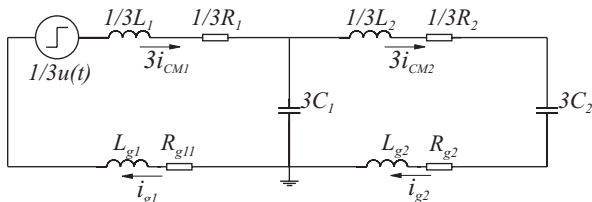


Fig. 3. One phase equivalent circuit for CM noises

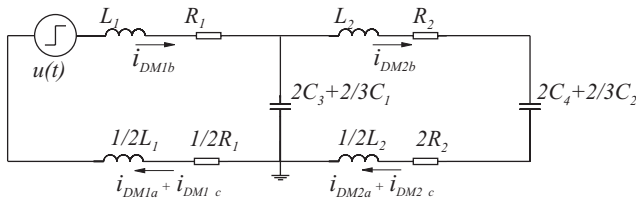


Fig. 4. One phase equivalent circuit for DM noises

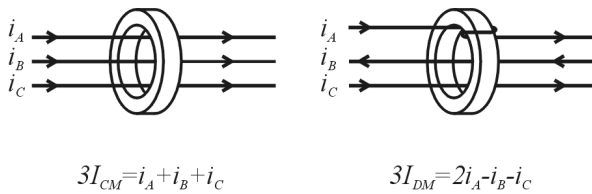


Fig. 5. Arrangement for DC CM and DM current measurement for 3-phase system

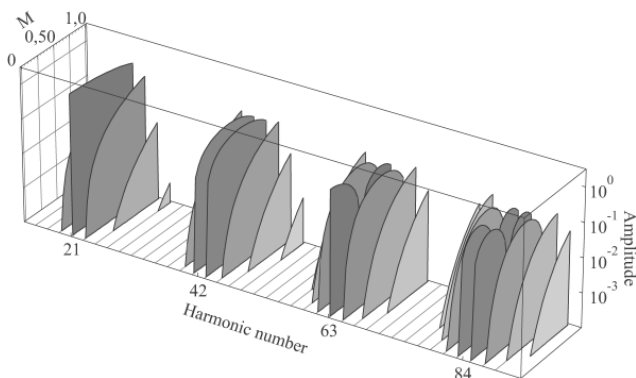


Fig. 6. Theoretical harmonic spectra of the phase voltage versus modulation index

$i_{CM}$ . This separation allows the building of single phase equivalent circuits for distinguished modes, Figure 3 and Figure 4.

The overall circuit mode can be verified by experimental tests carried out by means of HF current probes [1, 2, 6]. Figure 5, shows wire arrangements for a three-phase system [3].

### 3. ANALYTICAL DECOMPOSITION OF PHASE VOLTAGES IN CISPR A FREQUENCY BAND

Usually, EMI noises connected with control algorithms of PWM drive systems are located in CISPR A band because of typically used carrier frequencies in such systems [4]. To determine voltage excitations for both modes the double Fourier integral analysis could be applied. In this paper the decomposition has been done by means of the analysis of the sum of phase voltages. The concept has been verified for a three-phase natural sampled (sinusoidal) PWM using double Fourier series analysis.

For this algorithm phase voltages can be expressed by the well known expression [4]:

$$u_{iN}(t) = V_{DC} + V_{DC} M \cos(\omega_0 t + \theta_i) + \frac{4V_{DC}}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} J_0\left(\frac{m\pi}{2} M\right) \sin\left([m+n]\frac{\pi}{2}\right) \cos(m\omega_c t + n[\omega_0 t + \theta_i]) + \frac{4V_{DC}}{\pi} \sum_{m=1}^{\infty} \sum_{\substack{n=1 \\ n \neq 0}}^{\infty} \frac{1}{m} J_n\left(\frac{m\pi}{2} M\right) \sin\left([m+n]\frac{\pi}{2}\right) \cos(m\omega_c t + n[\omega_0 t + \theta_i]) \quad (1)$$

where:

- $i = a, b, c$  — phase leg identifiers for three phase inverter,
- $m, n$  — harmonic index variables,
- $V_{DC}$  — one-half of DC-link voltage,
- $J_n(x)$  — Bessel function of order  $n$  and argument  $x$ ,
- $\omega_c$  — angular frequency of carrier waveform,
- $\omega_0$  — angular frequency of fundamental component,
- $M$  — modulation index.

Figure 6 shows the theoretical harmonic spectra of the phase voltage for a three-phase natural sampled (sinusoidal) PWM versus modulation index.

The sum of phase voltage harmonics is given by:

$$u_{a,mn}(t) + u_{b,mn}(t) + u_{c,mn}(t) = \quad (2)$$

$$\begin{aligned} & \frac{4V_{DC}}{\pi} \frac{1}{m} J_n \left( m \frac{\pi}{2} M \right) \sin \left( [m+n] \frac{\pi}{2} \right) \cos \left( m\omega_c t + n[\omega_0 t + \theta_i] \right) \times \\ & \times \left[ \begin{aligned} & \cos \left( m\omega_c t + n(\omega_0 t) \right) + \cos \left( m\omega_c t + n \left[ \omega_0 t - \frac{2\pi}{3} \right] \right) \\ & + \cos \left( m\omega_c t + n \left[ \omega_0 t + \frac{2\pi}{3} \right] \right) \end{aligned} \right] = \\ & = \frac{4V_{DC}}{\pi} \frac{1}{m} J_n \left( m \frac{\pi}{2} M \right) \sin \left( [m+n] \frac{\pi}{2} \right) \left[ 1 + 2 \cos n \frac{2\pi}{3} \right] J \cos \left( m\omega_c t + n(\omega_0 t) \right) \end{aligned}$$

The common mode voltage source is equal to:

$$u_{CM}(t) = \frac{u_a(t) + u_b(t) + u_c(t)}{3} \quad (3)$$

where:

$u_a, u_b, u_c$  — phase voltages.

Sideband harmonics with even combination of  $m \pm n$  will not appear in the sum because of the term  $([m+n] \pi / 2)$ .

The elimination term  $\cos[1 + 2 \cos(n \cdot 2\pi / 3)]$  causes a cancellation of specific harmonics in the CM voltage. Harmonics, which are cancelled in the CM voltage, are represented in the DM voltage because of the term  $\sin n\pi / 3$ .

Using the above mentioned terms, common and differential mode components in phase voltages can be expressed as follows:

$$\begin{aligned} u_{DMi}(t) &= \frac{8V_{DC}}{\sqrt{3}\pi} \sum_{m=1}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{m} J_n \left( m \frac{\pi}{2} M \right) \sin \left( [m+n] \frac{\pi}{2} \right) \times \\ & \times \sin n \frac{\pi}{3} \cos \left( m\omega_c t + n\omega_0 t \right) \end{aligned} \quad (4)$$

$$\begin{aligned} u_{CMi}(t) &= \frac{4V_{DC}}{3\pi} \sum_{m=1}^{\infty} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{m} J_n \left( m \frac{\pi}{2} M \right) \times \\ & \times \sin \left( [m+n] \frac{\pi}{2} \right) \left[ 1 + 2 \cos n \frac{2\pi}{3} \right] \times \cos \left( m\omega_c t + n\omega_0 t \right) \end{aligned} \quad (5)$$

Figure 7 shows the result of the analytical decomposition of phase voltages into CM and DM components for arbitrarily selected parameters of the modulation (modulation index  $M=0.9, f_c/f_o=21$ ).

#### 4. SPECTRA OF COMMON AND DIFFERENTIAL MODE EMI CURRENTS

Figure 8 shows CM and DM current spectra measured on the motor side of the converter by means of the EMI receiver, fully compliant with CISPR 16, equipped with a current probe.

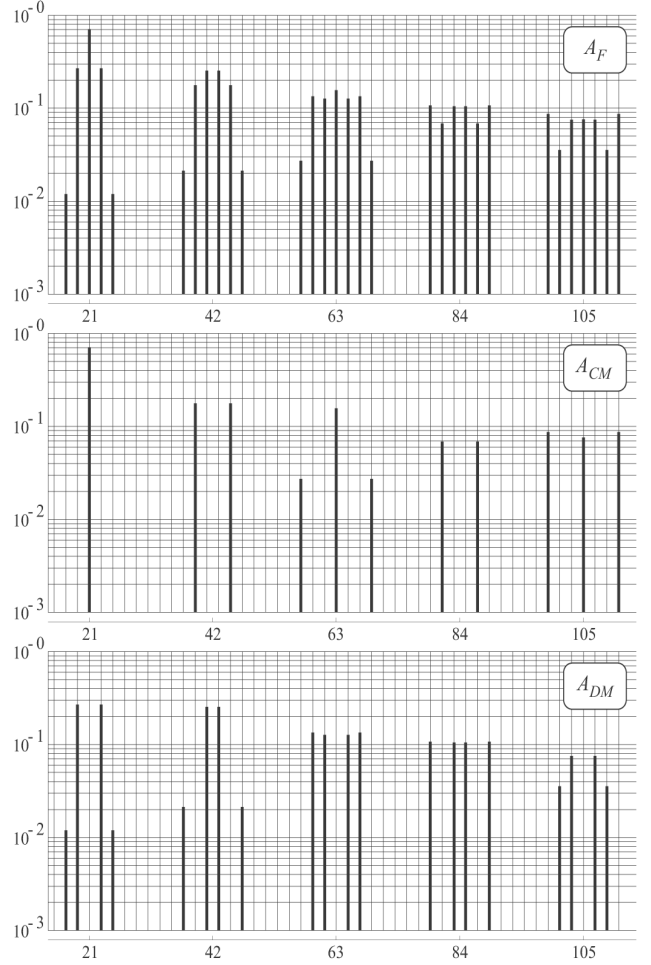


Fig. 7. Phase voltage spectrum, CM voltage component spectrum and DM voltage component spectrum

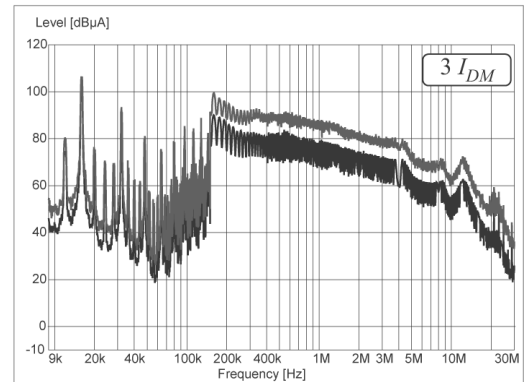
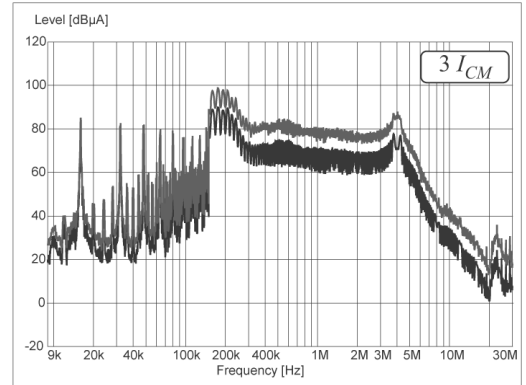


Fig. 8. Experimental spectra of CM and DM currents

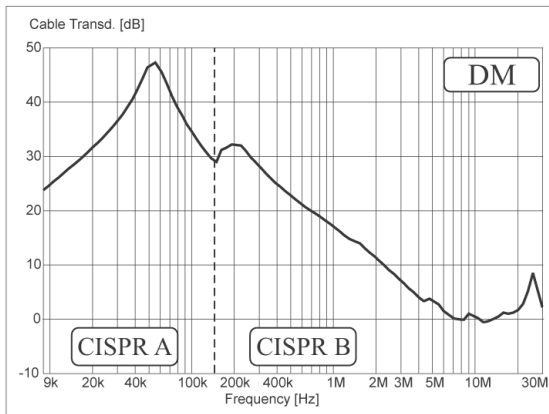
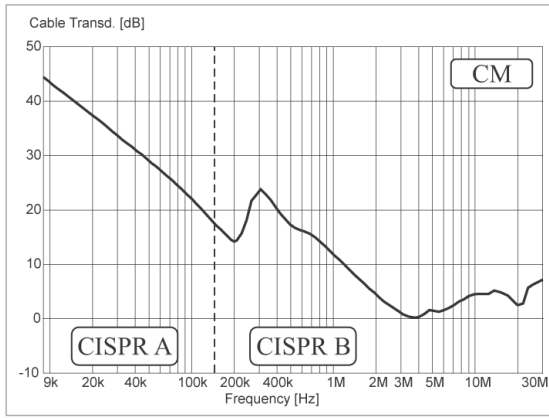


Fig. 9. Insertion losses of CM and DM paths

In a frequency domain CM and DM currents can be calculated as follows:

$$I_{CM}(\omega) = \frac{V_{CM}(\omega)}{Z_{CM}(\omega)} \quad (6)$$

$$I_{DM}(\omega) = \frac{V_{DM}(\omega)}{Z_{DM}(\omega)} \quad (7)$$

where:

$Z_{CM}(\omega), Z_{DM}(\omega)$  — impedance of  $i_{CM}$  and  $i_{DM}$  current path.

Figure 9 shows insertion losses of CM and DM paths on the motor side of the drive system.

In the CISPR A frequency band (9 kHz – 150 kHz) the CM path has capacitive nature and the CM current is proportional to the CM mode voltage component.

$$I_{CM}(\omega) = V_{CM}(\omega) \cdot j\omega C \quad (8)$$

Harmonics of the CM current spectrum could be approximately expressed as proportional to harmonics of CM voltage weighted by the number of harmonic group ( $m$ ):

$$A'_{mn}(\omega) = A_{mn}(\omega) m \quad (9)$$

where:

$A_{mn}$  — coefficients of Fourier expansion.

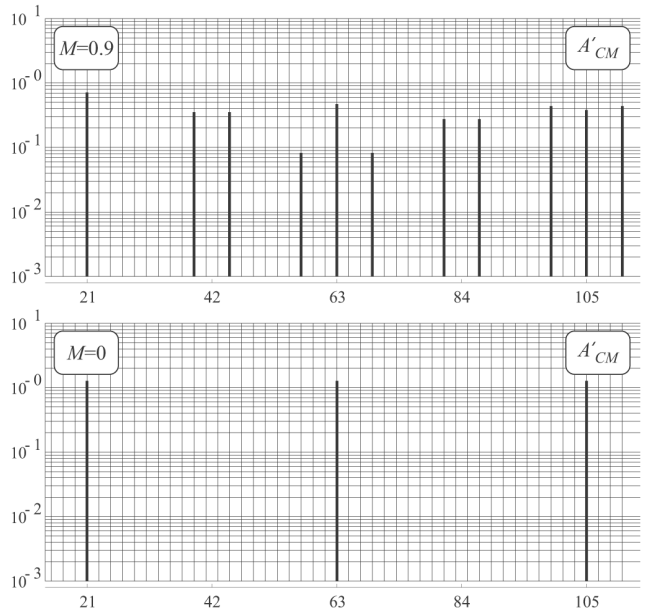


Fig. 10. Spectrum of weighted harmonic of CM voltage

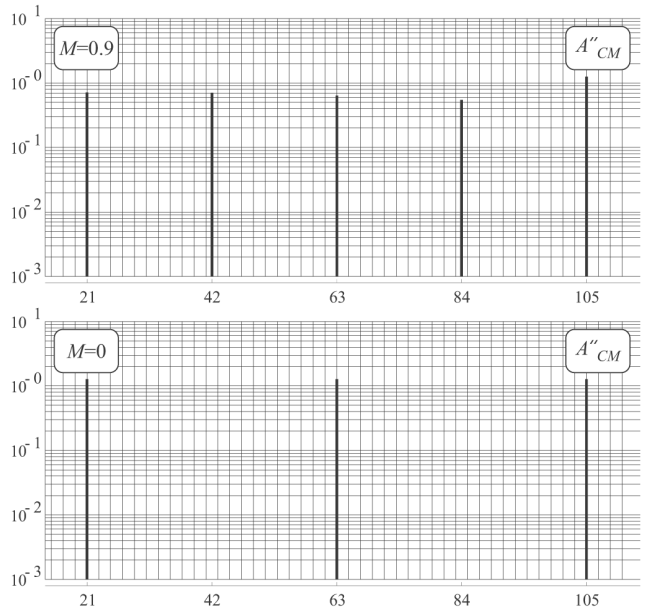


Fig. 11. Spectrum of weighted and aggregated harmonics of CM voltage

Figure 10 shows spectrum of weighted harmonics of CM voltage (modulation index  $M=0.9$  and  $M=0$ ).

A compliant EMI receiver has to fulfill requirements concerning selectivity. It means that measurements should be provided with the appropriate filter of the specified bandwidth for intermediate frequency (IF BW = 200 Hz for CISPR A band).

For this reason the use of analytical aggregation of sideband harmonics in the group region will assure comparison with results obtained using EMI receiver.

$$A''_{mn}(\omega) = \sum_{n=-\infty}^{n=\infty} A_{mn}(\omega) \cdot m \quad (10)$$

Figure 11 shows the spectrum obtained this way for modulation indexes  $M=0.9$  and  $M=0$ .

The shape of the analytical spectrum showed in Figure 11 (for  $M=0.9$ ) agrees with the experimentally obtained spectrum of the CM current in CISPR A frequency band, presented in Figure 8. This analytical approach can be useful in the comparative analysis of an influence of inverter control algorithms on the spectra of CM currents in a given drive system.

The nature of DM current paths in CISPR A frequency band changes from an inductive to a capacitive one (Fig. 9) that influences the spectrum of DM currents presented in Figure 8. A similar approach to the decomposition is also possible in this case, however, analyses are more complicated. Furthermore the main EMC problems in PWM drive systems are caused by CM noises.

The influence of control algorithms in the CISPR B band is less noticeable due to the requirements of the IF filter bandwidth equal to 9 kHz.

## 5. EXPERIMENTAL VALIDATION

Figure 12 shows the experimental spectra of CM currents in PWM inverter fed drives measured by an EMI receiver for two different values of inverter output frequency ( $f_0 = 50$  Hz and  $f_0 = 0$  Hz) which approximately correspond to theoretical spectra for modulation indexes  $M=0.9$  and  $M=0$  (Fig. 10). In the shapes of expanded peaks presented in magnified views (Fig. 12) there are visible sideband harmonics concentrated around carrier harmonics related to theoretical weighted spectra presented in Figure 10. The relative levels of common mode EMI noises show consistence with the theoretical spectra of weighted and aggregated harmonics presented in Figure 11 due to the way in which the EMI receiver "aggregates" harmonics by normalized IF BW of the filter.

## 6. CONCLUSION

In this paper the separation of CM and DM harmonic components from total EMI noise spectra of each phase through algebraic calculations and the Fourier transform has been presented. The results of calculations have been compared to EMI currents measured by the EMI receiver on the motor side of the PWM drive system taking into account the study of characteristics of CM and DM propagation paths.

The proposed approach can be useful in a comparative analysis of the influence of inverter control algorithms on spectra of CM currents in a given drive system.

## REFERENCES

1. Caponet M.C., Profumo F., Ferraris L., Bertoz A., Marzella D.: *Common and differential mode noise separation*. 32-nd Annual IEEE Power Electronics Specialist Conference PESC'01, Vol 3, pp. 1383–1388. Vancouver, Canada.
2. Caponet M.C., Profumo F.: *Devices for the separation of the common and differential mode noise: design and realization*. 17-th Applied Power Electronics Conference and Exposition pp.100–105., APEC 2002.

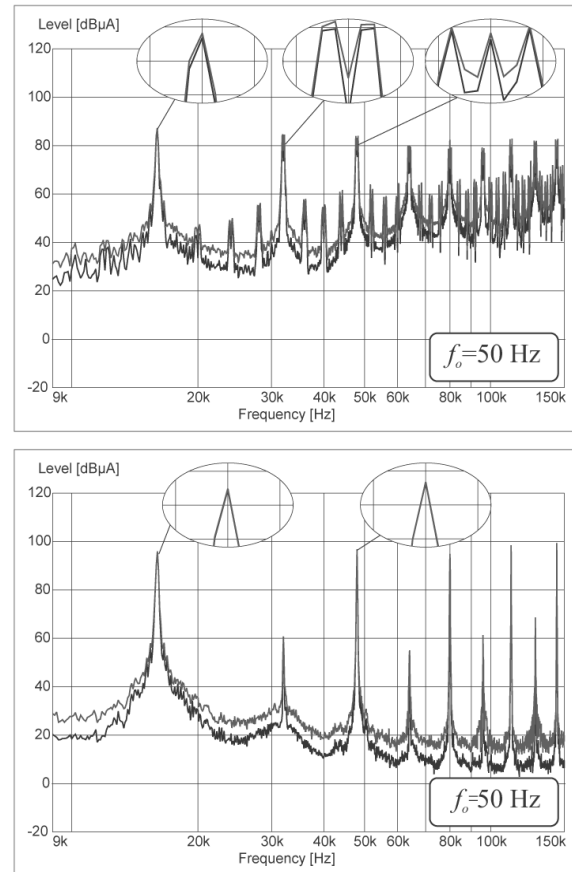


Fig. 12. Spectra of CM currents in PWM inverter fed drives for inverter output frequency equal to 50 Hz and 0 Hz

3. Grandi G., Casadei D., Reggiani U.: *Analysis of common- and differential-mode HF current components in PWM inverter-fed AC motors*. 29-th Annual IEEE Power Electronics Specialist Conference PESC'98, pp. 1146-1151 Vol. 2, Fukuoka, Japan.
4. Holmes G.D., Lipo T.A.: *Pulse width modulation for power converters*. Principles and practice, IEEE Press, 2003.
5. Kempski A., Smolenski R., Strzelecki R.: *Common mode current paths and their modeling in PWM inverter-fed drives*. 33-rd Annual IEEE Power Electronics Specialist Conference — pp. Vol. 3, pp.1551–1556, PESC'02, Cairns, Australia, 2002.
6. Lo Y.K., Chiu H.J., Song T.H.: *A software-based CM and DM measurement system for the conducted EMI*. IEEE Trans. on Industrial Electronics, Vol. 47, No. 4, August 2000.
7. Qu S., Chen D.: *Mixed-mode EMI noise and its implications to filter design in offline switching power supplies*. IEEE Trans. on Power Electronics, pp.502–507 Vol. 17, No. 4, July 2002.
8. Shen W., Wang F., Boroyevich D., Liu Y.: *Definition and acquisition of CM and DM EMI noise for general-purpose adjustable speed motor drives*. 35-th Annual IEEE Power Electronics Specialist Conference PESC'04, Aachen, Germany, s. 1028–1033.



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