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The Influence of Piers on the Risk of Flooding Upstream from a Bridge

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Abstract

The objective of this paper is to compare the risk of flooding areas upstream from a bridge without piers and a deck supported only on abutments, with a bridge of the same width with a deck supported on abutments and piers. The additional goal of this paper is to analyze the influence of the width and shape of piers on the risk of flooding. The paper begins with the formulation of performance function used in reliability analysis. Then, after the short characteristic of the methods of this analysis, one of them – the advanced first-order second-moments (AFOSM) method chosen for solution of the problem resulting from objectives of the paper – is presented. The largest part of the paper contains the solution of this problem for an exemplary bridge. After the description of this bridge two models used for computation of hydraulic losses, which are required for determination of the performance function, are characterized. The first is based on an energy equation (the standard step method) and used for a bridge without piers, the second one applied for computation of losses in the inner section of a bridge with piers is based on momentum equation. The solution of the problem was based on computer simulations performed for the compared variants of bridge projects. The computations results were presented graphically in the form of risk curves showing the relations between the return period defined as the reciprocal of risk and stage of water upstream from a bridge. It was found that the reduction of a return period due to: application of piers for support of bridge deck (as an alternative to its support on abutments alone), an extension of piers by 100% or replacing a favourable hydraulical shape (elliptical) of piers by an unfavourable one (with a square nose), is comparable and for considered bridge oscillates around 70% of the initial return period.

Key words: AFOSM method, bridge, piers, risk

1. Introduction

In most cases, bridges belong to the group of hydraulic structures, which decrease the width of natural river beds and their conveyances. This leads to increase in the flood risk of the areas located upstream of these structures. The term flood risk is understood, according to Yen (1970), as the probability of inundation of some area (upstream of a bridge in the case considered). It responds to the term flood hazard and is not a function of damages or losses caused by this flooding. By the assumed width of the bridge, the water level upstream is influenced by hydraulic losses caused by piers. These losses depend on geometric parameters of the piers (width and shape) and their numbers.

The objective of this paper is to compare the risk of flooding areas located upstream from a bridge without piers with a deck supported only on abutments, and a bridge of the same width with a deck supported on abutments and piers. The computation of the water level upstream of a bridge was by adopting two approaches (HEC-RAS Manual 1998):

- based on an energy equation for a bridge without piers,
- based on a momentum equation for the inner section of a bridge with piers.

The second approach is recommended for bridges in which piers are the dominant contributor to energy losses and the change in water source (HEC-RAS Manual 1998). The additional goal of this paper is to analyze the influence of the width and shape of piers on the risk of flooding The hydrologic risk and hydraulic uncertainties were considered by computation.

2. The Formulation of Performance Function

The risk of flooding upstream of a bridge can be defined (Yen 1970) as the probability of the event when a performance function Z assumes the negative value

$$RY = p(Z < 0). \tag{1}$$

Complement to the risk is reliability

$$RE = 1 - RY = p(Z > 0). \tag{2}$$

The performance function can be defined in terms of flows or stages. The second approach (Singh and Melching 1993, Sowiński and Marlewski 2003) although less popular seems to be more attractive from the practical point of view. Thus, Z is described as the difference between the target stage upstream from a bridge, H_T, and an upstream flood-stage, H_{UP}, computed by a hydraulic model

$$Z = H_T - H_{UP}. \tag{3}$$

The upstream stage, H_{UP}, is a function of the stage of water downstream from the bridge, H_{DW}, and hydraulic losses, ΔH due to flow through this bridge (Fig. 1)

$$Z = H_T - [\lambda_H \Delta H + \lambda_{RC} H_{DW}]. \tag{4}$$

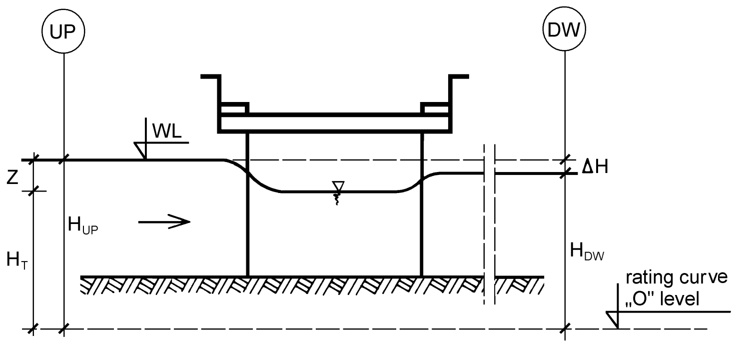

Fig. 1. Graphical interpretation of relation used for determining an upstream stage H_{UP}

Different forms of performance function depending on the model correction factor(s) can be adopted. In general, two different correction factors can be used: one for correction of the hydraulic model applied for the computation of hydraulic losses, λ_H, and the other for the correction of stages determined from a rating curve downstream from a bridge, λ_{RC}.

Assuming in the study, that both correction factors specified are equal i.e. $\lambda_{RC} = \lambda_H = \lambda$ the equation (4) can be written as

$$Z = H_T - [\lambda(\Delta H + H_{DW})]. \tag{5}$$

Stage H_{DW} is not influenced by a bridge structure. It can be evaluated from a rating curve developed for natural river bed downstream of a bridge. Hydraulic losses are determined by the usage of an adequate hydraulic model which uses two types of input variables: deterministic and stochastic. Values of deterministic variables can be assumed in analysis as constant or varied i.e. treated as parameters. In case of the second type of variables an uncertainty is taken into account. In the paper these variables are called the basic variables and are denoted by x_1, x_2,...x_N. Hydraulic losses, ΔH, hence the performance function, Z, can be expressed as their function

$$\Delta H = f(x_1, x_2, \ldots x_N = f(\mathbf{X}), \tag{6}$$

$$Z = g(\mathbf{X}), \tag{7}$$

where **X** is the vector of basic variables.

3. The Reliability Analysis – AFOSM Method

Several methods have been developed for reliability analysis, used in water engineering (Yen and Tung 1993, Lian and Yen 2003) . The direct integration methods assure high accuracy (Plate 1984, Tung and Mays 1980, Sowiński and Yusuff 1995), but are not often used for the solution of practical problems due to mathematical complexity. Among approximate methods first-order second-moments (FOSM) methods are probably the most frequently applied (Tang and Yen 1972, Ang and Tang 1984, Yen et al 1986). These methods can be divided into two groups: the mean-value first-order second-moments (MFOSM) methods and the advanced first-order second-moments (AFOSM) methods. Other large groups consist of the point-estimation methods including different variants, among which the most known are Harr (1989) approach and Li (1992) approach. Finally, one should mention the Monte Carlo method, which requires the large number of simulations and its improved version – the Latin Hypercube Sampling (LHS) method, where the significant reduction of simulations is allowed (Melching 1992, Sowiński 2004).

Assuming that the probability density function (PDF) of Z is normal, the risk can be measured in terms of the reliability index β, as

$$RY = \Phi(-\beta), \tag{8}$$

where Φ is the standard normal cumulative density function (CDF).

The reliability index β is defined as the ratio of expected value of the performance function $E[Z]$ and its standard deviation σ_Z

$$\beta = \frac{E[Z]}{\sigma_Z}. \tag{9}$$

In the advanced first-order second-moments (AFOSM) method the performance function Z is expanded in Taylor series on the likely failure point on a failure surface (Ang and Tang 1984, Melching and Yen 1986). Terms of higher order than the first order are neglected, hence the expected value of Z can be approximated by

$$E[Z] \ = \ g(X^*) + \sum_{i=1}^{m} \left(\overline{x}_i \ - \ x_i^*\right) c_i^*, \tag{10}$$

where $c_i^* \ = \ \partial g \ (X) / \partial x_i |_{x_i^*}$ – the partial derivative of a function $g(\mathbf{X})$ evaluated at a failure point $\mathbf{X}^*$.

The variance of the performance function can be computed

$$\begin{aligned}\operatorname{var}[Z] &= \operatorname{var}\left[\sum_{i=1}^{m}\left(x_i - x_i^*\right) c_i^*\right] = \\ &= \operatorname{var}\left[\sum_{i=1}^{m} c_i^* x_i\right] = \sum_{i=1}^{m}\sum_{j=1}^{m} c_i^* c_j^* \operatorname{cov}\left[x_i , x_j\right]\end{aligned} \tag{11}$$

in which cov $[x_i, x_j]$ – the covariance of variables x_i and x_j.

After transformation the term containing variance can be extracted

$$\operatorname{var}[Z] = \sum_{i=1}^{m}\left(c_i^*\right)^2 \operatorname{var}[x_i] + \sum_{i=1}^{m}\sum_{i\neq j}^{m} c_i^* c_j^* \operatorname{cov}\left[x_i , x_j\right]. \tag{12}$$

For uncorrelated variables their covariances are equal to zero hence the expression for the variance of performance function is simplified to

$$\operatorname{var}[Z] = \sum_{i=1}^{m}\left(c_i^*\right)^2 \operatorname{var}[x_i]. \tag{13}$$

Substituting equations (10) and (13) to equation (9) yields the formula for computation of reliability index

$$\beta = \frac{g(X^*) + \sum c_i^* \left(\overline{x}_i - x_i^*\right)}{\left\{\sum_{i-1}^{m} (c_i^*)^2 \operatorname{var}[x_i]\right\}^{1/2}}. \tag{14}$$

The performance function computed at the likely failure point on a failure surface $g(\mathbf{X}^*)$ is supposed to equal zero. But the location of failure point is not known a priori and iterations are required for its determination. In successive iterations the value of $g(\mathbf{X}^*)$ approaches zero. An iterative procedure for determining β proposed by Fiessler (Smith 1986), which uses reduced variables, was applied for solution of the problem presented in the paper.

4. Solution of Problem

Objectives formulated in the introduction can be achieved with solving a problem composed of tasks aimed on the investigation of the influence of bridge construction (with or without piers), the influence of contraction of cross-section under a bridge due to enlarged width of piers and influence of piers shape on the risk of flooding. A solution was obtained by means of comparative analysis of risk curves developed for different options of exemplary bridge project. Only subcritical (tranquil) flow was considered as the most frequently occurring in Polish conditions. This limitation implicates the direction of calculations for the standard step method applied within energy based balance approach. Computations start from the cross-section downstream of a bridge and are performed upstream of the river.

4.1. An Exemplary Bridge

For the study: a bridge constructed over a symmetrical river bed composed of three segments: the main channel and two overbank segments (Fig. 2) is chosen. Its plain view with computational cross-sections is also shown in Fig. 2.

4.2. Hydraulic Models

In order to compute the performance function it is necessary to determine hydraulic losses, which requires the application of hydraulic models. The model concept is based on the HEC-RAS model developed by the U.S. Army Corps of Engineers (HEC-RAS Manual 1998). As stated in the introduction, two approaches were applied for computation of hydraulic losses: the energy- and the momentum-based balance. Hence two relevant hydraulic models were developed: the first was applied for a bridge without piers and the second – for the inner section of a bridge with a deck supported on piers.

4.2.1. Energy Balance-Based Model

The approach applied in this model is based on an iterative solution carried out from section to section (known as the standard step method). Denoting the downstream cross-section by index 1 and the upstream by index 2, the energy equation can be written as follows

$$z_2 + h_2 + \frac{\alpha_2 v_2^2}{2g} = z_1 + h_1 + \frac{\alpha_1 v_1^2}{2g} + h_e, \tag{15}$$

where:

z_1, z_2	–	elevation of main channel invert,
h_1, h_2	–	depth of water at cross-section,
v_1, v_2	–	average velocity,
α_1, α_2	–	Saint Venant coefficients,
h_e	–	energy head loss,
g	–	acceleration due to gravity.

Energy head loss is computed as a function of friction losses and contraction or expansion losses

$$h_e = L\overline{S}_f + C \left| \frac{\alpha_2 v_2^2}{2g} - \frac{\alpha_1 v_1^2}{2g} \right|, \tag{16}$$

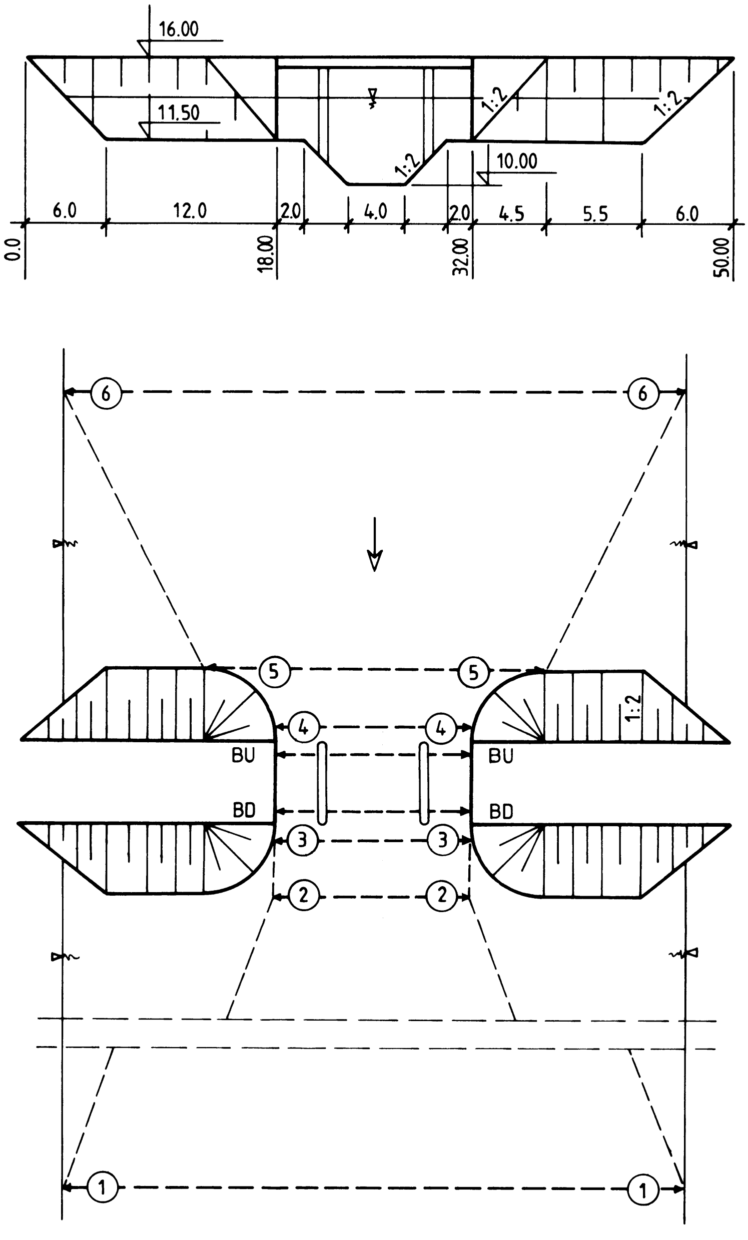

Fig. 2. Plan view and cross-section of river bed in the vicinity of the exemplary bridge

where:

- L – distance between cross-sections 1 and 2,
- $\overline{S}_f$ – average friction slope between two sections,
- C – expansion or contraction coefficients.

Friction slope is computed from the Chezy-Manning formula. According to HEC-RAS Manual (1998) six cross-sections are required to describe stream behaviour downstream (Nos. 1, 2 and 3) and upstream (Nos. 4, 5 and 6) from a bridge (Fig. 2). Cross-sections Nos. 1 and 6 are located sufficiently far from a bridge structure so that flow is not affected by it. Cross-sections Nos. 2 and 5 are placed at the downstream and upstream toe of embankment. In addition two cross-sections located at the downstream (No. 3) and upstream (No. 4) edges of a bridge are used to describe the geometry of its opening. In order to prevent effecting profile computations through a bridge by boundary conditions (e.g. determined from a rating curve) an additional cross-section No. 0 (not shown in Fig. 2) was introduced downstream, from a bridge.

4.2.2. Momentum Balance Based Model

This approach is based on the momentum balance for cross-sections between cross-sections Nos. 3 and 4. In the first step the momentum balance equation is written for cross-section Nos. 3 and BD, which are located, respectively, just downstream and upstream of the downstream edges of piers.

$$\beta_3 \frac{Q^2}{gA_3} + A_3 \overline{y}_3 - A_{pBD} \overline{y}_{pBD} = \beta_{BD} \frac{Q^2}{g A_{BD}} + A_{BD} \overline{y}_{BD}, \tag{17}$$

where:

- β_3, β_{BD} – momentum coefficient at sections 3 and BD, respectively,
- A_3, A_{BD} – active flow area at sections 3 and BD, respectively,
- $\overline{y}_3, \overline{y}_{BD}$ – vertical distance from water surface to centre of gravity of flow area A_3 A_{BD}, respectively
- A_{pBD} – obstructed area of piers in the cross-section BD,
- Y_{pBD} – vertical distance from water surface to centre of gravity of wetted pier area in the cross-section BD.

The second step is momentum balance from cross-section BU to cross-section BD just inside the bridge at its upstream edge

$$M_{BU}(y_{BU}) = M_{BD}(y_{BD}). \tag{18}$$

Assuming small longitudinal bed slope and short distance between cross-sections BU and BD (for a bridge deck of average width) the friction force and the weight force components can be neglected, hence only two terms (functions of depth y_{BU} and y_{BD} in respective cross-sections) appear on each side of momentum equation

$$\beta_{BU}\frac{Q^2}{gA_{BU}} + A_{BU}\,\overline{y}_{BU} = \beta_{BD}\,\frac{Q^2}{g\,A_{BD}} + A_{BD}\,\overline{y}_{BD}. \tag{19}$$

In the third step the momentum balance equation is written for cross-section BU and cross-section No. 4, which are located, respectively, downstream and upstream of the upstream edges of piers:

$$\beta_4\frac{Q^2}{gA_4} + A_4\,\overline{y}_4 = \beta_{BU}\frac{Q^2}{gA_{BU}} + A_{BU}\,\overline{y}_{BU} \;+\; \frac{1}{2}CD\frac{A_{pBU}Q^2}{gA_4^2}, \tag{20}$$

where CD – drag coefficient for pier.

4.3. Hydrologic Model

For the sake of simplicity, the hydrological model in this study is represented by the log-normal probability distribution of flood flow. Parameters of this distribution were computed assuming the mean maximal annual flow $\overline{Q} = 60$ m^3/s and standard deviation $\sigma_Q = 12$ m^3/s.

4.4. The Characteristic of Basic Variables

Hydraulic losses are the function of several variables from which only three were identified (Sowiński 2004) as significantly affecting the uncertainty of the stage upstream from a bridge and hence assumed as basic variables. These are: the discharge, Q, the correction factor, λ, and the coefficients of Manning roughness (MN_M for the main channel and MN_L and MN_R for left and right overbank areas, respectively). The first one describes a hydrologic uncertainty, the remainder – a hydraulic uncertainty. The hydraulic basic variables are assumed to be normally distributed due to the lack of information about their true distribution and for ease of illustration. Hence two parameters are required for the determination of a probability density function (PDF) of each variable: a mean and a standard deviation.

The correction factor λ_H takes into account potential errors resulting from the imperfect hydraulic model used for hydraulic loss computation and λ_{RC} – an inaccurate rating curve used for determining depth in the initial downstream section. Their mean values were assumed to be 1.0. The determination of standard deviation of λ_{RC} is based on an assumption that the maximum error of stage evaluated from a rating curve is ±10%. This implies the variation range of λ_{RC} between 0.9 and 1.1 which means that its range is $\Delta\lambda_{RC} = 0.2$. Assuming the normal distribution for λ_{RC} and applying the rule of 3σ, which is equivalent to the assumption that 99.7% of the area under PDF curve in the interval from

-3σ to $+3\sigma$ should belong to the above determined variation range, $\Delta\lambda_{RC}$, the standard deviation can be computed as $\sigma_{\lambda RC} = \Delta\lambda_{RC}/6.0 = 0.033$. For standard deviation of λ_H Singh and Melching (1993) proposed the value $\sigma_{\lambda H} = 0.10$. Two options were taken into account in the presented analysis, assuming the standard deviation of common correction coefficient $\sigma_\lambda = 0.05$ (implicated by $\sigma_{\lambda RC}$) and $\sigma_\lambda = 0.10$ (implicated by $\sigma_{\lambda H}$).

The mean values of the coefficient of Manning roughness were determined by the average maintenance conditions of the river bed (HEC-RAS Manual 1998), their standard deviations were computed assuming: coefficients of variation (cov) $\Omega_{MNM} = 0.10$ for the main section of the river bed and $\Omega_{MNL} = \Omega_{MNR} = 0.20$ for overbank areas. The higher value of the cov for overbank areas is justified by the higher variability of roughness of these areas (temporary flooded) in comparison with the main river bed section permanently covered by water.

The statistical characteristics of basic variables are presented in Table 1.

Table 1. The characteristics of basic variables

Variable	Units	Mean $\overline{x}$	Standard deviation σ_x	Coefficient of variation Ω_x
Q	[m^3/s]	60.0	12.0	
λ	[–]	1.0	0.05	0.05
			0.10	0.10
MN_M	[–]	0.030	0.003	0.10
MN_L	[–]	0.050	0.010	0.20
MM_R	[–]	0.050	0.010	0.20

In the absence of data from measurements, the equation of a rating curve in section No. 0 (not shown in Fig. 2), downstream of section No. 1, was developed based on the uniform flow formula. The longitudinal slope of river bed was assumed to be $S = 0.0004$. The equation of rating curve was approximated by polynomial of the third order

$$H_{DW} = b_0 + b_1 Q + b_2 Q^2 + b_3 Q^3, \tag{21}$$

where: b_0, b_1, b_2, b_3 – coefficients of regression.

4.5. Discussion of Results

The solution of the problem was based on computer simulations performed for the compared variants of bridge projects. For all variants the same width of bridge between abutments was assumed. The computation results were presented graphically in the form of risk curves showing relation between return period, defined as the reciprocal of risk and stage of water upstream from a bridge. The discussion of considered task results is presented below.

4.5.1. The Investigation of the Influence of Bridge Construction (with or without Piers) on Flood Risk

The task was solved by the comparison of risk curves obtained by two of the above specified constructions (Fig. 3).

As an alternative for a bridge without piers, with a deck supported only on abutments, a bridge supported additionally on two piers, $b = 0.8$ m width each, was considered. Because a significant influence of correction factor λ on the risk of flooding was confirmed by earlier research (Sowiński and Marlewski 2003, Sowiński 2004), for each variant of bridge project, computer simulations were performed for two values of standard deviation of λ, i.e. $\sigma_\lambda = 0.05$ and $\sigma_\lambda = 0.10$.

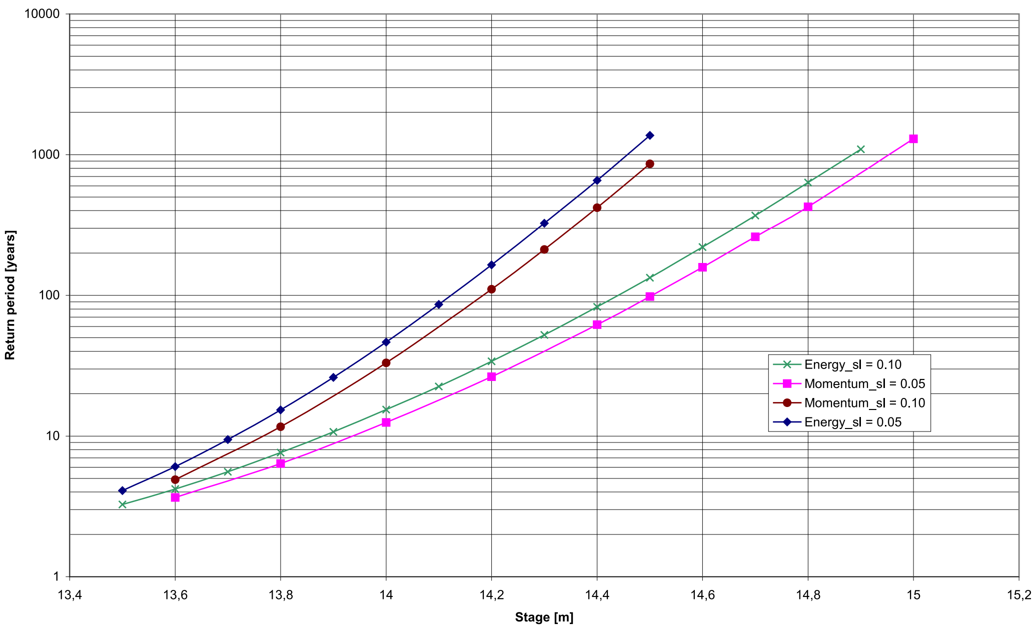

Fig. 3. Risk curves for a bridge with piers (momentum approach) and without piers (energy approach) for two values of σ_λ

Replacing the exemplary bridge, supported only on abutments, with the construction supported on two piers of $b = 0.8$ m width each, leads to the increase of risk or decrease of return period of flooding upstream from this bridge. For example, depending on standard deviation of correction coefficient, for the stage $H_T = 14.2$ m the return period decreases from $T = 164$ years to $T = 111$ years (i.e. by 68%) for $\sigma_\lambda = 0.05$, and from $T = 34$ years to $T = 26$ years (i.e. by 68%) for $\sigma_\lambda = 0.10$.

4.5.2. The Analysis of the Contraction Influence of Cross-Section under a Bridge on Flood Risk, due to Increased Width of Piers

The width of piers was considered to be a deterministic variable, i.e. as a parameter of a hydraulic model. The influence of piers width on risk of flooding was investigated by the comparison of risk curves obtained in successive simulations, increasing the width of piers initially from $b = 0.4$ m to 0.8 m and later from $b = 0.8$ m to 1.2 m. The analysis of the graph (Fig. 4) enables formulation of a conclusion that an increase of pier width leads to an increase in the risk of flooding or decrease of the return period.

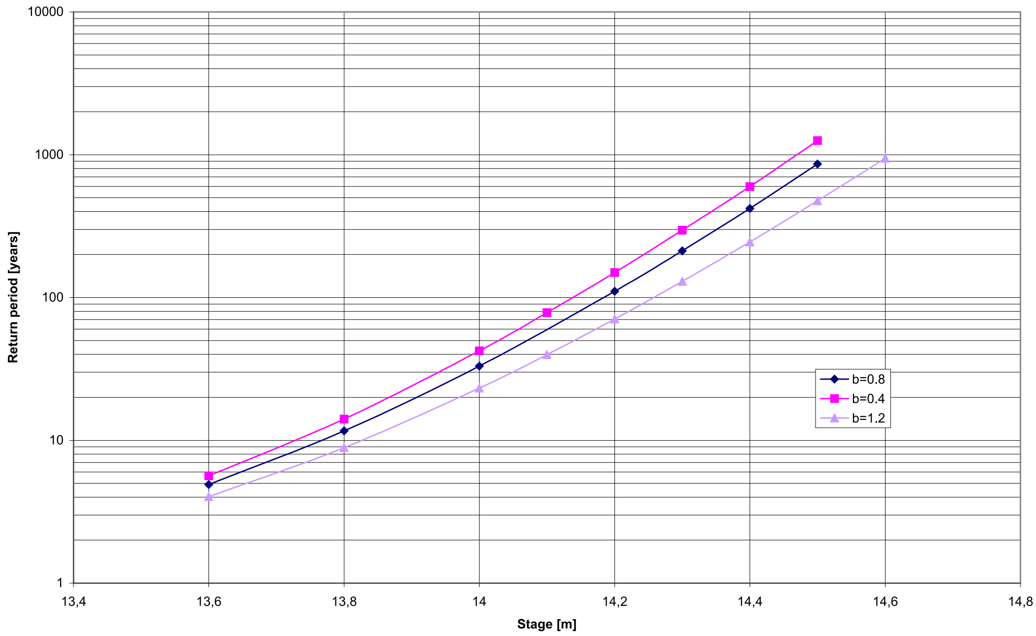

Fig. 4. The influence of piers width b on a risk curve

For example, for the assumed water stage $H_T = 14.2$ m, the increase in the width of piers from $b = 0.4$ m to $b = 0.8$ m results in the reduction of the return period from $T = 150$ years to $T = 111$ years i.e. by 74%. Further increasing the width of piers to $b = 1.2$ m leads to reduction of the return period to $T = 71$ years, i.e. by 64%.

4.5.3. The Analysis of Piers' Shape Influence on the Risk of Flooding

The influence of piers' shape on stream behavior is characterized by a drag coefficient, therefore this part of analysis was aimed at the investigation of the influence of statistical characteristics (mean and variance) of this coefficient on risk curves. Bases on data for the considered bridge, computer simulations were performed for two shapes of piers, characterized by the extreme values of drag coefficients: for elliptical piers with 2 : 1 length to width ratio and square nose piers, for which drag coefficients were assumed according to HEC RAS Manual (1998) recommendations as $CD = 0.60$ and $CD = 2.00$, respectively. In both cases, standard deviation was taken as $\sigma_{CD} = 0.25$. It was computed assuming a triangular probability density function with extreme values of its argument relevant to the specified above shape of piers, i.e. $CD_{\min} = 0.60$ and $CD_{\max} = 2.00$.

All three risk curves developed for different pier shapes shown in Fig. 5 are situated relatively closely.

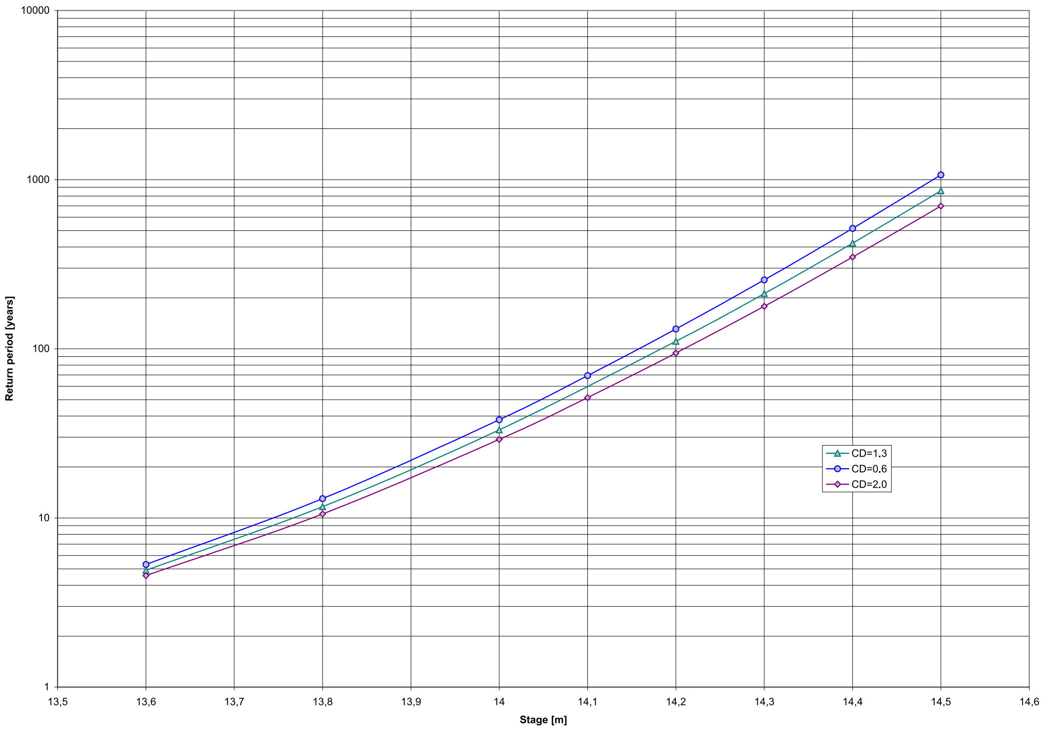

Fig. 5. The influence of piers drug coefficient CD on a risk curve

This means that an influence of piers' shape on the risk/return period of flooding is smaller than an influence of reduction of bridge opening by bridge piers. For examplary bridge, replacing elliptical piers by square nose piers, results in the decrease of the return period, for stage $H_T = 14.2$ m, from $T = 131$ years to $T = 95$ years, i.e. by 72%.

5. Conclusions

According to expectations, the increasing of hydraulic losses due to narrowing of effective cross-section under a bridge (as a result of piers application or extending their width) or modification of piers shape, leads to an increase in the risk or decrease in the return period of flooding upstream from a bridge. An analysis allowed for the quantitative evaluation of specified above factors influence on risk/return period of flooding for the exemplary bridge. It was found that the reduction of a return period due to: the application of piers for the support of the bridge deck (as an alternative to its support only on abutments), an extension of piers width by 100% or replacing favorable hydraulically shape (elliptical) of piers by an unfavorable one (with square nose), is comparable and for the considered bridge oscillates around 70% of the initial return period. It should be noticed that risk curves presented in Fig. 5 were obtained for the extreme values of drag coefficient and determined both boundaries of return period interval due to changes of piers' shape.

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