Control and Cybernetics

vol. 32 (2003) No. 2

Indirect adaptive neural controller of nonlinear systems using auto-tuning neuron

by

Wei-Der Chang[1], **Rey-Chue Hwang**[2], **Jer-Guang Hsieh**[3]

[1] Department of Computers and Communication
Shu-Te University
Kaohsiung, Taiwan 824, R.O.C.
E-mail: wdchang@mail.stu.edu.tw

[2] Department of Electrical Engineering
I-Shou University
Kaohsiung, Taiwan 840, R.O.C.
E-mail: rchwang@isu.edu.tw

[3] Department of Electrical Engineering
National Sun Yat-Sen University
Kaohsiung, Taiwan 804, R.O.C.

Abstract: In this paper, a novel indirect adaptive neural controller using only two auto-tuning neurons is developed for a class of nonlinear systems. Unlike traditional multi-layered neural controllers, the structure of the proposed controller is very simple and practicable. There are three adjustable parameters in each auto-tuning neuron. Two such auto-tuning neurons used in our proposed indirect adaptive controller are used to track on-line the desired signal. The adaptation law for adjusting these parameters is developed based on the Lyapunov approach. Moreover, the stability of the overall closed-loop system can be analyzed and guaranteed by introducing the additional supervisory controller and the technique of modified adaptation law with projection. Finally, the tracking control of the inverted pendulum system is presented to illustrate the proposed method.

Keywords: auto-tuning neuron, indirect adaptive control, Lyapunov approach, supervisory control, adaptation mechanism.

1. Introduction

The adaptive control by using neural networks has been widely used in many and adaptability. Two types of adaptive controls are generally investigated:

1) Indirect adaptive control: the models of the plant are first obtained and learned by using the neural networks, and then the feedback control law is designed based on these neural network models (Park, Choi, Lee, 1996; Horng, 1999; Khanmohammadi, Hassanzadeh, Sharifian, 2000).

2) Direct adaptive control: the neural networks are directly used as a controller in the feedback control system, i.e., the outputs of the neural networks are just the control inputs of the plant. Then the error between the desired and actual outputs is directly used to update on-line the adjustable parameters in the neural controller (Cui, Shin, 1993; Chen, Chang, 1996; Wu, Lee, Shih, 1998) based on certain adaptation law.

Generally, the architecture of the typical multilayer perceptrons (MLPs) consists of several layers, i.e., the input, hidden, and output layers. Each layer usually contains some neurons that are connected with those in the other layers by weights. The error value is then fed back level-by-level to the input layer in order to update the connection weights, so that the error is minimized, if the back-propagation algorithm is used. However, it is complicated and takes much time for computation because of many adjustable weights within the neural networks. In a real-line control process, a full-connected neural controller will affect the reaction time of the system. Consequently, the way to reduce the complexity of the neural controller for an on-line control system becomes more crucial and meaningful. Moreover, full-connected neural controller will undoubtedly increase the complexity for its hardware implementation.

In this paper, an indirect adaptive neural controller that is composed of only two auto-tuning neurons without any weight connection will be proposed. Completely different from the general multilayer neural controllers, the structure of the proposed controller is very simple and practicable. There are three adjustable parameters in each auto-tuning neuron. Two such auto-tuning neurons are adjusted online in order to track the dynamic behavior of the nonlinear systems according to the feedback linearization techniques. The main difference between the auto-tuning neuron in this study and the traditional one is that a new modified hyperbolic tangent function $a[1+\exp(-bx)]^{-1}[1-\exp(-bx)]$ is used as its activation function (Chen, Chang, 1996; Chang, Hwang, Hsieh, 1998; Duch, Jankowski, 1999). Chang et al. (1998) first used this type of auto-tuning neuron as a direct controller to control the two heights of the liquid-level systems. Note that the saturation level a and the slope value b are adjustable parameters, making the applicability of the proposed neural controller promising.

In the domain of stability analysis of the closed-loop neural and/or fuzzy control systems, a number of theoretical advances have been proposed in recent years. Thus, Wang (1994, 1996, 1997) first added a supervisory controller into the adaptive fuzzy control systems and proposed a modified adaptation law with projection, which is based on Lyapunov approach, for tuning the fuzzy using Gaussian networks for wind energy conversion systems is proposed by Mayosky, Cancelo (1999), and recently a direct adaptive PID control tuning has been also demonstrated (Chang, Hwang, Hsieh, 2002). Therefore, it is reasonable and practicable that an indirect neural control just using two auto-tuning neurons for a class of nonlinear systems be constructed based on the same techniques. The detailed adaptation law calculation and the analysis of stability will be described in the next section.

On the other hand, the tracking control of the inverted pendulum system has been already investigated and tested using various control strategies in the past. This problem is of interest, since it describes an inherently unstable system and is typical of a wide class of control problems with severe nonlinearity in a broad operation region. For example, an enhancing fuzzy controller with self-learning capability was proposed by Jang (1992). An optimal tracking controller based on multi-layered neural networks was demonstrated and discussed (Park, Choi, Lee, 1996). In addition, Wang (1996) provided an adaptive control technique based on fuzzy systems. In this study, we will also apply our proposed scheme to the inverted pendulum system to show the control performance.

2. Analysis and design of indirect adaptive neural controller

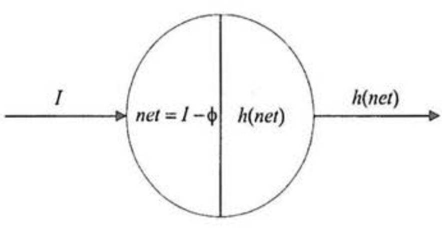
The structure of the auto-tuning neuron used in our proposed indirect adaptive neural controller is shown in Fig. 1 and is expressed mathematically as

$$net = I - \phi, \tag{1}$$

where I is the input of the neuron, ϕ is the threshold or bias, and net represents the internal state of the neuron. The output of the auto-tuning neuron is given by

$$h(net) = \frac{a[1 - \exp(-b \cdot net)]}{[1 + \exp(-b \cdot net)]}, \tag{2}$$

where the activation function $h(\cdot) : \Re \to \Re$ is a modified hyperbolic tangent function, and a and b are the saturation level and the slope value of the function,



respectively. It is noted that the output range and the curve shape of such an activation function in this auto-tuning neuron are mainly influenced by these two adjustable parameters a and b. For convenience, let $\theta = [\phi, a, b]^T \in \Re^3$ represent the vector of adjustable parameters. We wish to adjust these parameters in the indirect adaptive neural controller in such a way that the control objective can be achieved.

In this study, we consider an nth-order nonlinear system, with the input $u \in \Re$ and the output $y \in \Re$, described as

$$\begin{aligned} &\dot{x}_1 = x_2, \dot{x}_2 = x_3, \ldots, \dot{x}_{n-1} = x_n, \\ &\dot{x}_n = f(x_1, x_2, \ldots, x_n) + g(x_1, x_2, \ldots, x_n)u, \\ &y = x_1, \end{aligned}$$

or equivalently as

$$\begin{aligned} &x^{(n)} = f(x, \dot{x}, \ldots, x^{(n-1)}) + g(x, \dot{x}, \ldots, x^{(n-1)})u, \\ &y = x, \end{aligned} \tag{3}$$

where $f, g : \Re^n \to \Re$ are two unknown functions and, without loss of generality, $g(\cdot) > 0$ is assumed. In addition, it will be also assumed that there exist positive bounds $f^u(X)$, $g^u(X)$, and $g_l(X)$ so that $|f(X)| \le f^u(X)$ and $0 < g_l(X) \le g(X) \le g^u(X)$, where $X = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in \Re^n$ is the state vector of the system. As it is well known (Slotine, Li, 1991), if $f(X)$ and $g(X)$ of the system in (3) are known, then the feedback linearization technique can be employed to design a desired controller. Let $e = y_d - y$ $(= y_d - x)$ be the error between the desired and actual outputs. Define $Y_d = [y_d, \dot{y}_d, \ldots, y_d^{(n-1)}]^T$ and assume that $y_d, \dot{y}_d, \ldots, y_d^{(n-1)}$ are all bounded, i.e.,

$$\|Y_d\|_\infty = \sup_{t \ge 0} \|Y_d(t)\| < \infty.$$

Then the error vector of the system is

$$E = Y_d - X = [e, \dot{e}, \ldots, e^{(n-1)}]^T = [e_1, e_2, \ldots, e_n]^T.$$

Suppose that we choose a gain vector $K = [k_0, k_1, \ldots, k_{n-1}]^T$ such that all roots of

$$s^n + k_{n-1}s^{n-1} + \cdots + k_1 s + k_0 = 0$$

are in the open left-half complex plane. Now let the feedback control law be given by

$$u^* = g(X)^{-1}[-f(X) + y_d^{(n)} + K^T E]. \tag{4}$$

Substituting (4) into (3), we obtain

Consequently, we have $e(t) \to 0$ as $t \to \infty$, i.e., $y \to y_d$ asymptotically. Note that $f(X)$ and $g(X)$ of the system in (3) are assumed to be unknown in this study. Here we use two auto-tuning neurons denoted by $\hat{f}(X, \theta_f)$ and $\hat{g}(X, \theta_g)$ in place of $f(X)$ and $g(X)$ in the feedback control law of (4), respectively, where $\theta_f = [\phi_f, a_f, b_f]^T$ and $\theta_g = [\phi_g, a_g, b_g]^T$ represent the vectors of adjustable parameters in $\hat{f}(X, \theta_f)$ and $\hat{g}(X, \theta_g)$. Moreover, in order to satisfy the assumption $g(X) > 0$, equation (2) for $\hat{g}(X, \theta_g)$ should be modified as

$$h(net_g) = \frac{a_g[1 - \exp(-b_g \cdot net_g)]}{[1 + \exp(-b_g \cdot net_g)]} + a_g = \frac{2a_g}{[1 + \exp(-b_g \cdot net_g)]}, \tag{5}$$

where a_g is a positive value implying $h(net_g) > 0$, i.e., $\hat{g}(X, \theta_g) > 0$. For $\hat{f}(X, \theta_f)$, equation (2) is still used as its activation function. Hence, the resulting certainty equivalent controller based on these two auto-tuning neurons is

$$u_n = \hat{g}^{-1}(X, \theta_g)\lfloor -\hat{f}(X, \theta_f) + y_d^{(n)} + K^T E \rfloor. \tag{6}$$

Throughout the paper, the following assumption is made:

Assumption

Let the constraint sets Ω_x, Ω_{θ_f}, and θ_f for the state X and the adjustable parameter vectors θ_f and θ_g, respectively, be defined by

$$\begin{aligned} \Omega_x &= \{X \in \Re^n : \|X\| \le M_x\}, \\ \Omega_{\theta_f} &= \{\theta_f \in \Re^3 : \|\theta_f\| \le M_{\theta_f}\}, \\ \Omega_{\theta_g} &= \{\theta_g \in \Re^3 : \|\theta_g\| \le M_{\theta_g}, \text{ and } \phi_g, a_g, b_g \ge \varepsilon > 0\}, \end{aligned}$$

where M_x, M_{θ_f}, M_{θ_g}, and ε are pre-specified parameters and, for simplicity of analysis, we may choose $M_x \ge \|Y_d\|_\infty$.

The goal is to keep the state trajectory X and the adjustable parameter vector θ_f and θ_g inside the balls Ω_x, Ω_{θ_f}, and Ω_{θ_g}, respectively. First, to reach the objective $\|X\| \le M_x$, let the control input in (3) be

$$u = u_n + u_s, \tag{7}$$

where u_n is an indirect adaptive neural control in (6) and u_s is the supervisory control, which is activated only when $\|X\|$ of the system exceeds some bound. Now we will design u_s and obtain a proper adaptation law for θ_f and θ_g such that u_n approaches the feedback control law u^* of (4) based on the Lyapunov approach. In many papers, to implement such indirect adaptive controller as the one mentioned earlier, the neural networks or fuzzy systems were usually employed to learn or model the dynamic behavior of the nonlinear functions $f(X)$ and $g(X)$, respectively. Undoubtedly, in order to accomplish this task, it is necessary to use many adjustable parameters in the neural networks or fuzzy that their outputs of $\hat{f}(X,\theta_f)$ and $\hat{g}(X,\theta_g)$ may follow the dynamic behavior of nonlinear functions $f(X)$ and $g(X)$ of the plant. The main idea behind the proposed method is different from those of other methods.

By substituting (7) into (3), we have

$$\begin{aligned} x^{(n)} &= f(X)+g(X)(u_n+u_s) \\ &= f(X)+g(X)(u_n+u_s)+\hat{g}(X,\theta_g)u_n-\hat{g}(X,\theta_g)u_n \\ &= f(X)+g(X)(u_n+u_s)-\hat{f}(X,\theta_f)+y_d^{(n)}+K^TE-\hat{g}(X,\theta_g)u_n \\ &= y_d^{(n)}+K^TE+[f(X)-\hat{f}(X,\theta_f)]+[g(X)-\hat{g}(X,\theta_g)]u_n+g(X)u_s. \end{aligned}$$

This implies that

$$e^{(n)} = -K^TE+[\hat{f}(X,\theta_f)-f(X)]+[\hat{g}(X,\theta_g)-g(X)]u_n-g(X)u_s. \quad (8)$$

Let

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -k_0 & -k_1 & -k_2 & \cdots & -k_{n-2} & -k_{n-1} \end{bmatrix},$$

and

$$B_c = [\ 0 \ \ \cdots \ \ 0 \ \ 1\ ]^T, \quad (9)$$

be a companion form. From (8), we have

$$\dot{E} = A_cE+B_c[(\hat{f}(X,\theta_f)-f(X))+(\hat{g}(X,\theta_g)-g(X))u_n-g(X)u_s]. \quad (10)$$

Now consider the Lyapunov function candidate

$$V_e = 2^{-1}E^TPE, \quad (11)$$

where P is a positive definite symmetric matrix satisfying the Lyapunov equation

$$A_c^TP+PA_c = -Q, \quad (12)$$

and Q is a given positive definite symmetric matrix. In the following, we will choose Q such that $\lambda_{\min}(Q)>1$, where $\lambda_{\min}(Q)$ denotes the minimum eigenvalue of Q. Define

$$V_M = 2^{-1}\lambda_{\min}(P)(M_x-\|Y_d\|_\infty)^2. \quad (13)$$

Note that if $\|X\|\geq M_x$, then, from (11), we have

$$V_e \geq 2^{-1}\lambda_{\min}(P)\|E\|^2 \geq 2^{-1}\lambda_{\min}(P)(\|X\|-\|Y_d\|)^2$$

Hence if $V_e < V_M$, then $\|X\| < M_x$. The time derivative of V_e along the trajectories of the closed-loop system of (10) satisfies

$$\begin{aligned}
\dot{V}_e &= 2^{-1}E^T(A_c^TP + PA_c)E + E^TPB_c[(\hat{f}(X,\theta_f) - f(X)) \\
&+ (\hat{g}(X,\theta_g) - g(X))u_n - g(X)u_s] \\
&= -2^{-1}E^TQE + E^TPB_c[(\hat{f}(X,\theta_f) - f(X)) + (\hat{g}(X,\theta_g) \\
&- g(X))u_n - g(X)u_s] \\
&\leq -2^{-1}E^TQE + |E^TPB_c|(|\hat{f}(X,\theta_f)| + |f(X)| + |\hat{g}(X,\theta_g)u_n| + |g(X)u_n|) \\
&- E^TPB_cg(X)u_s. \qquad (14)
\end{aligned}$$

Define the indicator function I^* by $I^* = 1$ if $V_e \geq V_M$ and $I^* = 0$ if $V_e < V_M$. Hence, from the assumptions $|f(X)| \leq f^u(X)$ and $0 < g_l(X) \leq g(X) \leq g^u(X)$, if the supervisory controller is chosen as

$$\begin{aligned}
u_s &= I^*\mathrm{sgn}(E^TPB_c)g_l^{-1}(X)[|\hat{f}(X,\theta_f)| + f^u(X) \\
&+ |\hat{g}(X,\theta_g)u_n| + |g^u(X)u_n|], \qquad (15)
\end{aligned}$$

then we can guarantee that $\dot{V}_e < 0$ in (14) if $V_e \geq V_M$ (Wang, 1994).

On the other hand, in order to derive a proper adaptation law for $\theta_f = [\phi_f, a_f, b_f]^T$ and $\theta_g = [\phi_g, a_g, b_g]^T$, let θ_f^* and θ_g^* be two optimal parameters vectors such that the approximation error

$$w = (\hat{f}(X,\theta_f^*) - f(X)) + (\hat{g}(X,\theta_g^*) - g(X))u_n, \qquad (16)$$

is minimized. For simplicity of analysis, we may choose Ω_{θ_f} and Ω_{θ_g} large enough such that $\theta_f^* \in \Omega_{\theta_f}$ and $\theta_g^* \in \Omega_{\theta_g}$ for all t. By incorporating (16), equation (10) can be rewritten as

$$\begin{aligned}
\dot{E} &= A_cE - B_cg(X)u_s \\
&+ B_c[(\hat{f}(X,\theta_f) - \hat{f}(X,\theta_f^*)) + (\hat{g}(X,\theta_g) - g(X,\theta_g^*))u_n + w]. \qquad (17)
\end{aligned}$$

Now consider another Lyapunov function candidate containing the error of the system and the errors between θ_f and θ_f', θ_g and θ_g', given by

$$\begin{aligned}
V &= 2^{-1}E^TPE + (2\gamma_1)^{-1}(\theta_f - \theta_f^*)^T(\theta_f - \theta_f^*) \\
&+ (2\gamma_2)^{-1}(\theta_g - \theta_g^*)^T(\theta_g - \theta_g^*), \qquad (18)
\end{aligned}$$

where γ_1 and γ_2 are two positive constants determining the convergence speed. Using (17), we have

$$\begin{aligned}
\dot{V} &= -2^{-1}E^TQE - g(X)E^TPB_cu_s + E^TPB_cw \\
&+ E^TPB_c(\hat{f}(X,\theta_f) - \hat{f}(X,\theta_f^*)) + E^TPB_c(\hat{g}(X,\theta_g) - \hat{g}(X,\theta_g^*))u_n
\end{aligned}$$

Taking the Taylor series expansions of $\hat{f}(X,\theta_f)$ and $\hat{g}(X,\theta_g)$ around θ_f^* and θ_g^*, respectively, we obtain

$$\hat{f}(X,\theta_f)-\hat{f}(X,\theta_f^*)=(\theta_f-\theta_f^*)^T\frac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f}+O(\|\theta_f-\theta_f^*\|^2),$$
$$\hat{g}(X,\theta_g)-\hat{g}(X,\theta_g^*)=(\theta_g-\theta_g^*)^T\frac{\partial\hat{g}(X,\theta_g)}{\partial\theta_g}+O(\|\theta_g-\theta_g^*\|^2),$$

where $O(\cdot)$ represents the high-order term. From (19), we have

$$\begin{aligned}\dot{V}=&-2^{-1}E^TQE+\gamma_1^{-1}(\theta_f-\theta_f^*)^T\left[\dot{\theta}_f+\gamma_1E^TPB_c\frac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f}\right]\\&+\gamma_2^{-1}(\theta_g-\theta_g^*)^T\left[\dot{\theta}_g+\gamma_2E^TPB_c\frac{\partial\hat{g}(X,\theta_g)}{\partial\theta_g}u_n\right]-g(X)E^TPB_cu_s\\&+E^TPB_c[w+O(\|\theta_f-\theta_f^*\|^2)+O(\|\theta_g-\theta_g^*\|^2)u_n].\end{aligned}\tag{20}$$

From (15) and $g(X)>0$, we have $g(X)E^TPB_cu_s\geq 0$ and

$$\begin{aligned}\dot{V}\leq&-2^{-1}E^TQE+\gamma_1^{-1}(\theta_f-\theta_f^*)^T\left[\dot{\theta}_f+\gamma_1E^TPB_c\frac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f}\right]\\&+\gamma_2^{-1}(\theta_g-\theta_g^*)^T\left[\dot{\theta}_g+\gamma_2E^TPB_c\frac{\partial\hat{g}(X,\theta_g)}{\partial\theta_g}u_n\right]\\&+E^TPB_c[w+O(\|\theta_f-\theta_f^*\|^2)+O(\|\theta_g-\theta_g^*\|^2)u_n].\end{aligned}\tag{21}$$

In order to derive a proper adaptation law and simultaneously guarantee $\theta_f\in\Omega_{\theta_f}$ and $\theta_g\in\Omega_{\theta_g}$, a modified adaptation law with projection had been proposed (Wang, 1994). For θ_f, we have

$$\dot{\theta}_f=\begin{cases}-\gamma_1E^TPB_c\dfrac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f},\text{if}(\|\theta_f\|<M_{\theta_f})\text{ or}\\ \qquad\left(\|\theta_f\|=M_{\theta_f}\text{ and }E^TPB_c\theta_f^T\dfrac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f}\geq 0\right),\\ -\gamma_1E^TPB_c\dfrac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f}+\gamma_1E^TPB_c\dfrac{\theta_f}{\|\theta_f\|^2}\theta_f^T\dfrac{\partial\hat{f}(X,\theta_f)}{\partial\theta_f},\text{ otherwise.}\end{cases}\tag{22}$$

For θ_g, we have to avoid the certainty equivalent controller u_n of (6) being too large, resulting from a small value of $\hat{g}(X,\theta_g)$. Hence, if the ith element of θ_g, denoted by θ_{g_i}, for $i=1,2,3$, is equal to ε, then

$$\dot{\theta}_{g_i}=\begin{cases}-\gamma_2E^TPB_c\dfrac{\partial\hat{g}(X,\theta_g)}{\partial\theta_{g_i}}u_n, & \text{if }E^TPB_c\dfrac{\partial\hat{g}(X,\theta_g)}{\partial\theta_{g_i}}u_n<0,\\ 0 & \text{if }E^TPB_c\dfrac{\partial\hat{g}(X,\theta_g)}{\partial\theta_{g_i}}u_n\geq 0.\end{cases}\tag{23}$$

where $\partial \hat{g}(X,\theta_g)/\partial \theta_{g_i}$ is the ith element of $\partial \hat{g}(X,\theta_g)/\partial \theta_g$, else

$$\dot{\theta}_g = \begin{cases} -\gamma_2 E^T P B_c \dfrac{\partial \hat{g}(X,\theta_g)}{\partial \theta_g} u_n, \text{ if } (\|\theta_g\| < M_{\theta_g}) \text{ or} \\ \quad \left(\|\theta_g\| = M_{\theta_g} \text{ and } E^T P B_c \theta_g^T \dfrac{\partial \hat{g}(X,\theta_g)}{\partial \theta_g} u_n \geq 0 \right), \\ -\gamma_2 E^T P B_c \dfrac{\partial \hat{g}(X,\theta_g)}{\partial \theta_g} + \gamma_2 E^T P B_c \dfrac{\theta_g}{\|\theta_g\|^2} \theta_g^T \dfrac{\partial \hat{f}(X,\theta_g)}{\partial \theta_g} u_n, \text{ otherwise.} \end{cases} \tag{24}$$

From (22) to (24), these result in

$$(\theta_f - \theta_f^*)^T \left[\dot{\theta}_f + \gamma_1 E^T P B_c \frac{\partial \hat{f}(X,\theta_f)}{\partial \theta_f} \right] \leq 0,$$

and

$$(\theta_g - \theta_g^*)^T \left[\dot{\theta}_g + \gamma_2 E^T P B_c \frac{\partial \hat{g}(X,\theta_g)}{\partial \theta_g} u_n \right] \leq 0,$$

in (21). Hence, we have

$$\dot{V} \leq -2^{-1} E^T Q E + E^T P B_c [w + O(\|\theta_f - \theta_f^*\|^2) + O(\|\theta_g - \theta_g^*\|^2) u_n].$$

Let

$$\nu = [w + O(\|\theta_f - \theta_f^*\|^2) + O(\|\theta_g - \theta_g^*\|^2) u_n],$$

then we have

$$\dot{V} \leq -2^{-1} E^T Q E + E^T P B_c \nu. \tag{25}$$

THEOREM. *Consider the system of (3), subject to the control in (7) with (6), (15), and (22)-(24). If the initial state* $X(0) \in \Omega_x$, *the initial parameters* $\theta_f(0) \in \Omega_{\theta_f}$ *and* $\theta_g(0) \in \Omega_{\theta_g}$, *then* $X(t) \in \Omega_x$, $\theta_f(t) \in \Omega_{\theta_f}$, *and* $\theta_g(t) \in \Omega_{\theta_g}$, *and the tracking error satisfies*

$$\int_0^t \|E(\tau)\|^2 d\tau \leq p + q \int_0^t |\nu(\tau)|^2 d\tau, \quad t \geq 0, \tag{26}$$

where p *and* q *are constants. Furthermore, if* $\nu \in L_2$, *i.e.,* $\int_0^\infty |\nu(\tau)|^2 d\tau < \infty$, *then*

$$\lim_{t\to\infty} \|E(t)\| = 0$$

Proof. From (25), we have

$$\begin{aligned} \dot{V} &\leq -2^{-1} E^T Q E + E^T P B_c \nu \\ &\leq -2^{-1} \lambda_{\min}(Q) \|E\|^2 + 2^{-1} \|E\|^2 - 2^{-1} (\|E\|^2 - 2\|E\| \, \|P B_c \nu\| + \|P B_c \nu\|^2) \\ &\quad + 2^{-1} \|P B_c \nu\|^2 \\ &= -2^{-1} \lambda_{\min}(Q) \|E\|^2 + 2^{-1} \|E\|^2 - 2^{-1} (\|E\| - \|P B_c \nu\|)^2 + 2^{-1} \|P B_c \nu\|^2 \end{aligned}$$

Integrating both sides of (27) and recalling that $\lambda_{\min}(Q) > 1$, we get

$$V(t) - V(0) \le -2^{-1}[\lambda_{\min}(Q) - 1] \int_0^t \|E(\tau)\|^2 d\tau$$
$$+ 2^{-1}\|PB_c\|^2 \int_0^t |\nu(\tau)|^2 d\tau,$$

then

$$\int_0^t \|E(\tau)\|^2 d\tau \le 2[\lambda_{\min}(Q) - 1]^{-1}[V(0) - V(t)]$$
$$+ [\lambda_{\min}(Q) - 1]^{-1}\|PB_c\|^2 \int_0^t |\nu(\tau)|^2 d\tau$$
$$\le 2[\lambda_{\min}(Q) - 1]^{-1}[|V(0)| + |V(t)|]$$
$$+ [\lambda_{\min}(Q) - 1]^{-1}\|PB_c\|^2 \int_0^t |\nu(\tau)|^2 d\tau.$$

Define

$$p = 2[\lambda_{\min}(Q) - 1]^{-1}[|V(0)| + \sup_{t\ge 0} |V(t)|]$$
$$q = [\lambda_{\min}(Q) - 1]^{-1}\|PB_c\|^2,$$

then we have

$$\int_0^t \|E(\tau)\|^2 d\tau \le p + q \int_0^t |\nu(\tau)|^2 d\tau.$$

Notice that from (18) the $\sup_{t\ge 0} |V(t)|$ is finite because E, $\theta_f - \theta_f^*$, and $\theta_g - \theta_g^*$ are all bounded due to the supervisory controller in (15) and the modified adaptation law with projection in (22)–(24). Moreover, if $\nu \in L_2$, then from (26) we conclude that $E \in L_2$. Since all the variables on the right-hand side of (17) are bounded, we have $\dot{E} \in L_\infty$. According to the Barbalat's Lemma (Sastry, Bodson, 1989; Wang, 1994), if $E \in L_2 \cap L_\infty$ and $\dot{E} \in L_\infty$, then $\lim_{t\to\infty} \|E(t)\| = 0$. This completes our proof. ∎

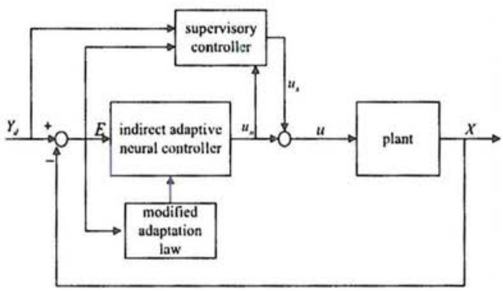


Fig. 2 shows the overall structure of the closed-loop feedback system with the supervisory controller and the modified adaptation laws with projection. In order to accomplish the adaptation laws in (22)–(24), $\partial\hat{f}(X,\theta_f)/\partial\theta_f$ and $\partial\hat{g}(X,\theta_g)/\partial\theta_g$ must be calculated. From (1), (2), and (5) with $\theta_f = [\phi_f, a_f, b_f]^T$ and $\theta_g = [\phi_g, a_g, b_g]^T$, we have

$$\frac{\partial\hat{f}(X,\theta_f)}{\partial\phi_f} = -\frac{a_f\cdot b_f}{2}\left[1+\frac{\hat{f}(X,\theta_f)}{a_f}\right]\left[1-\frac{\hat{f}(X,\theta_f)}{a_f}\right],$$
$$\frac{\partial\hat{f}(X,\theta_f)}{\partial a_f} = \frac{\hat{f}(X,\theta_f)}{a_f},$$
$$\frac{\partial\hat{f}(X,\theta_f)}{\partial b_f} = \frac{a_f\cdot net_f}{2}\left[1+\frac{\hat{f}(X,\theta_f)}{a_f}\right]\left[1-\frac{\hat{f}(X,\theta_f)}{a_f}\right],$$

and

$$\frac{\partial\hat{g}(X,\theta_g)}{\partial\phi_g} = -b_g\hat{g}(X,\theta_g)\left[1-\frac{\hat{g}(X,\theta_g)}{2a_g}\right],$$
$$\frac{\partial\hat{g}(X,\theta_g)}{\partial a_g} = \frac{\hat{g}(X,\theta_g)}{a_g},$$
$$\frac{\partial\hat{g}(X,\theta_g)}{\partial b_g} = net_g\hat{g}(X,\theta_g)\left[1-\frac{\hat{g}(X,\theta_g)}{2a_g}\right].$$

The overall design procedures can be summarized as follows.

Data: Plant in (3) and desired output y_d.

Goal: Design a control of (7), i.e., $u = u_n + u_s$, such that the plant output follows the desired output asymptotically.

Step 1: Determine functions $f^u(X)$, $g_l(X)$, and $g^u(X)$, such that $|f(X)| \leq f^u(X)$ and $0 < g_l(X) \leq g(X) \leq g^u(X)$.

Step 2: Choose the constraint parameters M_x, M_{θ_f}, M_{θ_f}, and parameter ε.

Step 3: Choose γ_1, γ_2, K, and Q with $\lambda_{\min}(Q) > 1$ and find the solution P of the Lyapunov equation in (12).

Step 4: Construct the supervisory control u_n in (15).

Step 5: Construct the indirect adaptive neural control u_n in (6) with adaptation laws in (22)–(24).

Step 6: The desired control of (7) is given by $u = u_n + u_s$.

Step 7: Stop.

3. Simulation

In this section, the tracking control of the inverted pendulum system is presented to illustrate the proposed method. The dynamics of the inverted pendulum system is described as follows (Wang, 1994)

$$\dot{x}_2 = \frac{g\sin(x_1) - \frac{ml x_2^2 \cos(x_1)\sin(x_1)}{m_c+m}}{l(\frac{4}{3} - \frac{m\cos^2(x_1)}{m_c+m})} + \frac{\frac{\cos(x_1)}{m_c+m}}{l(\frac{4}{3} - \frac{m\cos^2(x_1)}{m_c+m})}u,$$
$$y = x_1, \tag{28}$$

where x_1 is the angle of the pole with respect to the vertical axis, x_2 is the angular velocity of the pole, $g = 9.8m/s^2$ is the acceleration due to gravity, $m_c = 1kg$ is the mass of cart, $m = 0.1kg$ is the mass of pole, $l = 0.5m$ is the half length of pole, and u is the control force in Newtons. The Euler method with sampling period of 0.01 seconds is used to simulate the nonlinear differential

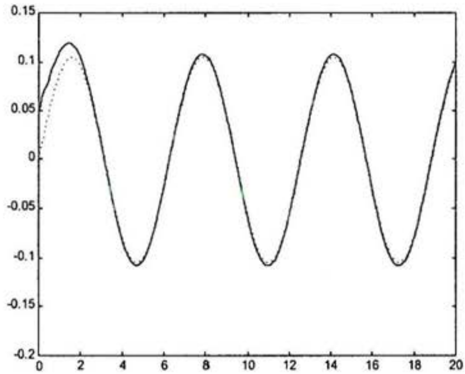
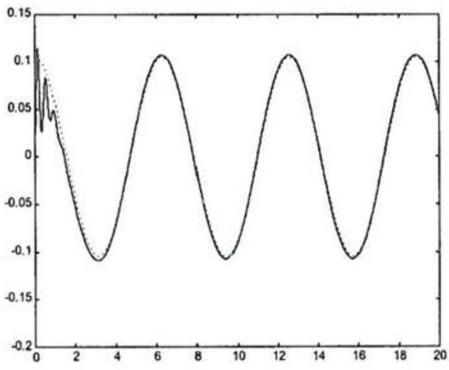


Fig. 3(a). Pole angle (solid line actual output; dashed line desired output).



equations. From (28), the corresponding $f^u(X)$, $g_l(X)$, and $g^u(X)$ are given as (Wang, 1994)

$$f^u(X) = 15.78 + 0.036x_2^2, \quad g_l(X) = 1.12, \text{ and } g^u(X) = 1.46.$$

Moreover, we choose $k_0 = 1$ and $k_1 = 2$ so that the roots of $s^2 + k_1 s + k_0 = 0$ are in the left-half complex plane. From (9), we have

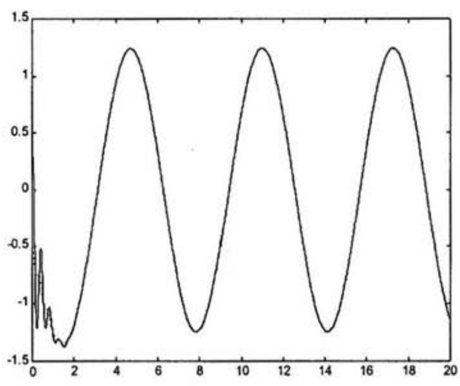
$$A_c = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}.$$

From (12) with $Q = diag[10,\ 10]$, we have

$$P = \begin{bmatrix} 15 & 5 \\ 5 & 5 \end{bmatrix}.$$

Moreover, we choose $M_x = \pi/10$, $M_{\theta_f} = 4$, $M_{\theta_g} = 4$, $\varepsilon = 0.1$, $\gamma_1 = 20$, and $\gamma_2 = 0.3$.

Our control objective is to make the pole angle x_1 of the inverted pendulum follow the desired output $y_d = \pi \sin(t)/30$ asymptotically. For instance, with the initial state $(\pi/60, 0)$, Fig. 3 shows the output responses of pole angle x_1 and its pole angular velocity x_2, respectively. It is clear that the control task can be successfully accomplished. The comparison between the certainty equivalent controller u_n in (6) and the feedback linearization controller u^* in (4) is shown in Fig. 4. From Fig. 4, we can easily see that these two curves are almost identical after about 1.8 seconds.



4. Conclusion and future research

In this paper, an indirect adaptive neural control based on only two auto-tuning neurons for a class of nonlinear systems has been proposed. The important contribution in our proposed scheme is that the complexity of the traditional neural controllers can be greatly reduced; only two auto-tuning neurons are utilized and adjusted such that their outputs can track the desired dynamic trajectories of the plant. The adaptation laws for adjustable parameters have been derived by using the Lyapunov approach, and the stability of the closed-loop systems can be guaranteed by introducing an additional supervisory controller and a modified adaptation law with projection. Finally, the tracking control of the inverted pendulum system has been employed to illustrate the use of the proposed indirect adaptive neural controller. From the simulation results, it is obvious that good performance can be achieved by using the proposed scheme. As to the future research, a possible direction is to use the genetic algorithms that are one of optimal techniques for searching unknown parameters, instead of the adaptation laws proposed in this study, to find the adjustable parameter vectors θ_f and θ_g.

References

CHANG, W.D., HWANG, R.C. and HSIEH, J.G. (2002) A self-tuning PID control for a class of nonlinear systems based on the Lyapunov approach. *Journal of Process Control*, **12**, 233–242.

CHANG, W.D., HWANG, R.C. and HSIEH, J.G. (1998) Adaptive control of multivariable dynamic systems using independent self-tuning neurons. *Proceedings of tenth international conference on tools with artificial intelligence*, Taipei, Taiwan, 68–73.

CHEN, C.T. and CHANG, W.D. (1996) A feedforward neural network with function shape autotuning. *Neural Networks*, **9** (4), 627–641.

CUI, X. and SHIN, K.G. (1993) Direct control and coordination using neural networks. *IEEE Transactions on Systems, Man, and Cybernetics*, **23**(3), 686–697.

DUCH, W. and JANKOWSKI, N. (1999) Survey of neural transfer functions. *Neural Computing Surveys*, **2**, 163–213.

HORNG, J.H. (1999) Neural adaptive tracking control of a DC motor. *Information Sciences*, **118**, 1–13.

JANG, J.S.R. (1992) Self-tuning fuzzy controllers based on temporal back propagation. *IEEE Transactions on Neural Networks*, **3** (5), 714–723.

KHANMOHAMMADI, S., HASSANZADEH, I. and SHARIFIAN, M.B.B. (2000) Modified adaptive discrete control system containing neural estimator and neural controller. *Artificial Intelligence in Engineering*, **14** (1), 31–38.

MAYOSKY, M.A. and CANCELO, G.I.E. (1999) Direct adaptive control of wind *on Neural Networks*, **10** (4), 898–905.

PARK, Y.M., CHOI, M.S. and LEE, K.Y. (1996) An optimal tracking neuron-controller for nonlinear dynamics systems. *IEEE Transactions on Neural Networks*, **7** (5), 1099–1110.

SASTRY, S. and BODSON, M. (1989) *Adaptive Control: Stability, Convergence, and Robustness.* Prentice-Hall International, Inc.

SLOTINE, J.J.E. and LI, W.P. (1991) *Applied Nonlinear Control.* Prentice-Hall International.

WANG, L.X. (1994) *Adaptive Fuzzy Systems and Control: Design and Stability Analysis.* PTR Prentice Hall.

WANG, L.X. (1996) Stable adaptive fuzzy controllers with application to inverted pendulum tracking. *IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics*, **26** (5), 677–691.

WANG, L.X. (1997) *A Course in Fuzzy Systems and Control.* Prentice-Hall International.

WU, M.C., LEE, L.C. and SHIH, M.C. (1998) Neuro-fuzzy controller design of the antilock braking system. *JSME International Journal*, **41**(4), 836–843.

ZHIHONG, M., WU, H.R. and PALANISWAMI, M. (1998) An adaptive tracking controller using neural networks. *IEEE Transactions on Neural Networks*, **9**(5), 947–955.