TESTING THE DYNAMICS OF THE ELECTRIC ENGINE BY MEANS OF BASIC SPLINES

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Summary

There is presented an algorithm of identification of the dynamic system described by means of differential equations. In order to describe the input and output signals there is used a regression function presented by means of the basic splines. The elaborated algorithm is used in testing the characteristics of the electric engine.

Keywords: regression function, splines, identification, dynamic systems

BADANIE DYNAMIKI SILNIKA ELEKTRYCZNEGO
ZA POMOCĄ BAZOWYCH FUNKCJI SKLEJANYCH

Streszczenie

Zaprezentowano algorytm identyfikacji układów dynamicznych opisanych równaniami różniczkowymi. Do opisu sygnałów wejścia i wyjścia wykorzystano funkcję regresji przedstawioną w postaci kombinacji bazowych funkcji sklejanych. Opracowany algorytm wykorzystano do badania charakterystyk silnika elektrycznego.

Słowa kluczowe: funkcja regresji, funkcje sklejane, identyfikacja, układy dynamiczne.

1. INTRODUCTION

The sudden technical progression creates the necessity of control system's identification. Majority of the objects can be treated as the dynamic systems. The acting, the effect of which is building the mathematical model acknowledged, due to the adopted criterion, for sufficiently describing the behaviour of the real object, will be called the *identification system.*

The following tasks belong to the identification of the systems:

a) the description of the input and output quantities;
b) the mathematical model of the relationships between input and output signals;
c) verification of the model.

In spite of many papers devoted dynamic identification of a system, new methods are still required, which are attractive for the electronic calculation technique.
Its development enables the complex system's identification and optimal controlling.

For signals description there have been used the basic splines. *B-splines* are polynomial smooth functions which are defined on the whole real axis, but they are different from zero in a certain interval.

2. REGRESSION

Assume that the real signal y is described by a function $y(t)$. On the basis of the data, $\widetilde{y}_k,\ k \in \overline{0,n_1} := \{0,1,\dots,n_1\}$ of that signal, measured in the moments $t_k^{'}$, $k \in \overline{0,n_1}$ of time i.e. $\widetilde{y}_k = \widetilde{y}(t_k^{'}),\ k \in \overline{0,n_1}$, we are going to find a function $y^* = y^*(t)$ which approximates a function $y = y(t)$. Let

$$\mathbf{B} = \left\{B_n^{-n}, \cdots, B_n^{N-1}\right\} \tag{1}$$

be a set of the so-called *basic splines of* n *-th order*, which can be determined recurrently by means of De Boor formula [4]:

$$\widetilde{B}_n^i(t) = \frac{n+1}{n}\left[\frac{t-t_i}{t_{i+n+1}-t_i}\widetilde{B}_{n-1}^i(t) + \frac{t_{i+n+1}-t}{t_{i+n+1}-t_i}\widetilde{B}_{n-1}^{i+1}(t)\right], \tag{2}$$

where $i \in \overline{-n, N-1}$, and for $n = 0$ the base $\widetilde{B}_0^i(t)$ is described as follows:

$$\widetilde{B}_0^i(t) = \frac{1}{h}\begin{cases}1 & \text{for } t \in [t_i, t_{i+1}),\\ 0 & \text{for } t \notin [t_i, t_{i+1}).\end{cases}$$

Next we intend to determine the regression function

$$\hat{y} = \sum_{i=-n}^{N-1} c_i \widetilde{B}_n^i(t) , \qquad (3)$$

such that for each fixed $t_k^{'} \in \overline{0,n_1}$ the random variable y_k has normal distribution with the mean $\hat{y}_k = \sum_{i=-n}^{N-1} c_i \widetilde{B}_n^i(t_k^{'})$ and the standard deviation σ, where $c_i \in$ R, $i \in \overline{-n, N-1}$, σ are unknown parameters which do not include t. Moreover we assume that the random variables y_k are independent. The coefficients $c_i, i \in \overline{-n, N-1}$ will be determined using the maximum likelihood method minimizing the density function

$$L = \frac{1}{(2\pi\sigma^2)^{n_1/2}} \exp(-\frac{1}{2\sigma^2} \sum_{k=0}^{n_1} [y_k - \sum_{i=-n}^{N-1} c_i \widetilde{B}_n^i(t_k^{'})]^2) \quad (4)$$

of the random variables $y_k, k \in \overline{0,n_1}$.

3. IDENTIFICATION OF THE COEFFICIENTS OF THE DIFFERENTIAL EQUATION .

The regression function will be used to identification the control system of the electric engine of direct current described by the equation

$$a_2\ddot{\omega}(t) + a_1\dot{\omega}(t) + a_0\omega(t) = \\ = b_1\dot{M}_0(t) + b_0 M_0(t) + U_t(t) \qquad (5)$$

which presents the dependence on the angular velocity $\omega(t)$ and the voltage of the armature $U_t(t)$ and also the external load torque $M_0(t)$ under assumption that the excitation voltage $\widetilde{U}_w$ and the excitation intensity $\widetilde{i}_w$ are constant. Let

$$U_t^*(t) = \sum_{j=-n}^{N-1} \alpha_j \widetilde{B}_n^j(t), \quad M_0^*(t) = \sum_{j=-n}^{N-1} \beta_j \widetilde{B}_n^j(t),$$
$$\omega^*(t) = \sum_{j=-n}^{N-1} \gamma_j \widetilde{B}_n^j(t)$$

denote the signals approximated on the basis of measure data of the armature voltage $\widetilde{U}_{tk} = U_t(t_k^{'})$, the external load torque $\widetilde{M}_{tk} = M_0(t_k^{'})$ and the angular velocity of the rotor $\omega_{tk} = \omega(t_k^{'})$ logged in the moments $t_k^{'}, k \in \overline{0,n_1}$.

The equation (5) can be rewritten in the form

$$A_4\ddot{\omega}(t) + A_3\dot{\omega}(t) + A_2\omega(t) + \\ + A_1\dot{M}_0(t) + A_0 M_0(t) = U_t(t) \qquad (6)$$

where $A_4 = a_2, A_3 = a_1, A_2 = a_0, A_1 = -b_1, A_0 = -b_0$.

We will define the identification of the system as a problem of choice the coefficients of (6) such as the functional

$$J(A_0,\dots,A_4) = (\int_{t_0}^{t_{n_1}} [A_4\ddot{\omega}^*(t) + A_3\dot{\omega}^*(t) + \\ + A_2\omega^*(t) + A_1\dot{M}_0^*(t) + A_0 M_0^*(t) - U_t(t)]^2 dt)^{0.5} \quad (7)$$

attains its minimum.

The problem of choice the optimal coefficients A_i^0 $,i \in \overline{0,4}$ which minimize the indices of the identification is carried out to solving the systems of linear equations:

$$\sum_{i=0}^{4} A_i^0 v_{ji} = d_j, \quad j \in \overline{0,4} , \qquad (8)$$

where:

$$v_{44} = \int_{t_0}^{t_{n_1}} (\ddot{\omega}^*(t))^2 \, dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \gamma_\mu \gamma_\mu e_{\nu\mu}^{22},$$

$$v_{43} = \int_{t_0}^{t_{n_1}} \dot{\omega}^*(t)\ddot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \gamma_\mu \gamma_\mu e_{\nu\mu}^{21},$$

$$v_{42} = \int_{t_0}^{t_{n_1}} \omega^*(t)\ddot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \gamma_\mu \gamma_\mu e_{\nu\mu}^{20},$$

$$v_{41} = \int_{t_0}^{t_{n_1}} \dot{M}^*(t)\ddot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \beta_\mu \gamma_\mu e_{\nu\mu}^{21}$$

$$v_{40} = \int_{t_0}^{t_{n_1}} M^*(t)\ddot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \beta_\mu \gamma_\mu e_{\nu\mu}^{20}$$

$$v_{33} = \int_{t_0}^{t_{n_1}} \dot{\omega}^*(t)\dot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \gamma_\mu \gamma_\mu e_{\nu\mu}^{11},$$

$$v_{32} = \int_{t_0}^{t_{n_1}} \omega^*(t)\dot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \gamma_\mu \gamma_\mu e_{\nu\mu}^{10},$$

$$v_{31} = \int_{t_0}^{t_{n_1}} \dot{M}^*(t)\dot{\omega}^*(t) dt = \sum_{\nu=-n}^{N-1} \sum_{\mu=-n}^{N-1} \beta_\mu \gamma_\mu e_{\nu\mu}^{11},$$

$$v_{30} = \int_{t_0}^{t_{n1}} M^*(t)\dot{\omega}^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\gamma_\mu e^{10}_{\nu\mu},$$

$$v_{22} = \int_{t_0}^{t_{n1}} \omega^*(t)\omega^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\gamma_\mu\gamma_\mu e^{00}_{\nu\mu},$$

$$v_{21} = \int_{t_0}^{t_{n1}} M^*(t)\omega^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\gamma_\mu e^{00}_{\nu\mu},$$

$$v_{20} = \int_{t_0}^{t_{n1}} M^*(t)\omega^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\gamma_\mu e^{00}_{\nu\mu},$$

$$v_{11} = \int_{t_0}^{t_{n1}} \dot{M}_0^*(t)\dot{M}_0^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\beta_\mu e^{11}_{\nu\mu},$$

$$v_{10} = \int_{t_0}^{t_{n1}} \dot{M}^*(t)M^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\beta_\mu e^{10}_{\nu\mu},$$

$$v_{00} = \int_{t_0}^{t_{n1}} M^*(t)M^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\beta_\mu\beta_\mu e^{00}_{\nu\mu},$$

$$v_{ij} = v_{ji}, \quad \text{for} \quad i,j \in \overline{0,4},$$

$$c_4 = \int_{t_0}^{t_{n1}} U^*(t)\ddot{\omega}^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\alpha_\mu\gamma_\mu e^{20}_{\nu\mu},$$

$$c_3 = \int_{t_0}^{t_{n1}} U^*(t)\dot{\omega}^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\alpha_\mu\gamma_\mu e^{10}_{\nu\mu},$$

$$c_2 = \int_{t_0}^{t_{n1}} U^*(t)\omega^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\alpha_\mu\gamma_\mu e^{00}_{\nu\mu},$$

$$c_1 = \int_{t_0}^{t_{n1}} U^*(t)\dot{M}_0^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\alpha_\mu\beta_\mu e^{10}_{\nu\mu},$$

$$c_0 = \int_{t_0}^{t_{n1}} U^*(t)M_0^*(t)\,dt = \sum_{\nu=-n}^{N-1}\sum_{\mu=-n}^{N-1}\alpha_\mu\beta_\mu e^{00}_{\nu\mu},$$

$$e^{ij}_{\nu\mu} = \int_{t_0}^{t_{n1}} \frac{d^i}{dt^i}B^\nu_n(t)\frac{d^j}{dt^j}B^\mu_n(t)\,dt, \quad i,j \in \overline{0,2}.$$

The coefficients $e^{ij}_{\nu\mu}$ can be determined in analytic way [3] and then the above written formulas do not include the determined integrals. It has a great meaning in numeric calculations. This way the time of computation is much more shorter and computation error is smaller.

For the described algorithm there was elaborated a computer program in TURBO PASCAL. The program concerns the basic splines of the 3rd order.

4. NUMERICAL EXAMPLE

For the numerical example there are used the measure data) of the excitation voltage $\widetilde{U}_w$, the excitation intensity $\widetilde{i}_w$, the armature voltage $\widetilde{U}_t$, the armature intensity $\widetilde{i}_t$, the load torque $\widetilde{M}_0$ and the angular velocity $\widetilde{\omega}$ of the electric engine of direct currency logging the values every 0.002 s.

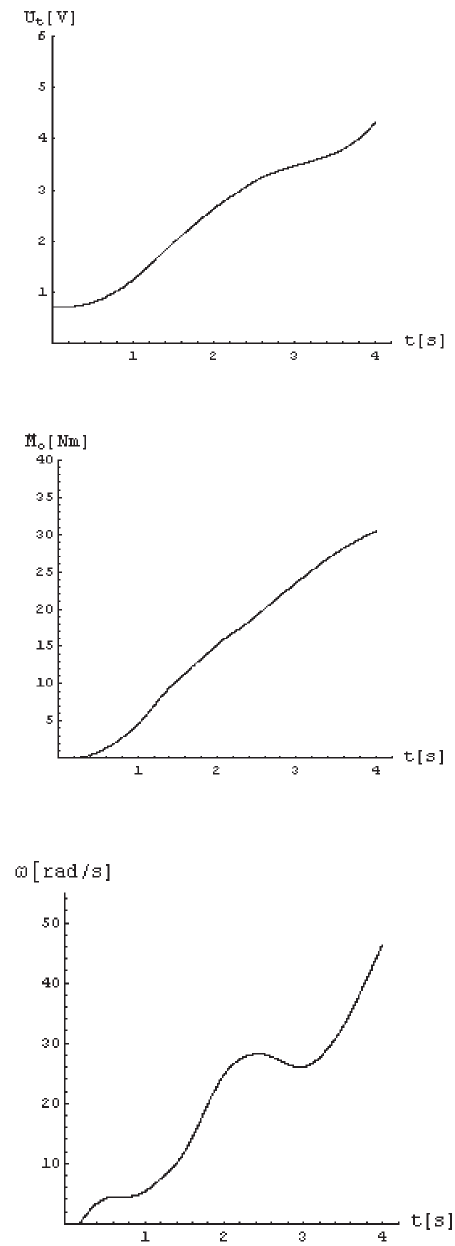

Fig.1. Measurements of the armature voltage, the load torque and the angular velocity.

Identification of the parameters of the differential equation (9) was made in the time interval [2, 4] with constant $\widetilde{U}_w$ and $\widetilde{i}_w$. To describe the signal there was used the base of 30 basic splines of the 3rd order. For the approximated signals, the determination coefficient R_y was greater than 0.9999, what testify for the accuracy of description of the measure data.

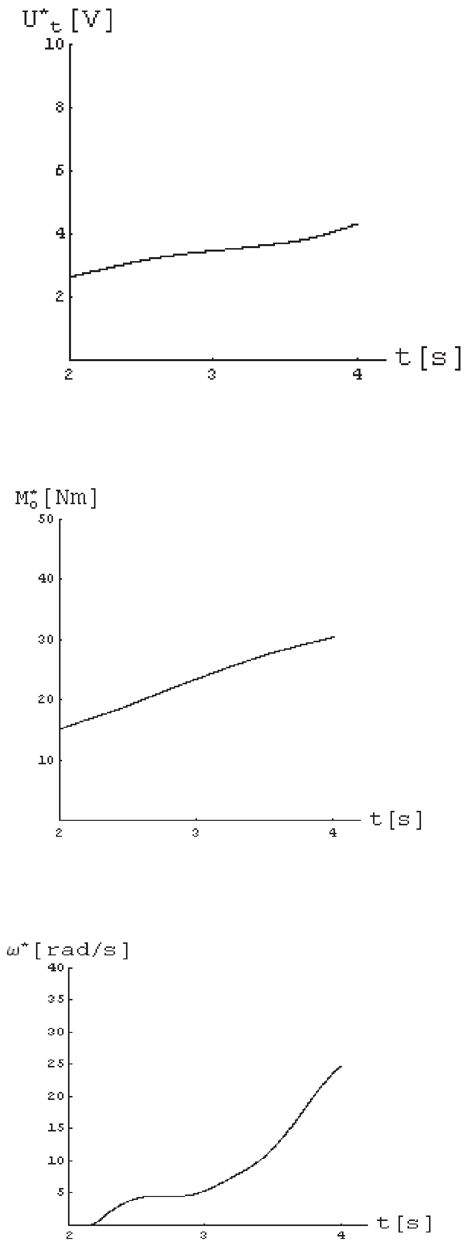

Fig.2. Approximated runs of the armature voltage $U^*(t)$, the load torque $M_o^*(t)$, and the angular velocity $\omega^*(t)$ on the basis of the measurements from Fig.1.

Using the computer program we obtained the identified form of the differential equation

$$\begin{aligned} a_2^0\ddot{\omega}(t) + a_1^0\dot{\omega}(t) + a_0^0\omega(t) &= \\ = b_1^0\dot{M}_0(t) + b_0^0 M_0(t) + U_t(t) \end{aligned} \quad (9)$$

where:

$$a_0^0 = 5.6705640186E-02,$$
$$a_1^0 = 2.0071258248E-02,$$
$$a_2^0 = 1.7905154994E-02,$$
$$b_0^0 = -1.9914915828E-02,$$
$$b_1^0 = -2.9281487907E-01$$

For the obtained solution, the coefficient of residual variation is $v_\omega =$ 4.07 %.

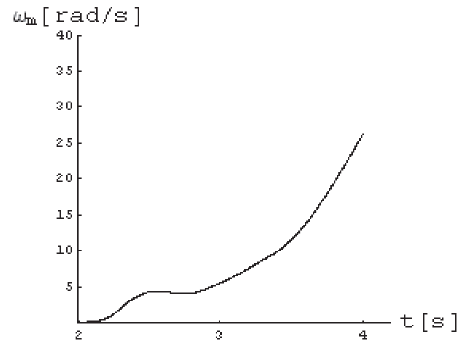

Fig.3. The solution $\omega_m(t)$ of the equation (9).

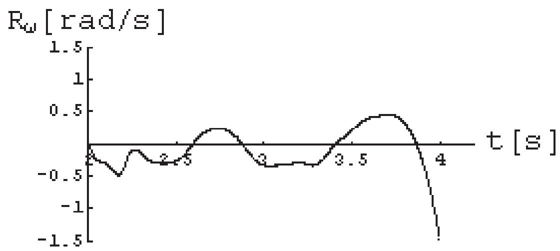

Fig.4. The graph of the residuals $R_\omega(t_k^{`}) := \tilde{\omega}(t_k^{`}) - \omega_m(t`_k)$

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